Quality Education for Change, Peace and Progress

FORM 3

MATHEMATICS

TEXTBOOK

ACKNOWLEDGEMENT

This F3 Textbook has been prepared by the CURRICULUM DEVELOPMENT UNIT with assistance from the following members of JICA Mathematics Volunteers .Without their sacrifice, commitment, support and valuable contributions, this book would not become a reality.

1. Ms. Akiko Hasegawa – Laucala Bay Secondary School

2 Mr. Kota Nasu – Dilkusha Methodist High School

3 Ms. Michiko Nakane – John Wesley College

4 Mr. Shin Tokuno – Sila Central High School

5 Mr. Takashi Muroi – DAV Girls' College

6 Ms. Tomoyo Ikeda – Suva Sangam Secondary School

7 Mr. Emosi Lutunaika – Mathematics Advisor, Curriculum Development Unit

This book would also not have been possible without the support of the 2011 Mathematics Form 4 CAT Moderators in all the moderation centres. They must also be acknowledged for their suggestions and recommendations and dedication in editing.

TO THE TEACHER

This book contains the entire content of the Form three Math prescriptions. The notes and examples given are written in the most simplified way to assist all students in the learning and understanding of the various concepts covered in Form 3 Mathematics.

It is understood that this book will be used by the Form 3 students in the classroom for the coverage of the year's work.

This textbook has been thoroughly vetted and proofread to avoid errors.

TO THE STUDENT

This book is prepared to guide and assist you in the learning and understanding of Mathematics at Form 3 level. Unit 1 begins with a revision of the work you studied at Form 2. Units 2 -8 cover the full content of the prescription and are prepared in such a way to make you clearly understand the learning concepts studied at Form 3.

Please ensure that you understand the worked examples fairly well before attempting the exercises.

Exercises and their answers are provided to assist you in understanding the mathematical concepts better.

Good luck

TABLE OF CONTENTS

UNIT 1

REVISION

UNIT 1 **REVISION**

NUMBERS AND NUMBER OPERATIONS

The Commutative Property – This is true for addition and multiplication, not true for subtraction and division. Changing the order of the **operands** does not change the end result.

For any numbers p and q	p + q = q + p
For any numbers p and q	p x q = q x p

e.g. $5 + 6 = 6 + 5$ $\quad$ $2 \times 4 = 4 \times 2$

$11 = 11$ $\quad$ $8 = 8$

The Associative Property – This is true for addition and multiplication, not true for subtraction and division.

$$(a + b) + c = a + (b + c)$$

$$(a \times b) \times c = a \times (b \times c)$$

e.g.

[Addition]		[Multiplication]	
(8 + 3) + 2	8 + (3 + 2)	(2 × 6) × 7	2 × (6 × 7)
= 11 + 2	= 8 + 5	= 12 × 7	= 2 × 42
= 13	= 13	= 84	= 84

The Distributive Property

$$p \times (q + r) = (p \times q) + (p \times r)$$

$$p \times (q - r) = (p \times q) - (p \times r)$$

e.g.

[Addition]		**[Subtraction]**	
$2 \times (4 + 7)$	$(2 \times 4) + (2 \times 7)$	$5 \times (2 - 1)$	$(5 \times 2) - (5 \times 1)$
$= 2 \times 11$	$= 8 + 14$	$= 5 \times 1$	$= 10 - 5$
$= 22$	$= 22$	$= 5$	$= 5$

Exercise 1

1. Rewrite each of the following expressions using the commutative property.
   (a) $2 + 5$ (b) 3×1
   (c) $6 + 9$ (d) 4×6

2. Rewrite the expressions below showing the associative property.
   (a) $5 \times (2 \times 8)$ (b) $10 + (4 + 7)$
   (c) $(25 + 2) + 4$ (d) $(6 \times 8) \times 2$

3. Use the distributive law to expand each of the following expressions.

(a) $2 \times (4 + 6)$ (b) $3 \times (8 + 7)$

(c) $12 \times (5 - 2)$ (d) $8 \times (9 - 6)$

4. Use the appropriate property (commutative, associative, distributive) to find the answer to the following questions.

(a) $2 + 9 + 8 + 1$ (b) $28 + 19 - 18 - 19$ (c) $8 \times 9 \times 5$

(d) $98 \times 23 \times \frac{2}{49} \times \frac{5}{23}$ (e) $(58 + 26) + 24$ (f) $(68 \times 5) \times 2$

(g) $12 \times \left(\frac{1}{3} - \frac{1}{4}\right)$ (h) $100 \times (0.2 - 0.03)$ (i) $36 \times \left(\frac{5}{12} + \frac{7}{18}\right)$

(j) $\frac{1}{2} \times \left(\frac{1}{2} + \frac{1}{3}\right) \times 12$ (k) $\frac{1}{2} \times 47 \times 20$ (l) $6 \times 13 + 6 \times 87$

(m) $\frac{3}{4} \times 13 + \frac{3}{4} \times 11$

COMMON MULTIPLES AND COMMON FACTORS

COMMON MULTIPLES

The **Multiple** of a number is the result of multiplying that number with a whole number.

When we are asked to find the **common multiples**, it means to find the intersection of two multiples. The numbers that are present in both multiples are known as the **common multiples.**

The **Lowest Common Multiple (LCM)** of two numbers is the lowest number in the set of the common multiples.

Example 1

M {4} = $\{4\times1, 4\times2, 4\times3, 4\times4, 4\times5, 4\times6, 4\times7, 4\times8, 4\times9, 4\times10, \cdots\}$

= {4, 8, 12, 16, 20, 24, 28, 32, 36, 40 . . .}

M{4} means **the set of the multiples of 4**.

Similarly,

M {6} = {6, 12, 18, 24, 30, 36, 42, 48, 54, 60 . . .}

Common multiples of 4 and 6

= M {4} ∩ M {6}

= {12, 24, 36 . . .}

The three dots show the sequence does not end and there is no limitation (infinity).

The LCM of 4 and 6 = 12

COMMON FACTORS

The **factor** of a number is a number that can be divided into this number without any remainder.

When we are asked to find a **common factor**, it means to find a number that divides two or more numbers evenly, i. e, without any remainder. In other words it is a factor that is common to two or more numbers.

The **Highest Common Factor (HCF)** of two numbers is the highest number of the common factors.

Example 2

F {40} = {1, 2, 4, 5, 8, 10, 20, 40}

F {40} means **the set of the factors of 40**.

F {48} = {1, 2, 3, 4, 6, 8, 12, 16, 24, 48}

Common factors of 40 and 48

= F {40} ∩ F {48}

= {1, 2, 4, 8 }

The HCF of 40 and 48 = 8

Exercise 2

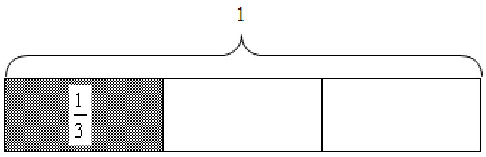
1. Find the first four terms of the common multiples and the LCM.
   (a) M {5} ∩ M {8} (b) M {12} ∩ M {10}
   (c) M {9} ∩ M {4} (d) M {6} ∩ M {5}

2. Find the common factors and the HCF of the following.
   (a) F {12} ∩ F {15} (b) F {24} ∩ F {30}
   (c) F {60} ∩ F {48} (d) F {72} ∩ F {54}

FRACTIONS AND OPERATIONS ON FRACTIONS

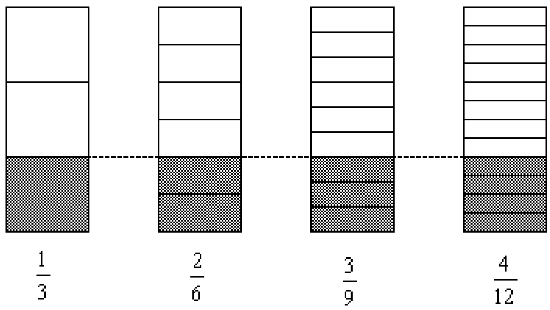
A fraction is part or portion of a whole item and it is normally written in the form

$$\text{Fraction} = \frac{\text{Numerator}}{\text{Denominator}}$$

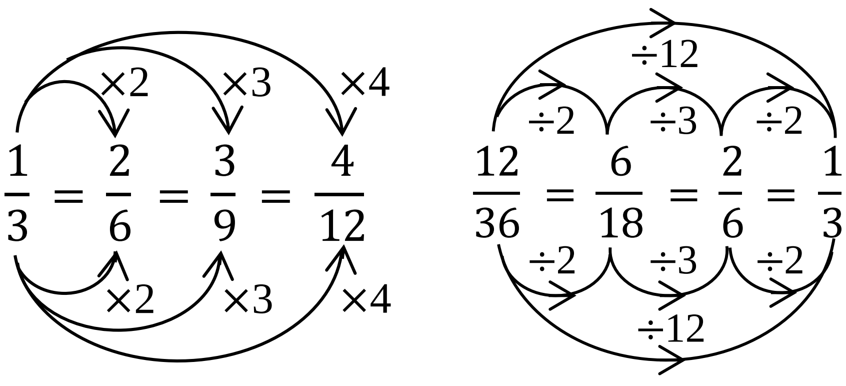


Equivalent fractions

Equivalent fractions are fractions that have different numbers but same values. Look at the diagram below.



The numerator and denominator must be multiplied or divided by the same number to get an equivalent fraction.



Here are some more equivalent fractions.

If we keep dividing until we can't go any further, then we have **simplified** the fraction (made it as simple as possible).

Exercise 3

Fill in the following blanks.

(a) $\dfrac{2}{3} = \dfrac{\square}{6} = \dfrac{6}{\square} = \dfrac{\square}{18} = \dfrac{20}{\square}$

(b) $\dfrac{48}{60} = \dfrac{24}{\square} = \dfrac{\square}{20} = \dfrac{\square}{15} = \dfrac{4}{\square}$

<u>Conversion of the fraction</u>

A proper fraction is the fraction that the numerator is smaller than the denominator.

$$\text{i. e.) } \frac{1}{2}, \frac{5}{9}, \frac{4}{13}$$

An improper fraction is the fraction that the numerator is bigger than the denominator.

$$\text{i. e.) } \frac{3}{2}, \frac{22}{9}, \frac{55}{13}$$

A mixed number is the combination of a whole number and a fraction.

$$\text{i. e.) } 1\frac{1}{2}, 2\frac{4}{9}, 4\frac{3}{13}$$

Example 3

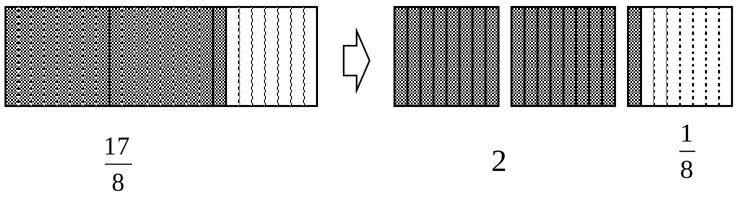
Convert the following fractions.（from an improper fraction to a mixed number or from a mixed number to an improper fraction）

(a) $\frac{17}{8}$　　　　(b) $1\frac{2}{7}$

[Answer]

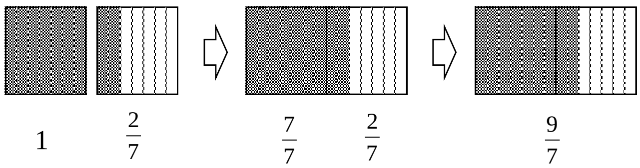
(a) $\frac{17}{8} = \mathbf{2\frac{1}{8}}$

$$\begin{array}{r} 2 \\ 8\,\overline{)\,17} \\ \underline{16} \\ 1 \end{array}$$



(b) $1\frac{2}{7} = \mathbf{\frac{9}{7}}$

$$1\frac{2}{7} = 1\frac{2}{7} = \frac{1 \times 7 + 2}{7} = \frac{9}{7}$$



Exercise 4

Convert the following fractions.（from an improper fraction to a mixed number or from a mixed number to an improper fraction）

(a) $\frac{5}{3}$　　(b) $\frac{28}{5}$　　(c) $1\frac{1}{4}$　　(d) $3\frac{5}{6}$

Addition and Subtraction of Fractions

When we add or subtract fractions, we must make the two denominators the same.

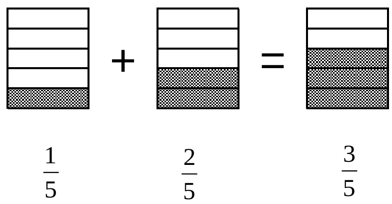
Example 4

Work out the following fractions.

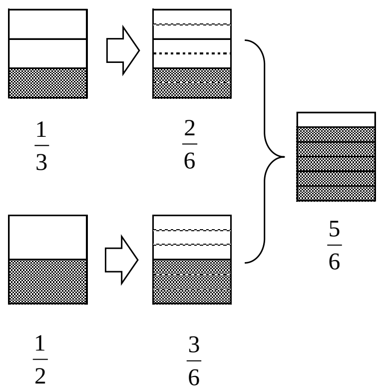
(a) $\frac{1}{5} + \frac{2}{5}$ (b) $\frac{1}{3} + \frac{1}{2}$ (c) $\frac{2}{3} + \frac{5}{6}$

[Answer]

(a) $\frac{1}{5} + \frac{2}{5} = \frac{1+2}{5} = \mathbf{\frac{3}{5}}$



(b) $\frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6} = \frac{2+3}{6} = \mathbf{\frac{5}{6}}$



(c) $\frac{2}{3} + \frac{5}{6} = \frac{4}{6} + \frac{5}{6} = \frac{4+5}{6} = \frac{9}{6} = \mathbf{\frac{3}{2}}$ or $\mathbf{1\frac{1}{2}}$

The answer must be the simplest fraction.

Exercise 5

Work out the following fractions.

(a) $\frac{2}{3}+\frac{1}{4}$ (b) $\frac{1}{2}+\frac{1}{10}$ (c) $\frac{2}{5}+\frac{1}{6}$ (d) $\frac{3}{8}+\frac{7}{10}$

(e) $\frac{3}{4}-\frac{5}{8}$ (f) $\frac{6}{7}-\frac{1}{4}$ (g) $\frac{2}{5}-\frac{1}{15}$ (h) $\frac{8}{7}-\frac{7}{8}$

Multiplication and Division of Proper Fractions

When we multiply fractions, we multiply the numerators together and the denominators together.

Example 5

Work out the following fractions.

(a) $\frac{3}{5} \times \frac{2}{7}$ (b) $\frac{3}{4} \div \frac{2}{5}$

[Answer]

(a) $\frac{3}{5} \times \frac{2}{7} = \frac{3 \times 2}{5 \times 7} = \mathbf{\frac{6}{35}}$

(b) $\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times \frac{5}{2}$

$= \frac{3 \times 5}{4 \times 2}$

$= \mathbf{\frac{15}{8}}$ **or** $\mathbf{1\frac{7}{8}}$

> In division of fractions, follow the steps given below
>
> 1. Change the division sign (÷) to multiplication sign (×).
> 2. Invert the second fraction.
> 3. Multiply the numerators together and the denominators together.

Exercise 6

Work out the following expressions.

(a) $\frac{1}{2} \times \frac{3}{5}$ (b) $\frac{5}{9} \times \frac{2}{7}$ (c) $\frac{7}{15} \times \frac{5}{8}$ (d) $\frac{5}{12} \times \frac{3}{10}$

(e) $\frac{1}{4} \div \frac{1}{3}$ (f) $\frac{6}{7} \div \frac{3}{4}$ (g) $\frac{15}{16} \div \frac{5}{4}$ (h) $\frac{8}{9} \div \frac{2}{9}$

Example 6

Work out the following expressions.

(a) $1\frac{1}{2} + 3\frac{1}{4}$ (b) $\frac{10}{3} \div 5\frac{1}{2}$

[Answer]

(a) $\frac{3}{2} + \frac{13}{4} = \frac{2\times 3+1\times 13}{4} = \mathbf{\frac{19}{4}}$ **or** $\mathbf{4\frac{3}{4}}$

(b) The mixed numbers are to be converted to improper fractions before being multiplied or divided.

$$\frac{10}{3} \div \frac{11}{2} = \frac{10}{3} \times \frac{2}{11} = \mathbf{\frac{20}{33}}$$

Exercise 7

Work out the following expressions.

(a) $1\frac{7}{8} + 2\frac{1}{6}$ (b) $2\frac{7}{15} - 1\frac{1}{3}$

(c) $3\frac{3}{7} \times 2\frac{1}{6}$ (d) $1\frac{3}{4} \div 2\frac{4}{5}$

DECIMALS AND OPERATIONS ON DECIMALS

To convert a decimal to a fraction

$0.1 = \frac{1}{10}$, $0.01 = \frac{1}{100}$, $0.001 = \frac{1}{1000}$, …

Example 7

Convert (a) 0.6 to a fraction

(b) 0.025 to a fraction

[Answer]

(a) $0.6 = \frac{6}{10} = \mathbf{\frac{3}{5}}$ (written in its simplest fraction)

(b) $0.025 = \frac{25}{1000} = \mathbf{\frac{1}{40}}$

To convert a fraction to decimal

Example 8

Convert $\frac{3}{8}$ to decimal

[Answer]

$\frac{3}{8} = 3 \div 8 = \mathbf{0.375}$

$$
\begin{array}{r|llll}
 & 0. & 3 & 7 & 5 \\
\hline
8 & 3. & 0 & & \\
 & 2 & 4 & & \\
\hline
 & & 6 & 0 & \\
 & & 5 & 6 & \\
\hline
 & & & 4 & 0 \\
 & & & 4 & 0 \\
\hline
 & & & & 0
\end{array}
$$

Addition and Subtraction of Decimals

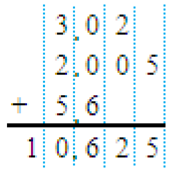
Example 9

Work out the following expressions.

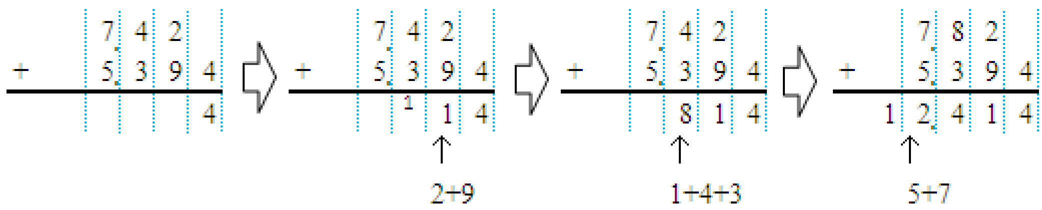
(a) 3.02 + 2.005 + 5.6 (b) 7.42+5.394 (c) 6.29 – 3.654

[Answer]

(a) These 3 decimal numbers are to be arranged first before they are added



(b)

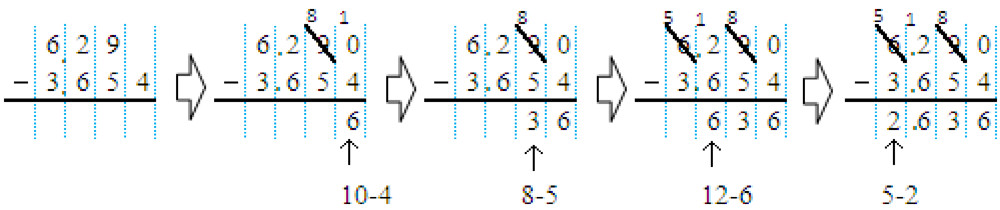


The decimal points are lined up vertically.

Calculate in the same way as addition of whole numbers.

Put the decimal points on the same place.

(c)



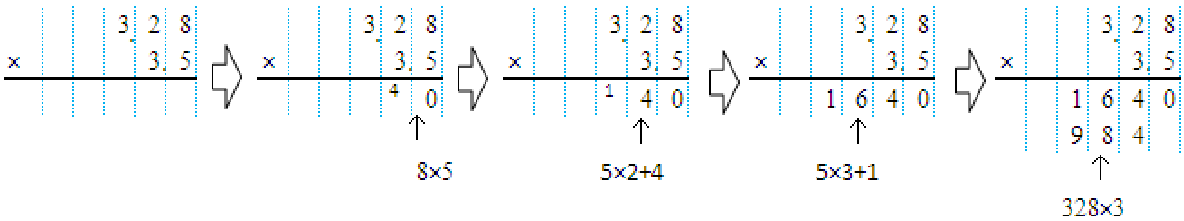
Multiplication and Division of Decimals

Example 10

Work out 3.28×3.5

[Answer]

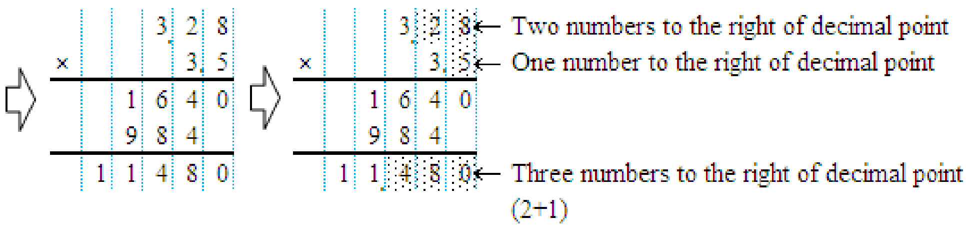
Line up the numbers on the right, multiply each digit in the top number by each digit in the bottom number (like whole numbers), add the products, and mark off decimal places in the numbers being multiplied



Write 8 and 5 vertically.

Calculate in the same way as multiplication of whole numbers.

Put the decimal point on the same place.



Add products

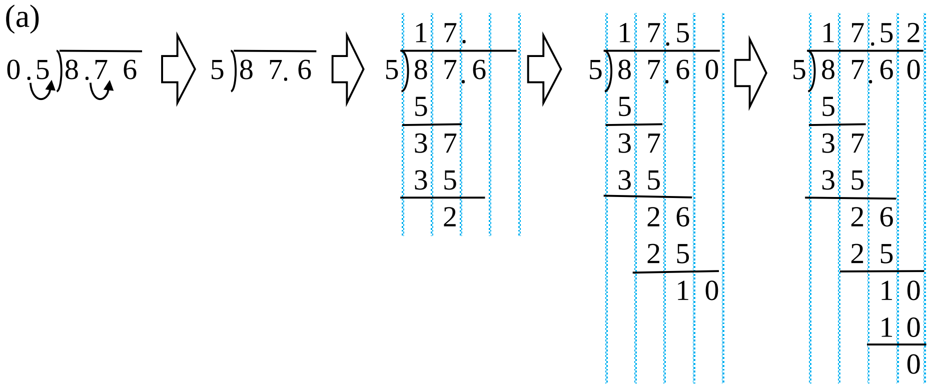
Dividing Decimal numbers

Example 11

Work out (a) 8.76 ÷ 0.5 (b) 16.9 ÷ 6.5

[Answer]

(a)



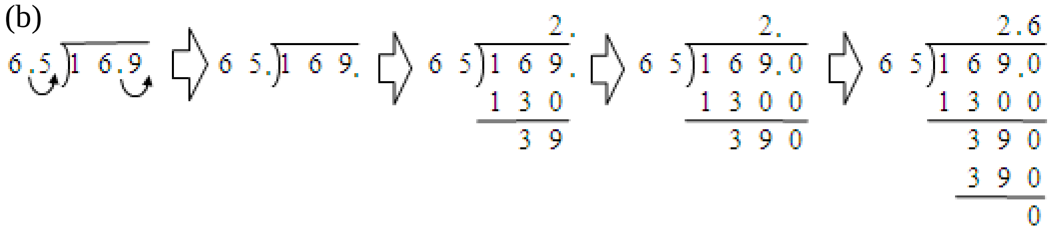
Move the decimal point one place to the right, which makes the divisor a whole number. Also move the decimal point in the dividend one place to the right.

Put the decimal point just above the decimal point in the devidend.

Bring down the 0

8.76 ÷ 0.5 = **17.52**

(b)



16.9 ÷ 6.5 = **2.6**

Exercise 8

1. Express in its simplest fraction

   (a) $\frac{7}{21}$ (b) $\frac{8}{40}$ (c) $\frac{24}{36}$ (d) $\frac{50}{75}$

2. Work out the value of p.

   (a) $\frac{4}{5} = \frac{p}{15}$ (b) $\frac{2}{3} = \frac{6}{p}$ (c) $\frac{8}{40} = \frac{p}{120}$

3. Work out the following fractions.

   (a) $\frac{1}{2} + \frac{3}{10}$ (b) $\frac{5}{9} + \frac{2}{7}$ (c) $\frac{6}{10} + \frac{3}{5}$

   (d) $\frac{3}{8} + \frac{2}{8}$ (e) $\frac{4}{13} + \frac{5}{13}$

4. Work out the following fractions.

   (a) $\frac{7}{8} - \frac{6}{8}$ (b) $\frac{1}{2} - \frac{3}{10}$ (c) $\frac{8}{9} - \frac{2}{5}$

   (d) $\frac{7}{9} - \frac{2}{9}$ (e) $\frac{8}{15} - \frac{7}{15}$

5. Convert the following fractions to decimals

   (a) $\frac{2}{5}$ (b) $\frac{3}{10}$ (c) $\frac{8}{32}$ (d) $\frac{12}{60}$

6. Convert the following decimals to simplest fractions

   (a) 0.002 (b) 0.32 (c) 0.65 (d) -0.375

7. Work out the following fractions.

   (a) $\frac{7}{8} \times \frac{1}{2}$ (b) $\frac{3}{5} \times \frac{4}{7}$ (c) $\frac{2}{3} \times \frac{6}{13}$

8. Work out the following fractions

(a) $\frac{3}{4} \div \frac{2}{5}$ (b) $\frac{6}{7} \div \frac{5}{8}$ (c) $\frac{3}{4} \div \frac{2}{11}$

9. Work out the following fractions.

(a) $\frac{1}{2} \times \frac{2}{3} + \frac{2}{7}$ (b) $\frac{3}{5} \div \frac{4}{7} - \frac{2}{9}$ (c) $\frac{5}{8} - \frac{8}{9} \times \frac{2}{7}$

10. Convert the following improper fractions to mixed numbers.

(a) $\frac{5}{4}$ (b) $\frac{6}{5}$ (c) $\frac{8}{3}$ (d) $\frac{9}{2}$

11. Convert the following mixed numbers to improper fractions.

(a) $2\frac{1}{3}$ (b) $4\frac{2}{5}$ (c) $5\frac{1}{2}$

(d) $1\frac{5}{6}$ (e) $5\frac{1}{6}$ (f) $2\frac{4}{7}$

12. Work out the following fractions.

(a) $3\frac{1}{5} - 1\frac{2}{3}$ (b) $1\frac{4}{5} + 2\frac{1}{6}$ (c) $3\frac{2}{7} - 2\frac{1}{2}$

(d) $8\frac{3}{8} + 2\frac{1}{9}$ (e) $5\frac{1}{3} - 2\frac{1}{4}$ (f) $4\frac{1}{3} + 1\frac{2}{5}$

13. Work out the following fractions.

(a) $2\frac{2}{5} \times 5\frac{1}{6}$ (b) $1\frac{4}{5} \div 3\frac{1}{7}$ (c) $6\frac{2}{9} \times 1\frac{2}{5}$

(d) $6\frac{1}{3} \div 1\frac{2}{7}$ (e) $2\frac{1}{3} \times 4\frac{1}{2}$ (f) $3\frac{4}{7} \div 2$

14. Work out the following decimals.

(a) 2.003 + 12.05 (b) 9.561 + 1.03 (c) 2.01 + 3.4 + 5.225

(d) 3.67 – 1.009 (e) 8.856 – 6.2 (f) 7.661 – 0.32

(g) 5.89 – 2.0007 (h) 9.45 $-$ 2.8 (i) 4.002 – 1.6

15. Work out the following decimals.

(a) 2.012 $\times$ 0.4 (b) -5.6×2.3 (c) 6.21 $\times$ 4.2

(d) 65.78 $\div$ 0.5 (e) 6.234 $\div$ 0.12 (f) 6.54 $\div$ 1.5

16. Work out.

(a) $\frac{1}{4}$ of 10kg (b) $\frac{1}{3}$ of 12 metres

(c) $\frac{1}{6}$ of $42.00 (d) $\frac{3}{4}$ of 8 hours

(e) $\frac{3}{5}$ of 35 km / hr (f) $\frac{3}{5}$ of 1 minute

17. In a class of 40 students, $\frac{5}{8}$ of the students are boys. How many boys are there in this class?

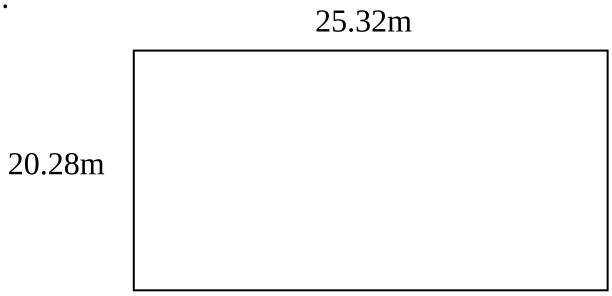
18. In a school of 800 students, $\frac{1}{10}$ of the students are involved in athletics.

(a) How many students are involved in athletics?

(b) How many students are not involved in athletics?

19. In a Hardware Store, any customer who wishes to buy a special 3 piece Lounge Suite at cash price of $1, 200.00 has to pay a deposit of $\frac{1}{8}$ of the cash price. How much does a customer have to pay as a deposit for these 3 pieces Lounge Suite?

20. The length and width of a rectangular piece of land are 25. 32 m and 20. 28m respectively.

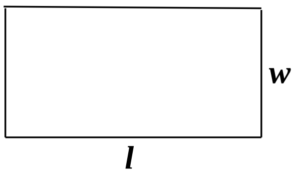
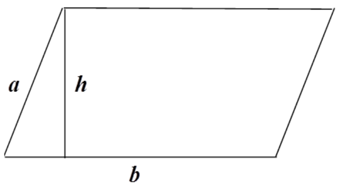
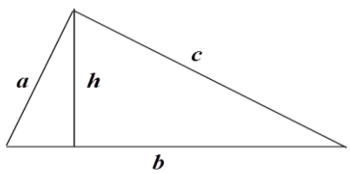
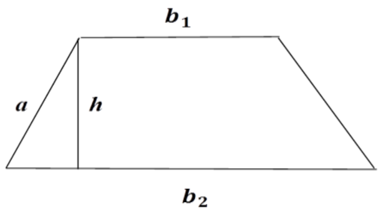
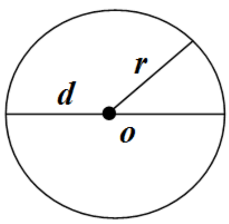


(a) Work out the perimeter of the piece of land.

(b) Work out the area of the rectangular piece of land. Round your answer to 2 decimal places.

AREAS AND VOLUMES OF BASIC SHAPES

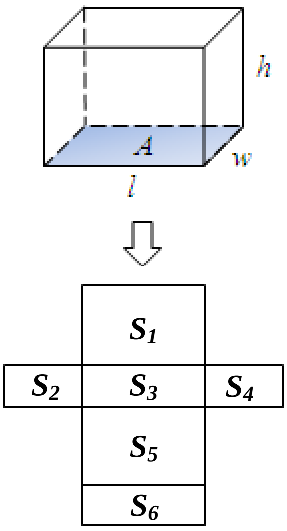
Areas and Perimeters

shape	Area	Perimeter
Rectangle 	Length × Width $A = l\,w$	2×(Length + Width) $P = 2(\,l + w)$
Parallelogram 	Base × Height $A = b\,h$	$P = 2a + 2b$
Triangle 	$\frac{1}{2}$ × Base × Height $A = \frac{1}{2}\,bh$	$P = a + b + c$
Trapezoid 	$\frac{1}{2}$ × (Sum of parallel sides) × Vertical height $A = \frac{1}{2}(b_1 + b_2)h$	$P = a + b_1 + b_2 + c$
Circle 	Radius × Radius × π (Radius)^2 × π $A = \pi r^2$	Diameter × π 2 × Radius × π $P = \pi d$ $= 2\pi r$

Volumes and Surfaces

<table>
<tr>
<td>

Rectangular Solid



</td>
<td>

Volume

Length × Width × Height
= Base Area (Rectangular) × Height

$$V = lwh = Ah$$

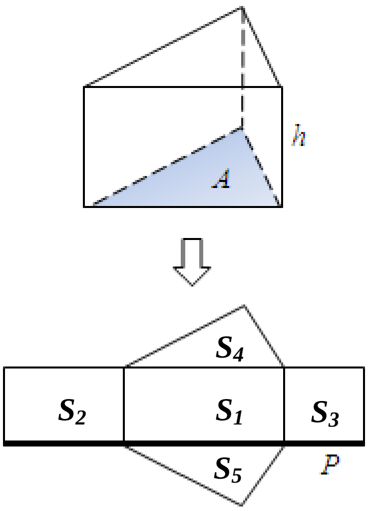
Surface

$$S = S_1 + S_2 + S_3 + S_4 + S_5 + S_6$$

$$= 2lw + 2wh + 2lh$$

</td>
</tr>
<tr>
<td>

Prisms



</td>
<td>

Volume

Base Area (Triangle) × Height

$$V = Ah$$

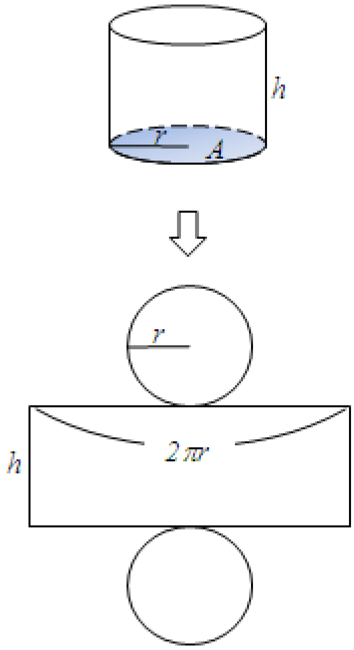
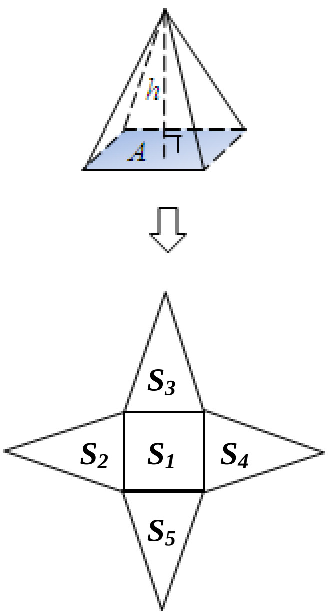
Surface

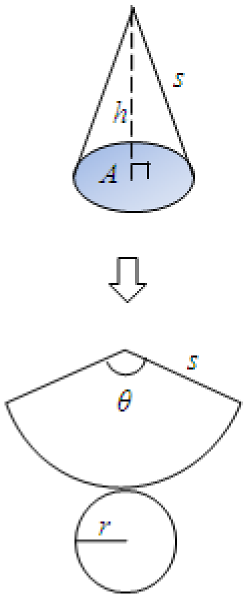
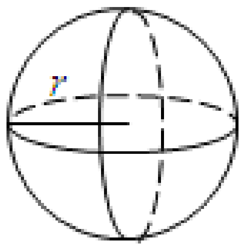
$$S = S_1 + S_2 + S_3 + S_4 + S_5$$

$$= 2A + Ph$$

(P is the perimeter of the base)

</td>
</tr>
</table>

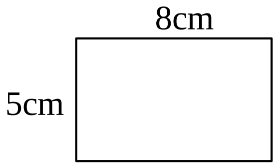
Cylinder  	**<u>Volume</u>** Base Area (Circle) × Height = (Radius)^2 × π × Height $V = Ah = \pi r^2 h$ **<u>Surface</u>** $S = 2\pi rh + 2\pi r^2$
Pyramid  	**<u>Volume</u>** $\frac{1}{3}$ × Base Area (Rectangle) × Height $V = \frac{1}{3}Ah$ **<u>Surface</u>** $S = S_1 + S_2 + S_3 + S_4 + S_5$

Cones 	**Volume** $\frac{1}{3}$ × Base Area (Circle) × Height = $\frac{1}{3}$ × (Radius)^2 × π × Height $$V = \frac{1}{3}Ah = \frac{1}{3}\pi r^2 h$$ **Surface** $$S = \pi r^2 + \pi r s$$
Sphere 	**Volume** $$V = \frac{4}{3}\pi r^3$$ **Surface** $$S = 4\pi r^2$$

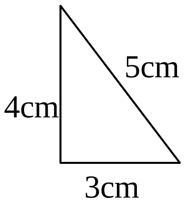
Exercise 9

1. Find the area of each of the following figures.

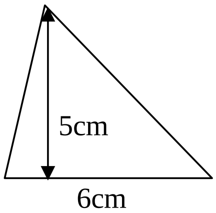
(a)



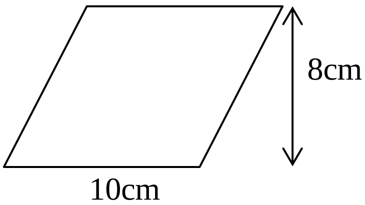
(b)



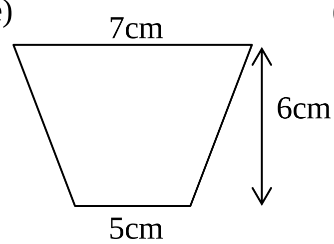
(c)



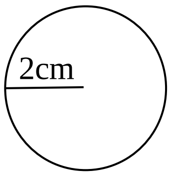
(d)



(e)

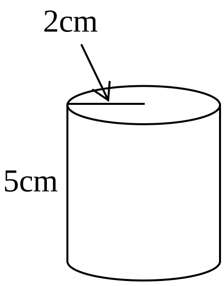


(f)

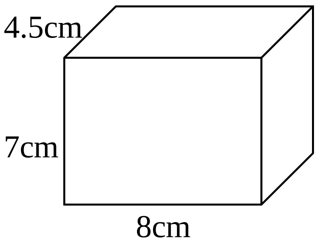


2. Find the volume of the following figures.

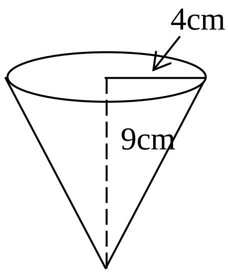
(a)



(b)

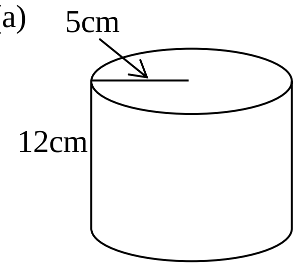


(c)

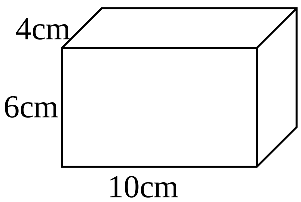


3. Find the surface area of the figures given below.

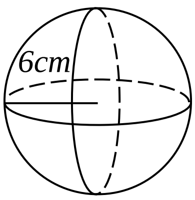
(a)



(b)



(c)



PROPERTIES OF TRIANGLES AND POLYGONS

Properties of a Square

(i) All the four sides are equal

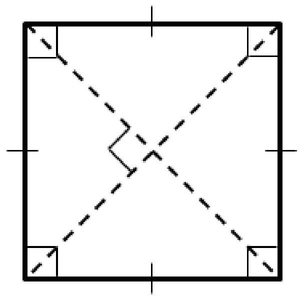
(ii) All angles are right angles or equal to 90^o

(iii) Opposite sides are parallel

(iv) The two diagonal lines are equal and bisect one another at right angles

(v) The diagonals bisect the corner of a square

(vi) The diagonals bisect the area of a square



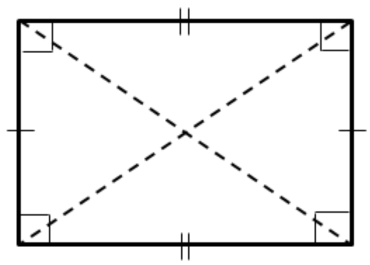
Properties of a Rectangle

(i) A rectangle has four equal angles (right angles)

(ii) Opposite sides are equal and parallel

(iii) The diagonals are equal and bisect one another

(iv) The diagonals bisect the area



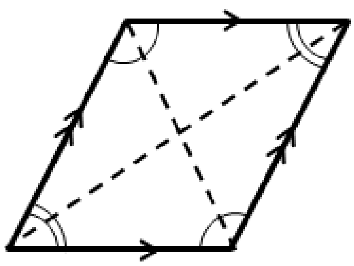
Properties of a Parallelogram

(i) Opposite sides are equal and parallel

(ii) The diagonals bisect one another

(iii) Opposite angles are equal

(iv) The diagonals bisect the area



Properties of a Rhombus

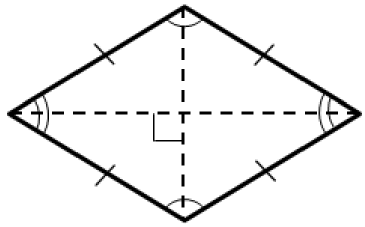
(i) All the sides are equal

(ii) Opposite sides are equal and parallel

(iii) Diagonals bisect one another at right angles

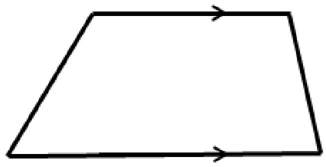
(iv) Diagonals bisect the area

(v) Opposite angles are equal



Properties of a Trapezium

One pair of parallel sides

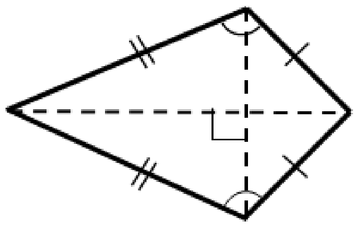


Properties of a Kite

(i) Two pairs of equal sides

(ii) One pair of opposite angles are equal

(iii) The main diagonal which is the axis of symmetry bisects the other diagonal at right angles



(iv) The diagonal (axis of symmetry) also bisects the two corners through which it passes

(v) The diagonal(axis of symmetry) bisects the area

UNIT 2

DIRECTED NUMBERS

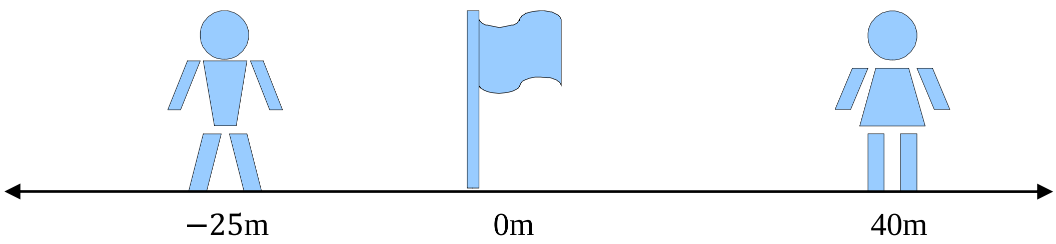
<u>**UNIT 2**</u>

<u>DIRECTED NUMBERS</u>

<u>DIRECTED NUMBERS</u>

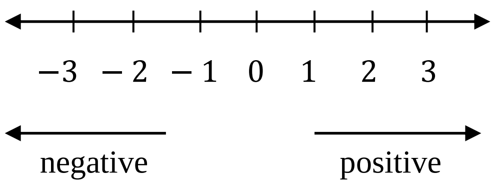
> A directed number has both magnitude [size] and direction from zero
> The direction of a positive number is 'up' and 'forward'
> The direction of a negative number is 'down' and 'backward'

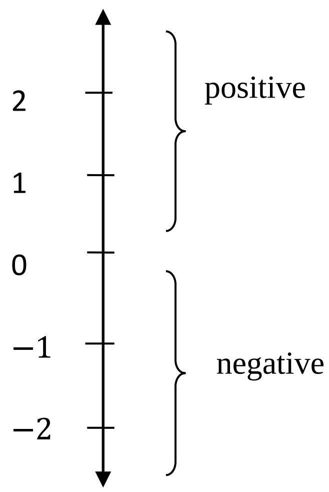
In the diagram shown below, a boy is standing 25 metres on the left and a girl is standing 40 metres on the right of the flagpole.



These are examples of **directed numbers**. They have magnitude (distance of the boy and the girl from the flagpole) and direction (left and right of the flagpole).

In mathematics, the direction of a number is shown by the positive (+) and negative (-) signs which is placed in front of the number. If the number is placed on the **right** or **above** the starting point, it is regarded as a positive number (+), whereas if it placed to the **left**, **below** the starting point, it is regarded as a negative number (-).





Examples of positive numbers are 2, 6, 18, 116, etc. Normally the positive sign (+) is not placed in front of the number.

For negative numbers, the negative sign (-) is placed in front of the number.
Examples of negative numbers are -1, -4, -34, -2278, etc .

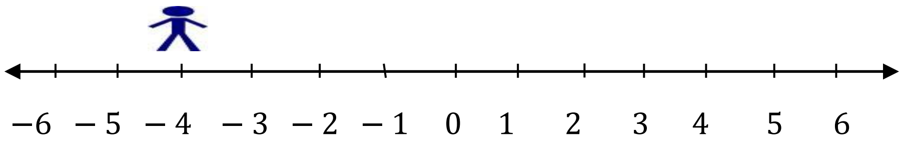
Example 1

Negative numbers		**Positive Numbers**	
1 m down	-1m	1m up	+1m
3 m to the left	-3m	3m to the right	+3m
5 m backward	-5m	5m forward	+5m
$15 withdraw	-$15	$15 deposit	+$15

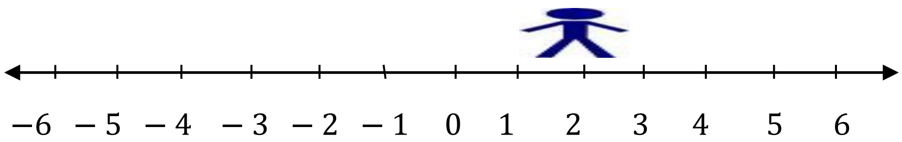
Exercise 1

1. Use directed numbers to tell the distance of a boy from zero.

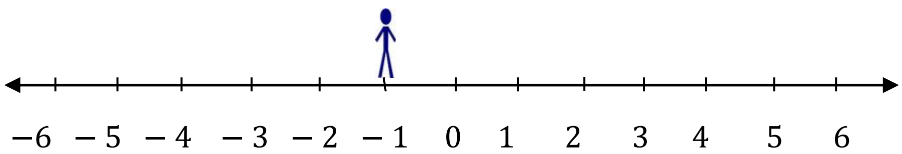
(a)



(b)



(c)



2. Use a directed number to indicate the magnitude and direction of each number.

(a) 10 m below sea level

(b) 30 m above the ground

(c) 30 km west of the City

(d) Sam withdrew \$300.00 from the bank

NUMBERS

All numbers can be classified into any of the following sets

(a) **N = {Natural numbers} –** the set of counting numbers

Example 1, 2, 3, . . .

(b) **W= {Whole numbers}** – the set of natural numbers plus 0

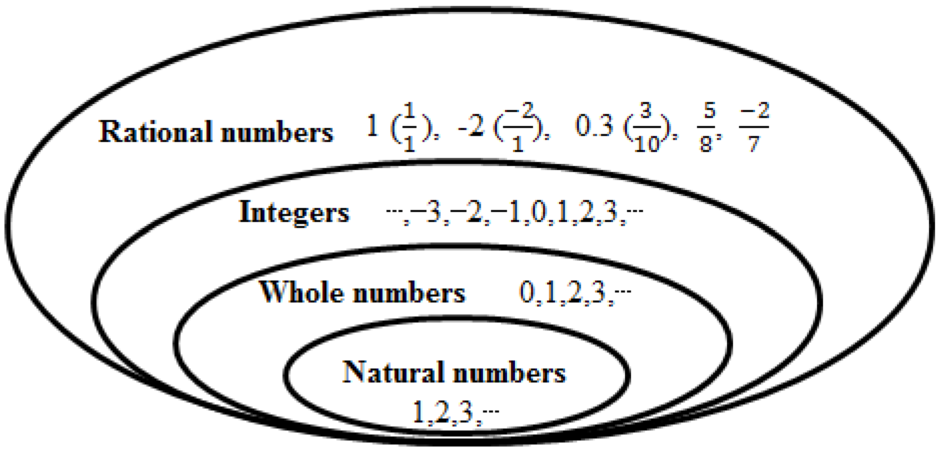
Example. 0, 1, 2, 3, . . .

(c) **I = {Integers}** – the set of positive and negative whole numbers

Example . . . −3, −2, −1, 0, 1, 2, 3, . . .

(d) **Q = {Rational numbers}** – the set of **all** numbers that can be expressed as fractions.

Example 1 $(\frac{1}{1})$, -2 $(\frac{-2}{1})$, 0.3 $(\frac{3}{10})$, $\frac{5}{8}$, $\frac{-2}{7}$



Example 2

Find out whether the mathematical statements shown below is true or false

(a) -1 is a Whole number Answer: **False**

(b) -3 is an Integer Answer: **True**

(c) -2 is a Natural number Answer: **False**

Exercise 2

1. Use the list of numbers given to answer the following questions

$0.3, \quad -5, \quad -6, \quad 4, \quad -0.2, \quad \frac{1}{7}, \quad 0, \quad \frac{-6}{5}, \quad 12,$

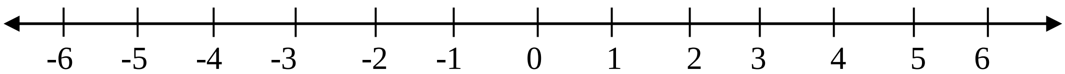
(a) Which numbers are rational numbers?.

(b) Which numbers are integers?

(c) Which numbers are negative rational numbers?.

2. Place the following numbers in their right positions on the number line given below.

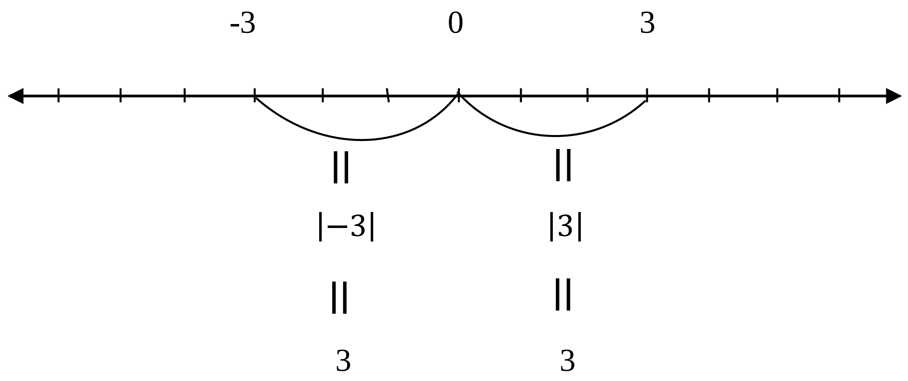
$-3, \quad -2.5, \quad \frac{7}{2}, \quad +4.5$



ABSOLUTE VALUE

The absolute value of a number is the distance of the number from zero

The symbol $|\ |$ is used to represent the absolute value of a number



$$|3| = 3$$

$$|-3| = 3$$

Exercise 3

Work out the following.

(a) $|5|$ (b) $|-4|$ (c) $|-1.2|$ (d) $\left|\frac{2}{3}\right|$ (e) $|0|$

Example 3

Work out the following.

(a) $|-3| + |-5|$ (b) $|6-2|$ (c) $|11-3| - |4-2|$

[Answer]

(a) $|-3| + |-5| = 3 + 5 = 8$

(b) $|6-2| = |4| = 4$

(c) $|11-3| - |4-2| = |8| - |2| = 8 - 2 = 6$

Exercise 4

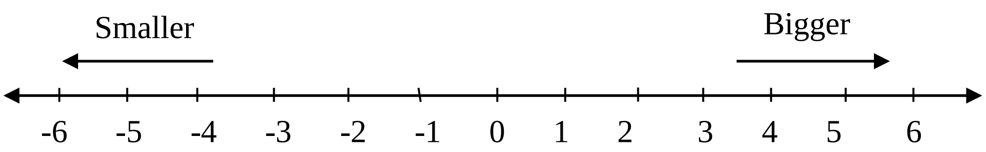
Work out the following.

(a) $|5| + |4|$ (b) $|-6| \div |-3|$ (c) $|6-4|$

INEQUALITIES AND NUMBER LINE

$>$	Greater than/ bigger than	$5 > 3$
$<$	Less than/ smaller than	$4 < 7$
$\geq$	Greater than or Equal to / Bigger than or equal to	$x \geq 3$
$\leq$	Less than or equal to / smaller than or equal to	$x \leq 6$

THE NUMBER LINE



Example 4

(a)	$3 < 7$	$(7 > 3)$	-	3 is smaller than 7
(b)	$-4 < 2$	$(2 > -4)$	-	-4 is smaller than 2
(c)	$-5 < -1$	$(-1 > -5)$	-	-5 is smaller than -1

Exercise 5

1. Choose the inequality sign (>, <) to go in the box in order to make the mathematical statement true

   (a) 7 ☐ 6 (b) -5 ☐ -3 (c) 0 ☐ -2

   (d) -0.1 ☐ -0.01 (e) $-\frac{1}{2}$ ☐ -2

2. Sort the numbers into ascending order

   (a) 0, +1, -2 (b) -6, +3, -8 (c) +6, -9, -2.5

ADDITION OF INTEGERS

Example 5

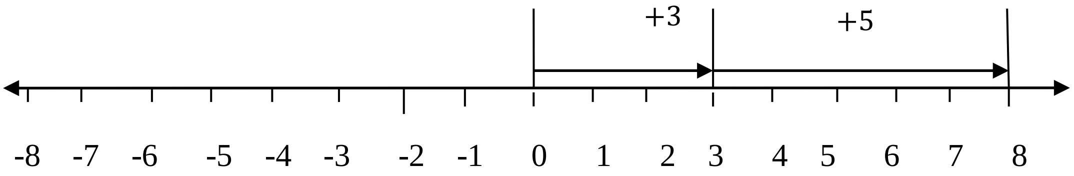
(a) First 3 m to the East and then 5 m to the East

(b) First 3 m to the East and then 5 m to the West

(c) First 3 m to the West and then 5 m to the West

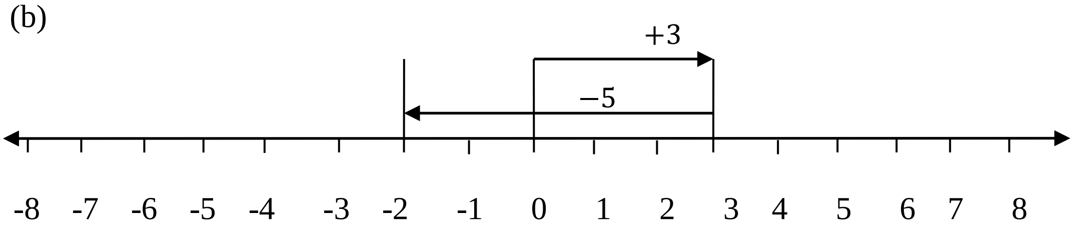
[Answer]

(a)



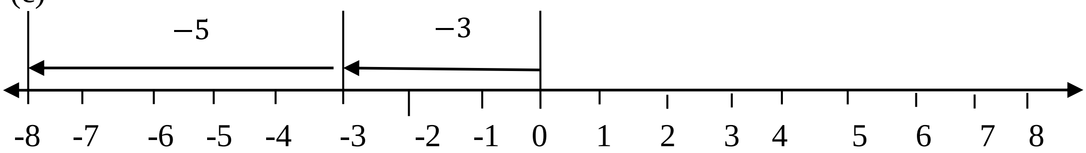
It means (+3) + (+5) = +8 **8 m to the east**

(b)



It means (+3) + (−5) = −2 **2 m to the west**

(c)



It means (-3) + (-5) = −8 **8 m to the west**

Exercise 6

Work out the following.

(a)	(+2) + (+7)	(b)	(−2) + (−4)	(c)	(+9) + (−4)
(d)	(−5) + (+5)	(e)	(−10) + (+4)	(f)	(+7) + (−9)

THE RULES OF ADDITION BETWEEN TWO NUMBERS

(i) Two numbers having the same sign (+ +/ - -)

Sign: the same sign as two numbers

Absolute value: the total of the two numbers

$(+5) + (+8) = +(5 + 8) = +13$

$(-5) + (-8) = -(5 + 8) = -13$

(ii) Two numbers have different signs (+ -)

Sign: the same sign as the number which has bigger absolute value

Absolute value: the difference between two numbers

$(+5) + (-8) = -(8 - 5) = -3$

$(-5) - (+8) = +(8 - 5) = +3$

Exercise 7

Work out the following.

(a) $(+21) + (-26)$

(b) $(-35) + (+38)$

(c) $(-27) + (-12)$

(d) $(-12) + (-12)$

(e) $0 + (-33)$

(f) $(-29) + (+17)$

(g) $(-0.4) + (+1.7)$

(h) $\left(\frac{1}{7}\right) + \left(-\frac{3}{7}\right)$

SUBTRACTION OF INTEGERS

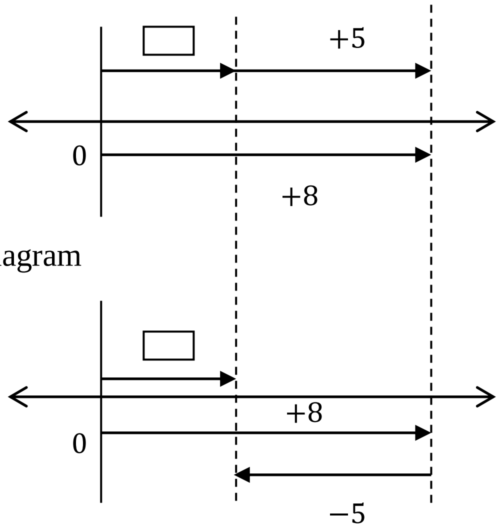
Example 6

(a) □ + (+5) = +8

□ = (+8) – (+5), from this diagram

= (+8) + (−5)

= +3



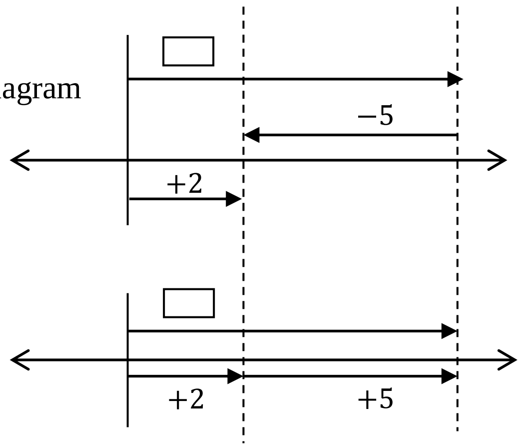
We can change subtraction into addition

(b) □ **+ (**−5) = +2

□ = (+2) – (−5), from this diagram

= (+2) + (+5)

= +7



THE RULE OF SUBTRACTION

The subtraction is same as addition of the opposite sign number

Example 7

Work out the following.

(a) (+3) – (+7)
= (+3) + (−7)
= −4

(b) (−3) – (−8)
= (−3) + (+8)
= +5

Exercise 8

Work out the following.

(a) (−2) – (+9) (b) (+8) – (−4)

(c) (+3) – (−5) (d) (+7) – (+5)

ADDITION AND SUBTRACTION OF INTEGERS

Example 8

Work out the following.

(a) $6 - (-9) - 3$ (b) $4 - 7 + 9 - 5$

[Answer]

(a) $6 - (-9) - 3$

$= 6 + (+9) - 3$

$= 15 - 3$

$= 12$

(b)

Method 1	**Method 2**
$4 - 7 + 9 - 5$	$4 - 7 + 9 - 5$
$= (4-7)+(9-5)$	$= 4 + 9 - 7 - 5$
$= -3+4$	$= (4+9)+(-7-5)$
$= 1$	$= 13 + (-12)$
	$= 1$

Exercise 9

Work out the following.

(a) $-4 + 12 - 9$ (b) $-5 + 3 - 2 + 6$

(c) $2 - 8 + 7 - 2 + 3$ (d) $-17 - (-28) + 0 - 19$

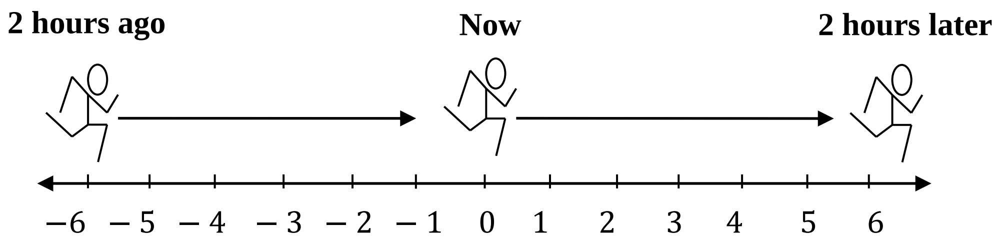
MULTIPLICATIONS OF INTEGERS

Example 9

(i) Sami runs to the East with a speed of 3km/ hr. How far does he travel

(a) 2 hours later

(b) 2 hours ago



[Answer]

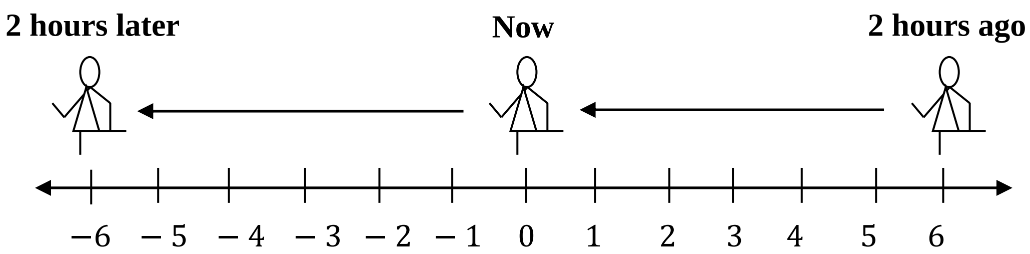
(a) $3 \times 2 = 6$

(b) $(-3) \times 2 = -6$

(ii) Sunita runs to the West with a speed of 3km/ hr. How far does she travel

(a) 2 hours later

(b) 2 hours ago



[Answer]

(a) $(-3) \times 2 = -6$

(b) $(-3) \times (-2) = 6$

RULES FOR MULTIPLICATION & DIVISION

1. Two numbers have the same sign $\rightarrow$ *the result has a positive sign*
2. Two numbers have different signs $\rightarrow$ *the result has a negative sign*

e.g.

1. $3 \times 2 = 6$ $(-3) \times (-2) = 6$ $6 \div 3 = 2$ $(-6) \div (-3) = 2$

2. $3 \times (-2) = -6$ $(-3) \times 2 = -6$ $6 \div (-3) = -2$ $(-6) \div 3 = -2$

Exercise 10

1. Work out the following.

   (a) 2×3 (b) $(-6) \times (-5)$ (c) 5×7 (d) $(-4) \times (-7)$

2. Work out the following.

   (a) $3 \times (-7)$ (b) $(-6) \times 9$ (c) $(-5) \times (-8)$ (d) $(-9) \times 0$

3. Work out the following.

   (a) $(-8) \times (-3)$ (b) 8×10 (c) $7 \times (-8)$

   (d) $(-11) \times 3$ (e) $(-5) \times 20$ (f) $(-1) \times (-6)$

   (g) $(+15) \times (+4)$ (h) $17 \times (-2)$

Exercise 11

1. Work out the following.

   (a) $12 \div 3$ (b) $(-30) \div (-5)$ (c) $35 \div (-7)$ (d) $(-24) \div (-4)$

2. Work out the following.

   (a) $60 \div (-5)$ (b) $(-72) \div (-9)$ (c) $(-52) \div 13$ (d) $0 \div (-6)$

3. Work out the following.

(a) $(-8) \div (-8)$ (b) $75 \div (-25)$ (c) $(-64) \div (-8)$

(d) $(-21) \div 3$ (e) $(-80) \div (-20)$ (f) $(-9) \div 3$

(g) $100 \div (-5)$ (h) $(-32) \div (-2)$

Review Exercise

1. Use directed numbers to indicate the magnitude and direction of each of the following statements.

(a) A boy moves 10m above the ground.

(b) Philip jumps 30m down into the swimming pool.

(c) Samisoni runs 45 m backwards.

(d) Avinesh throws a ball 20 m upwards.

2. Work out the following

(a) $|-2|$ (b) $|8|$ (c) $|-16|$

(d) $|-2| - |-5|$ (e) $|5| + |-3|$

(f) $|-3| + |-10|$ (g) $7 - |-3|$

(h) $|32| + |-4|$ (i) $|-25| - |-4|$

(j) $|-6| - 2$ (k) $|-3| + |-6|$

3. Work out the following

(a) $-2-4$ (b) $4-(+5)$ (c) $-2-(-8)$

(d) $-4+(-6)$ (e) $4-(-5)$ (f) $5+(-7)$

4. Work out the following

(a) $2 \times (-5)$ (b) -18×3 (c) $10 \div (-5)$

(d) $-8 \times (-6)$ (e) $-12 \div 6$ (f) $-24 \div (-6)$

5. Work out the following

(a) $-5 \times 4 - 11$ (b) $-2 - 4 \times 8$ (c) $-12 \div 3 - 5$

(d) $-16 \div (-4) + 3$ (e) $-3 - 2 \times -5$ (f) $7 - 12 \div (-6)$

6. Choose the inequality sign (<, >) to go in the box in order to make the mathematical statement true.

(a) $7 \ \square \ 6$ (b) $-5 \ \square \ -3$

(c) $-5 + 8 \ \square \ 7$ (d) $-2 - 6 \ \square \ -10$

UNIT 3

BASIC ALGEBRA

UNIT 3 **BASIC ALGEBRA**

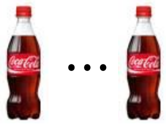
How much?

You can buy one bottle of coke for 2 dollars.

$2 $\times$ 1 = $2

$2 $\times$ 2 = $4

$2 $\times$ 3 = $6



$2 $\times$?

In accordance with this example, the total price can be expressed as

$$\$2 \times (the\ number\ of\ bottles\ of\ coke)$$

The number of bottles of coke is expressed as variable *x* where *x* is an element of 1, 2, 3, ⋯ . The total price can be expressed as

$$\$2\ \times x$$

NUMERALS AND PRONUMERALS

Numbers are referred to as **numerals** whereas variables that are used to replace numbers are known as **pronumerals**.

For **Algebra**, variables or pronumerals are used instead of numbers as we move towards more complex mathematics. It also makes our work faster and calculations easier.
A mathematical statement in algebraic form is known as **Algebraic Expression**.

The rules of Algebra

1. Leave out the $\times$ signs.
   e.g. $a \times b = ab,\ 6 \times x = 6x$

2. The number is written before the pronumerals. However, 1 is not written.
   e.g. $a \times 3 = 3a,\ 1 \times a = a,\ -1 \times a = -a$

3. An index shows how many times the same number or the same pronumeral is multiplied.
   e.g. $5 \times 5 \times 5 = 5^3, \quad a \times a \times a \times a \times a = a^5$

Let's try!! Express each of following sentences as mathematical statement.

1. You can buy one burger for 7 dollars. If Emosi buys *x* burgers, how much should he pay?

2. You can buy one notebook for 2 dollars and one calculator for 13 dollars. If Shayal buys *y* notebooks and one calculator, how much should she pay?

3. You can buy one bundle of dalo for 5 dollars and one chicken for 10 dollars. If Jone buys *p* bundles of dalo and *q* chickens, how much should he pay?

ALGEBRAIC EXPRESSIONS

The number beside the variable is known as the **coefficient**.

e.g. The coefficient of y in $3y$ is 3.

e.g. The coefficient of x^2 in $-2x^2$ is -2.

$3\,y$	$-2x^2$
↑ coefficient	↑ coefficient

A **term** can be an algebraic expression or part of it.

The terms of an algebraic expression are the parts of the expression that are connected by a plus (+).

e.g. $3p + 7m - 2n - 6$

$= 3p + 7m + (-2n) + (-6)$

$\therefore$ **The terms are** $\mathbf{3p, 7m, -2n, -6}$

There are different types of algebraic expressions depending on the number of terms in the expression.

(i) **Monomial** – 'mono' means **one**. There is one term in the expression.

e.g. $2x, 3p$ and x^2

(ii) **Binomial** – 'bi' means **two**. There are two terms in the expression.

e.g. $x – 4$, $x^2 + y^2$ and $2x-y$

(iii) **Trinomial** – 'tri' means **three**. There are three terms in the expression.

e.g. $x + y – 3$, $x^2 – x – 8$ and $p + 4q– 3r$

(iv) **Polynomial** – 'poly' means **many**. An expression that has two or more terms is known as polynomial.

Exercise 1

1. Find each coefficient of the following expressions.

(a) $2a$ (b) $-5b^2$ (c) x^3 (d) $\frac{3}{5}y$

2. Find the terms and each coefficient of a and b of $6a - b + 5$.

SIMPLIFICATION OF ALGEBRAIC EXPRESSIONS

Adding and Subtracting Algebraic Expressions

Terms that contain the same variables with same index are known as **like terms**.
Terms that contain the different variables or same variables with different indices are known as **unlike terms.**

e.g. $2a$ and $-5a$ are like terms. $3a$ and $7b$ are unlike terms. $4x$ and $4x^2$ are unlike terms.

Suppose tables and chairs.

3 chairs + 2 chairs = 5 chairs

 +  = 

Here the chairs can be added together since they are the same kind of objects.

Likewise, $3a + 2a = 5a$.

Like terms are the only terms that can be added and subtracted.

2 chairs + 4 tables

 + 

2 chairs and 4 tables cannot be added together since they are not the same kind of object.

Likewise, $2a + 4b$ cannot be added.

To combine like terms

(i) Determine which terms contain the same variable or groups of variables with same index.

(ii) Add or subtract the numerical coefficients.

(iii) Attach the variable and index.

$$3x+5x$$
$$=(3+5)x$$
$$=8x$$

Example 1

Simplify the following expressions.

(a) $3y + 7y – 5y – 3y$

(b) $2x^2 - 5x^2$

(c) $3x^2 + x^2 – 4x - 6x$

[Answer]

(a) $3y + 7y – 5y – 3y = (3 + 7 – 5 – 3)\ y$

$= \mathbf{2y}$

(b) $2x^2 - 5x^2 = (2 - 5)\ x^2$

$= \mathbf{-3x^2}$

(c) $3x^2 + x^2 – 4x - 6x = 3x^2 + x^2 – 4x - 6x$

$= (3 + 1)\ x^2 + (– 4 - 6)\ x$

$= \mathbf{4x^2 - 10x}$

Exercise 2

Simplify the following expressions.

(a) $4x + 5x$

(b) $3x + y - 2x + 3y$

(c) $x^2 - 3xy - 3x^2 + 4xy$

(d) $x + 12y - 5x + 6y + 17$

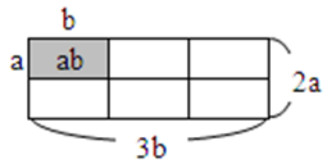
(e) $\frac{2}{3}p - 4q + \frac{5}{3}p + 7q$

(f) $2x^2 + 8x - 4x^2 - 5x + x$

(g) $7a^2b + 4ab^2 - 6ab^2 - 2a^2b + 9ab$

Multiplying Algebraic Expressions

When multiplying algebraic expressions such as 2a × 3b, we can separate the coefficients from the variables. We multiply the numbers and then followed by the multiplication of the variables.



The area of the rectangle is $2a \times 3b$

$2a \times 3b$

$= 2 \times a \times 3 \times b$

$= 2 \times 3 \times a \times b$

$= 6ab$ (variables are to be in alphabetical order)

Multiply the coefficients first and then followed by the variables.

$$\boxed{a^m \times a^n = a^{m+n}}$$

e.g. $a^3 \times a^2 = \underbrace{(a \times a \times a)}_{\text{3units}} \times \underbrace{(a \times a)}_{\text{2units}} = a^5$ [3 + 2]

$$\boxed{a^m \div a^n = a^{m-n}}$$

e.g. $a^6 \div a^4 = \frac{a^6}{a^4} = \frac{a \times a \times a \times a \times a \times a}{a \times a \times a \times a} = a^2$ [6 − 4]

$$\boxed{(a^m)^n = a^{m \times n}}$$

e.g. $(a^2)^3 = (a^2) \times (a^2) \times (a^2) = (a \times a) \times (a \times a) \times (a \times a) = a^6$ [2 × 3]

$$(a\times b)^m = a^m \times b^n$$

e.g. $(a\times b)^3 = (a\times b)\times(a\times b)\times(a\times b) = a\times a\times a\times b\times b\times b = a^3\times b^3$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

e.g. $\left(\frac{a}{b}\right)^3 = \left(\frac{a}{b}\right)\times\left(\frac{a}{b}\right)\times\left(\frac{a}{b}\right) = \frac{a\times a\times a}{b\times b\times b} = \frac{a^3}{b^3}$

$$a^1 = a \qquad a^0 = 1$$

e.g. $a^3 \div a^3 = \frac{a^3}{a^3} = \frac{a\times a\times a}{a\times a\times a} = 1$ otherwise $a^3 \div a^3 = a^{3-3} = a^0$ so $\mathbf{a^0 = 1}$

Property

$$a^m \times a^n = a^{m+n}$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$$a^m \div a^n = a^{m-n}$$

$$a^1 = a$$

$$(a^m)^n = a^{m\times n}$$

$$(a\times b)^m = a^m \times b^n$$

$$a^0 = 1$$

Example 2

Simplify the following expressions.

(a) $2m^3 \times 6m^5$ (b) $-5p \times 7q$

(c) $6g^5 \div 3g^3$ (d) $2ab \times (-4a^2b^3)$

[Answer]

(a) $2m^3 \times 6m^5$

$= 2 \times m^3 \times 6 \times m^5$

$= 2 \times 6 \times m^3 \times m^5$

$= 12 \times m^{3+5}$

$= 12m^8$

(b) $-5p \times 7q$

$= -5 \times 7 \times p \times q$

$= -35pq$

(c) $6g^5 \div 3g^3 = (6 \div 3)(g^5 \div g^3)$

$= 2g^{5-3}$

$= 2g^2$

Or $6g^5 \div 3g^3 = \dfrac{\not{6} \times \not{g} \times \not{g} \times g \times g \times g}{\not{3} \times \not{g} \times \not{g} \times g}$

$= 2g^2$

(d) $2ab \times (-4a^2b^3)$

$= 2 \times (-4) \times a \times a^2 \times b \times b^3$

$= -8a^{1+2}b^{1+3}$

$= -8a^3b^4$

Exercise 3

Simplify the following expressions.

(a) $x^3 \times 5x^2$

(b) $(-2x^2) \times (-3x^3)$

(c) $12a^5 \div (-3a^4)$

(d) $2x^2 \times (-3x^2y^3)^3$

(e) $2t^3 \times (-t^5) \div (3t^3)^2$

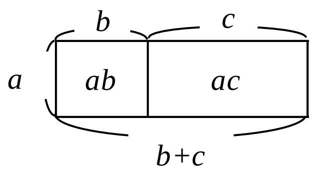
(f) $(-3x^2y) \times 6xy^4 \div (-2xy)^3$

DISTRIBUTIVE PROPERTY

Distributive property is normally used to expand brackets.

The distributive property states that the term outside the bracket is multiplied to each of the terms inside the bracket.

a(b + c) = ab + ac



Example 3

Simplify $2p\,(q - 3r)$

[Answer]

Remove the bracket by multiplying the term outside the bracket to the two terms inside the bracket.

$2p \times q + 2p \times -3r$

$= \mathbf{2pq - 6pr}$

Exercise 4

Simplify the following expressions.

(a) $2(x - 3y)$

(b) $-x(2x^2 - x + y)$

(c) $2 + 3(x - 1) - x$

(d) $2(5x - 2) - x(x - 7)$

Practice 1

1. Simplify the following expressions.

(a) $2a + 5a$ (b) $7H^2 + 8H^2$ (c) $15w^2 - 7w^2$

(d) $3t - 8t$ (e) $15n - 8n + 3n$ (f) $8t^2 - 15t^2 + 2t^2$

(g) $8k^3 - 5k^3 - 3k^3$ (h) $2x + 3 - x + 4$ (i) $5x + y - 3x - 4y$

(j) $a + 4b - 3 + 2a - 7b - 2$ (k) $\frac{2}{3}x - 1 + \frac{x}{3} - 9$ (l) $\frac{x}{3} - \frac{y}{2} - \frac{x}{6} + \frac{y}{4}$

(m) $2t^2 - 5st + 4 - 3t^2 + 7st - 4$ (n) $3a^2 - 2ab - 4b^2 - 5a^2 + 2ab - 8b^2$

2. Simplify the following expressions.

(a) $7u \times 3v$ (b) $-12a \times 4b$ (c) $4e \times (-7f) \times 2g$

(d) $n^2 \times n^3$ (e) $4y^7 \times (-9y^3)$ (f) $2t^2 \times 3t^5 \times (-t^3)$

(g) $3x^2y \times (-2x^3y^2)$ (h) $(-2ab^3)^2$ (i) $w^8 \div w^5$

(j) $6d^8 \div 2d^3$ (k) $12j^5 \div (-4j^4)$ (l) $24p^6q^4 \div (2p^2q)^3$

(m) $8v^6 \times 3v^5 \div 6v^{11}$ (n) $2x^3y \times (-5xy^2) \div (-2xy)^3$

(o) $(-3a^2b)^3 \div 2a^3b \times (-2ab^2)^2$

3. Simplify the following expressions.

(a) $2(x-3)$ (b) $-(5-4x)$ (c) $3(x^2-2x-8)$

(d) $p(q+1)$ (e) $a(2b-3c)$ (f) $\frac{1}{2}x^2(-10x-4)$

(g) $-3p^2(-p+2p^4)$ (h) $\frac{2}{3}a^2(-9a^3-\frac{1}{2}a^2+\frac{3}{2}b)$ (i) $2+2(y-4)$

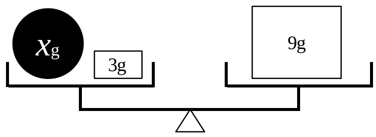
(j) $-(x-3y)+3(2x-y-1)$ (k) $a^2(b+3ab-2)+b(a^3-3a^2+2)-3$

LINEAR EQUATIONS

An **expression** can only become an **equation** if the equal (=) sign is present. In an **algebraic equation**, there is an unknown which is normally represented by a variable.

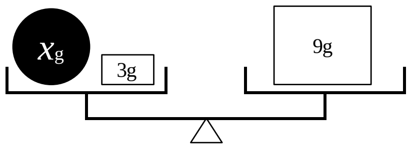
$x + 3$ is an **expression**

$x + 3 = 9$ is an **equation**

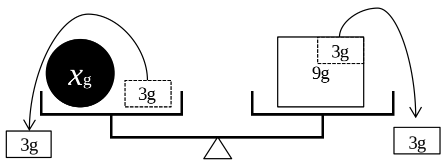


When both sides of an equation are added, subtracted, multiplied or divided by the same number, both sides are equal.

There is a scale. On the left side there are 3g and *x* g weights. On the right side there is 9g weight. Both sides are equal. What is the weight of *x* ?

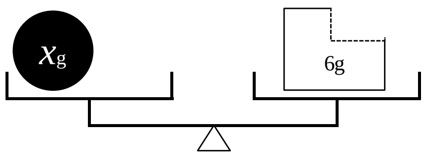


xg + 3g = 9g



Subtract 3g from the both sides.

xg + 3g -3g= 9g-3g



Both sides are equal.

Therefore **x g = 6g**

Example 4

Solve these equations.

(a) $x + 5 = 12$ (b) $2y - 3 = 9$ (c) $\frac{x}{2} + 3 = 8$

[Answer]

(a)

$$x + 5 = 12$$
$$x + 5 - \mathbf{5} = 12 - \mathbf{5}$$
$$x + 0 = 7$$
$$\boldsymbol{x = 7}$$

To remove +5 from the left hand side, place −5 on the both sides.

(b)

$$2y - 3 = 9$$
$$2y - 3 + 3 = 9 + 3$$
$$2y + 0 = 12$$
$$2y = 12$$
$$\frac{\cancel{2}y}{\cancel{2}} = \frac{12}{2}$$
$$\boldsymbol{y = 6}$$

To remove – 3 from the left hand side, place +3 on the both sides.

To leave *y* alone, divide both sides by 2.

(c)

$$\frac{x}{2} + 3 = 8$$
$$\frac{x}{2} + 3 - 3 = 8 - 3$$
$$\frac{x}{2} = 5$$
$$\frac{x}{2} \times 2 = 5 \times 2$$
$$\boldsymbol{x = 10}$$

To remove + 3 from the left hand side, place −3 on the both sides.

To leave *x* alone, multiply both sides by 2.

To check whether this answer is correct or not, substitute $x = 10$ into the original equation and find out whether left hand side equals to the right hand side or not.

Check LHS $= \frac{10}{2} + 3$

$= 5 + 3$

$= 8$

$=$ RHS

Since LHS = RHS, $x = 10$ is correct.

Exercise 5

Solve these equations.

(a) $x+6=13$ (b) $y-8=5$ (c) $5x+4=19$

(d) $7y-2=12$ (e) $\frac{x}{5}+2=17$ (f) $\frac{3}{5}y-3=12$

Example 5

Solve the equation $4(x-2) = 12$

[Answer]

There are **two methods** of solving this equation.

Method 1

$$4(x-2) = 12$$
$$4x-8=12$$
$$4x-8+8 = 12+8$$
$$4x = 20$$
$$\frac{\cancel{4}x}{\cancel{4}} = \frac{20}{4}$$
$$\boldsymbol{x = 5}$$

Method 2

$$4(x-2) = 12$$
$$\frac{\cancel{4}(x-2)}{\cancel{4}} = \frac{12}{4}$$
$$x-2 = 3$$
$$x-2+2 = 3+2$$
$$\boldsymbol{x = 5}$$

Example 6

Solve for t $\frac{2t+3}{3}=5$

[Answer]

$$\frac{2t+3}{3}=5$$
$$\frac{2t+3}{3}\times 3=5\times 3$$
$$2t+3=15$$
$$2t+3-3=15-3$$
$$2t=12$$
$$\frac{2t}{2}=\frac{12}{2}$$
$$\boldsymbol{t = 6}$$

Exercise 6

Solve these equations.

(a) $5(x-4)=30$ (b) $-2(7+x)=3(1-2x)$

(c) $x-2=\dfrac{x+8}{6}$ (d) $\dfrac{2}{3}x+1=\dfrac{x-1}{4}$

Transposition

$$4x - 15 = 9$$

$$4x - 15 + 15 = 9 + 15$$

$$4x + 0 = 9 + 15$$

$$4x = 24$$

$$\frac{4x}{4} = \frac{24}{4}$$

$$\boldsymbol{x = 6}$$

$$4x - 15 = 9$$

$$4x = 9 + 15$$

$$4x = 24$$

$$\frac{4x}{4} = \frac{24}{4}$$

$$\boldsymbol{x = 6}$$

When -15 moves from the left hand side to the right hand side of the equation, change -15 to +15.

Generally, a term on one hand side can be moved to the other side with changing the sign.

This method is called **Transposition**.

SOLVING SIMPLE INEQUATIONS

Let's try!! Use appropriate inequality signs either < or >.

- $10 □ $2
- dog's weight □ cow's weight
- Australia's Land mass □ Fiji's Land mass

The methods for solving inequations are similar to the methods used for solving equations.

= is used for equation whereas either $<, >, \leq, or\ \geq$ is used for inequation.

The inequality sign does not change when you **add** or **subtract** any term on both sides.

e.g.

3	<	7		5	<	10	
3+4	?	7+4	(add 4 to both sides)	5-4	?	10-4	(subtract 4 from both sides)
7		11		1		6	
7	<	11	(leave the inequality sign)	1	<	6	(leave the inequality sign)

Example 7

Solve the inequation $x - 4 < 2$

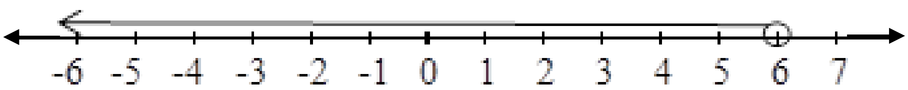
[Answer] $x - 4 < 2$

$x - 4 + 4 < 2 + 4$ Add 4 (+ 4) on both sides of the equation.

$x - \cancel{4} + \cancel{4} < 2 + 4$

$\boldsymbol{x < 6}$

The solution set is 'all numbers less than 6'. The solutions can be displayed on the number line shown below.



Exercise 7

Solve the following inequations.

(a) $x-5>3$ (b) $3+x>11$ (c) $a - 4 \leq -7$ (d) $-5+b \geq 8$

> The inequality sign does not change when **multiplying** or **dividing** on both sides by a **positive** number.
>
> However, **reverse the inequality sign** when dividing or multiplying both sides by a **negative** number.

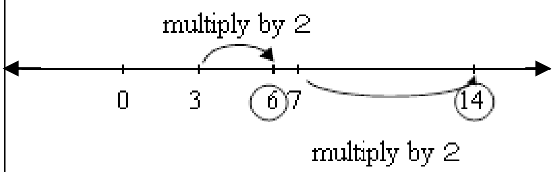
e.g.

$3 < 7$

$3\times 2 \quad ? \quad 7\times 2$ multiply both sides by 2

$6 \quad 14$

$6 < 14$ leave the inequality sign

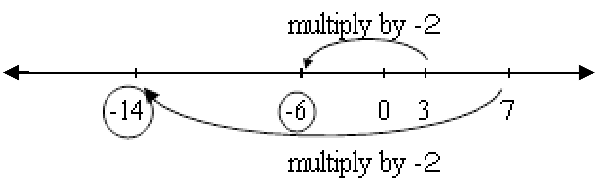


$3 < 7$

$3\times(-2) \quad ? \quad 7\times(-2)$ multiply both sides by -2

$-6 \quad -14$

$-6 > -14$ **reverse** the inequality sign



$6 < 9$

$\frac{6}{3} \quad ? \quad \frac{9}{3}$ (divide by 3 both sides)

$2 \quad 3$

$2 < 3$ leave the inequality sign

$6 < 9$

$\frac{6}{-3} \quad ? \quad \frac{9}{-3}$ (divide by -3 both sides)

$-2 \quad -3$

$-2 > -3$ **reverse** the inequality sign

Example 8

Solve the following inequations.

(a) $3x > 9$ (b) $\frac{x}{4} \le -3$ (c) $-2x \ge 6$ (d) $-\frac{x}{5} > 4$

[Answer]

(a) $3x > 9$

$$\frac{3x}{3} > \frac{9}{3}$$ (divide by 3 both sides)

$$\boldsymbol{x > 3}$$

(b) $\frac{x}{4} \le -3$

$$\frac{x}{4} \times 4 \le 3 \times 4$$ (multiply 4 both sides)

$$\boldsymbol{x \le 12}$$

(c) $-2x \ge 6$

$$\frac{-2x}{-2} \le \frac{6}{-2}$$ (divide both sides by -2)

(reverse the sign)

$$\boldsymbol{x \le -3}$$

(d) $-\frac{x}{5} > 4$

$$-\frac{x}{5} \times (-5) < 4 \times (-5)$$ (multiply by -5 both sides)

(reverse the sign)

$$\boldsymbol{x < -20}$$

Exercise 8

Solve the following inequations.

(a) $2x > 6$ (b) $-5x \leq -15$ (c) $\frac{x}{2} \geq 6$ (d) $-\frac{x}{4} < -\frac{3}{2}$

Example 9

Solve the following inequations and show the solutions on a number line.

(a) $3x + 1 \geq 10$ (b) $\frac{2-x}{3} > 1$

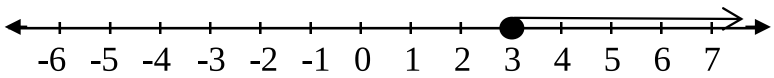
[Answer]

(a) $3x + 1 \geq 10$

$3x \cancel{+1} \cancel{-1} \geq 10 - 1$ (subtract 1 from both sides)

$3x \geq 9$

$\frac{\cancel{3}x}{\cancel{3}} \geq \frac{9}{3}$ (divide both sides by 3)



$x \geq 3$

(b) $\frac{2-x}{3} > 1$

$\cancel{3} \times \frac{2-x}{\cancel{3}} > 1 \times 3$ (multiply both sides by 3)

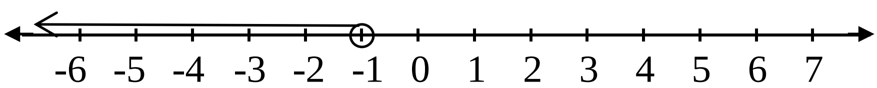
$2 - x > 3$

$\cancel{2} - x \cancel{-2} > 3 - 2$ (subtract 2 from both sides)

$-x > 1$

$\frac{-x}{-1} < \frac{1}{-1}$ (divide both sides by -1)
(reverse the sign)

$x < -1$



Exercise 9

Solve the following inequations.

(a) $5x + 8 \geq 23$ (b) $-6x - 5 \leq -17$

(c) $\frac{3+2y}{5} < -7$ (d) $8 - \frac{2y}{3} \geq -4$

Practice 2

1. Solve the following equations.

   (a) $t - 2 = 6$ (b) $r + 7 = 19$ (c) $y - 4 = 3$

2. Solve the following equations.

   (a) $2y - 3 = 9$ (b) $3h + 6 = 9$ (c) $5m - 2 = 8$

3. Solve the following equations.

   (a) $\frac{x}{4} = 3$ (b) $\frac{t}{3} = -4$ (c) $\frac{u}{2} = 7$

4. Solve the following equations.

   (a) $\frac{a}{2} - 5 = 4$ (b) $\frac{w}{3} + 3 = 8$ (c) $\frac{p}{5} - 2 = 1$

   (d) $\frac{3}{4} - y = 6$ (e) $3(w - 4) = 9$ (f) $2(2 - p) = 10$

   (g) $5(3 + h) = 33$ (h) $\frac{n-3}{2} = 10$ (i) $\frac{2r+2}{3} = 12$

5. Solve the following equations.

(a) $\frac{x+3}{2}=0$ (b) $\frac{y-4}{3}=0$ (c) $\frac{3-r}{2}=0$

(d) $\frac{t}{3}=\frac{2}{5}$ (e) $\frac{3}{2}=\frac{k}{4}$ (f) $\frac{-1}{2}=\frac{h}{8}$

(g) $\frac{t}{3}-\frac{2}{3}=0$ (h) $\frac{3}{5}+\frac{k}{6}=0$ (i) $\frac{-1}{3}-\frac{h}{9}=0$

6. Solve the following inequations and show the solutions on number line.

(a) $2t-1<5$ (b) $p+2>3$ (c) $2-r\leq 5$

7. John is two years older than his brother Edward. If their combined age is 18years, how old is Edward?

8. Asish collected three times as much money as Vikash did in their school organised fundraising. If Vikash collected $80.00, how much money was collected by Asish?

9. Maciu bought three times more lollies than his cousin Epeli. The shopkeeper gave Maciu 4 more lollies as a gift. If Maciu had 22 lollies altogether, how many lollies did Epeli buy?

Review Exercise

1. Simplify the following expressions.

(a) $3x + 4x$ (b) $9x - x$ (c) $-8x + 5x + 3x$

(d) $5x + 7y + 3x + 2y$ (e) $-2a - 3b - 8a + 7b$ (f) $\frac{6}{7}x - y - \frac{1}{2}x + \frac{1}{3}y$

2. Simplify the following expressions.

(a) $(-5a) \times (-2a)$ (b) $27x^3y^2 \div 9xy^2$ (c) $(-2x^2)^3$

(d) $(3x)^2 \times (-xy^2)^3$ (e) $-2xy \div \left(-\frac{4}{3}xy^2\right) \times 8x^2y$ (f) $\frac{64x^6y^3}{16x^4y^3}$

3. Simplify the following expressions.

(a) $2(3x - y)$ (b) $a^2(b - c^2)$ (c) $3x + 9 - 2(x - 1)$

(d) $x^2(3x^2 + 5x - 4) - 2x^2(x^2 + 3x + 5)$ (e) $12\left(\frac{x-2}{4} - \frac{2x-3}{3}\right)$

4. Solve the following equations.

(a) $x + 5 = 13$ (b) $p - 1 = 8$ (c) $q - 6 = 9$

(d) $\frac{x}{7} = 3$ (e) $-\frac{y}{4} = -5$ (f) $6r = -18$

5. Solve the following equations.

(a) $6y - 8 = 10$ (b) $3 - 4n = 11$ (c) $2 - 5c = 27$

6. Solve the following equations.

(a) $4+\frac{h}{2}=10$ (b) $2-\frac{c}{5}=3$ (c) $\frac{2}{3}-\frac{d}{6}=\frac{1}{3}$

(d) $5(x+2)=-15$ (e) $-3(2p-5)=9$ (f) $8-3(2x-1)=11$

(g) $\frac{3k-1}{2}=10$ (h) $\frac{9n-5}{6}=\frac{11}{3}$ (i) $\frac{x+3}{3}=0$

7. Luisa wants to buy suru. If she buys 7 metres, she is 1 dollar short. If she buys 5 metres, 5 dollars is left over. How much does she have?

8. Solve the following inequations.

(a) $x+3<4$ (b) $t-5 \geq -3$ (c) $\frac{s}{5}\leq 3$ (d) $-\frac{u}{2}\geq\frac{3}{4}$

9. Solve the following inequations and show the solutions on number line.

(a) $2-3m < -4$ (b) $1-2t \geq 3$ (c) $20+7a > -1$

(d) $\frac{x+7}{2}\geq 3$ (e) $\frac{7x-3}{5}\leq 5$ (f) $\frac{8-3x}{4}\leq -1$

10. You have \$30 to buy muffins. However, at the shop you can buy 1 muffin for \$4.

(a) Write an inequality from the above information.

(b) Solve this inequality to find the number of muffins you can buy.

(c) How much money will you have left over?

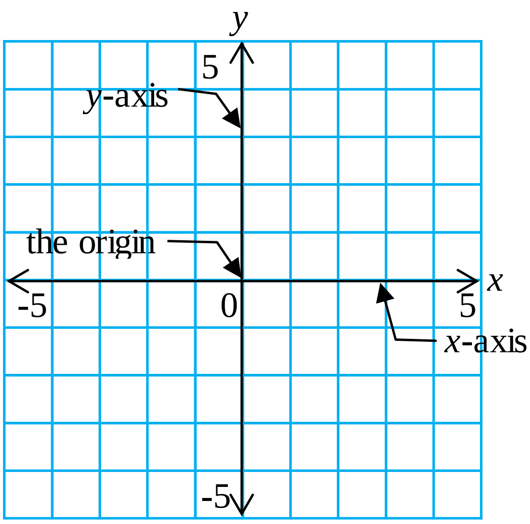
UNIT 4

SIMPLE LINEAR EQUATIONS AND INEQUATIONS

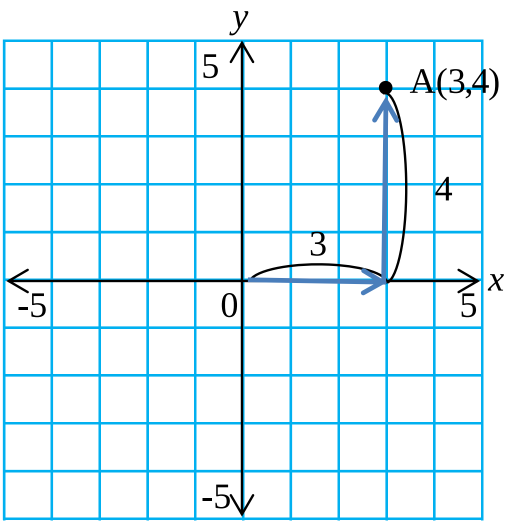
UNIT 4 SIMPLE LINEAR EQUATIONS AND INEQUATIONS

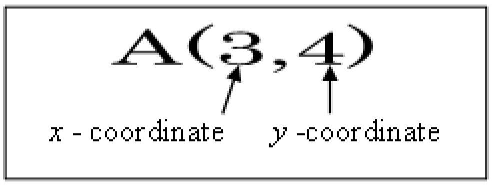
THE CARTESIAN PLANE

The locations of points can be sketched on a **Cartesian Plane ($x - y$ plane).**
The **Cartesian Plane** has two axes. The horizontal number line is called the **x – axis** and the vertical number line is called the **y – axis.**



The location of a point is represented as an ordered pair (x, y).
The x is known as the **x – coordinate** and the y is known as the **y – coordinate.**



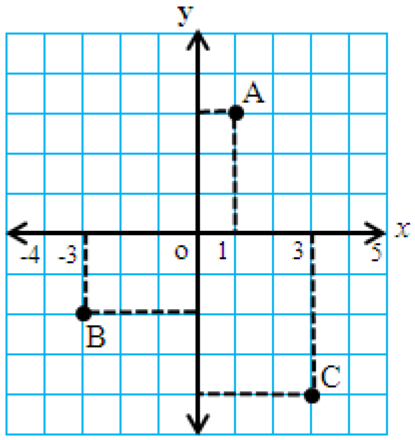


Origin is the point where the x and y – axes intersect. The coordinates of the origin are (0, 0).
The **x – coordinate** is **always** written first then followed by the **y – coordinate.**

Example 1

On a Cartesian Plane below,

A (1,3) B (-3,-2) C (3,-4) O (0,0)



Exercise 1

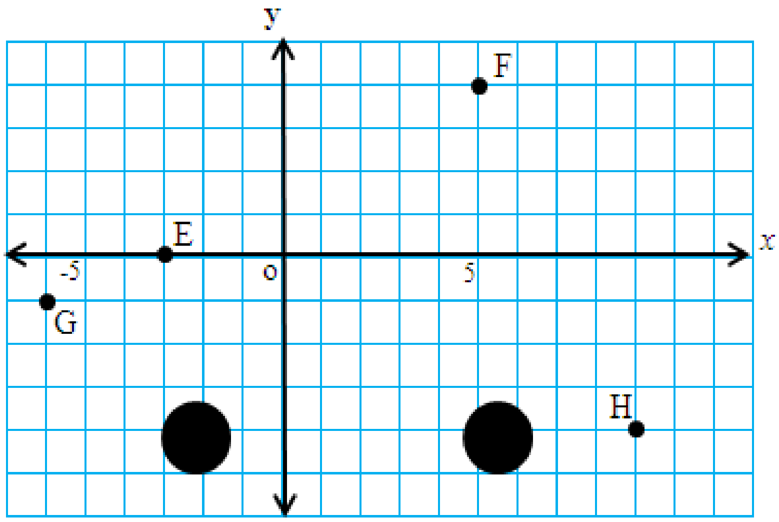
1. Plot the following points on the Cartesian Plane below.

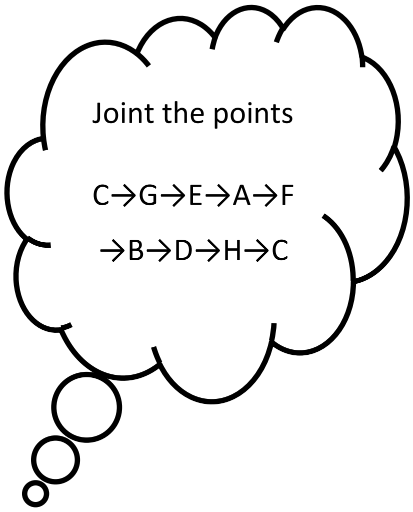
   A (-2,4) B (7,0) C (-5,-4) D (10,-1)

2. Find the coordinates of the following points.

   E (▭ , ▭) F (▭ , ▭)

   G (▭ , ▭) H (▭ , ▭)

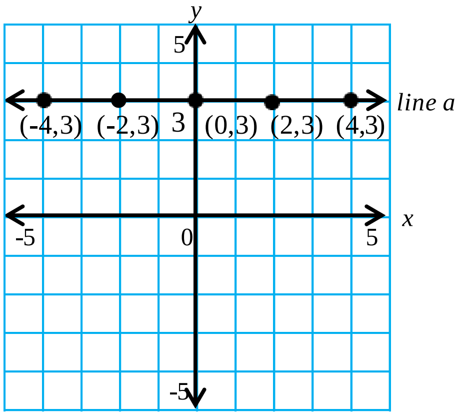




VERTICAL AND HORIZONTAL LINES

Example 2

(1)

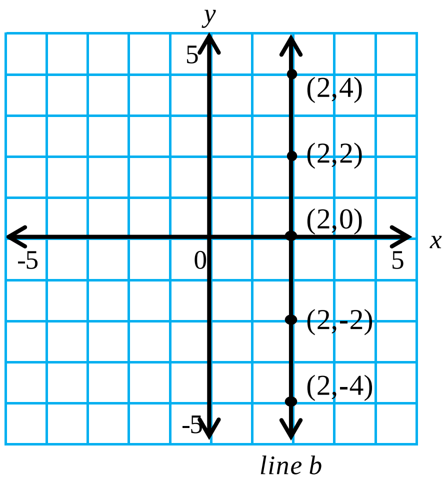


For all *x*, the *y* coordinates are 3. The equation of ***line a*** is *y* = 3.

y = 3 is a horizontal line parallel to the *x* – axis that cuts the ***y*** **– axis** at the point (0, 3).

The point is called ***y*** **– intercept.**

(2)



For all *y*, the *x* coordinates are 2. The equation of ***line b*** is *x* = 2.

x = 2 is a vertical line parallel to the *y* – axis that cuts the ***x*** **- axis** at the point (2, 0).

The point is called ***x*** **– intercept.**

Horizontal line;	*y* = a
Vertical line;	*x* = b

x **– intercept** is where the graph intersects the *x* – axis.

y **– intercept** is where the graph intersects the *y* – axis.

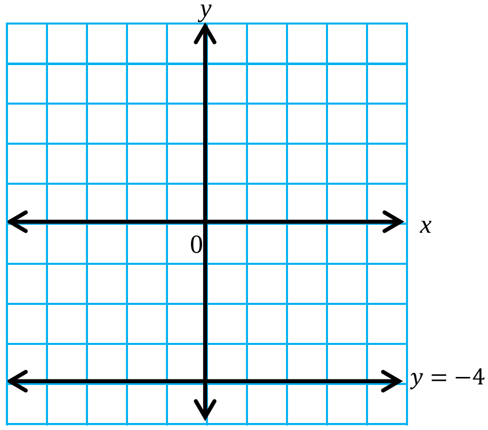
Exercise 2

(a) Sketch the graph of $y = -1$

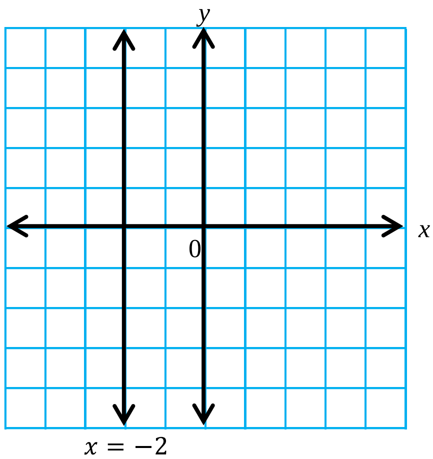
(b) Sketch the graph of $x = 1$

(c) Draw the graph of a line parallel to the x – axis and passing through the point $(0,-2)$.

(d) For the graph given below, find the coordinates of the y – intercept.



(e) For the graph shown below, find the coordinates of the x – intercept.



REGIONS INDICATED BY INEQUALITIES

The values of ***x*** can be shown on a number line which has already been explained earlier and as well as on a **Cartesian Plane.**
On a Cartesian plane we express the inequation in the form of ordered pairs

$$\text{e.g. } \{ (x, y): x \leq 2 \}$$

Graphing of Inequalities

The following are steps in graphing in inequalities on Cartesian plane

Step1

(i) Identify the axis involved.

(ii) Locate the number on the axis through which the lines would be drawn.

(iii) Decide whether the line drawn through the number located.

$<$	or	$>$	a dotted line	....
$\leq$	or	$\geq$	a solid line	______________

Step2

The shading of the graph is determined by the sign $<$ or $>$.
The equal sign only determines the dotted or solid line.

Example 3

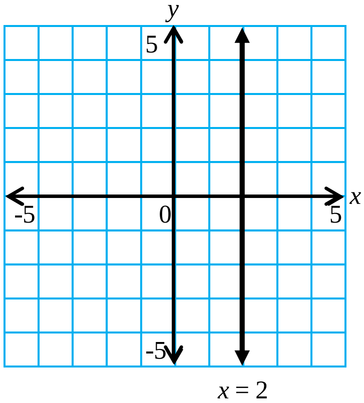
(a) Draw the graph of $\{(x, y): x \leq 2\}$

(b) Draw the graph of $\{(x, y): y > -1\}$

[Answer]

(a) <u>Step1</u> Draw a line $x = 2$

$x \leq 2$ has $\leq$ sign so the line is solid.

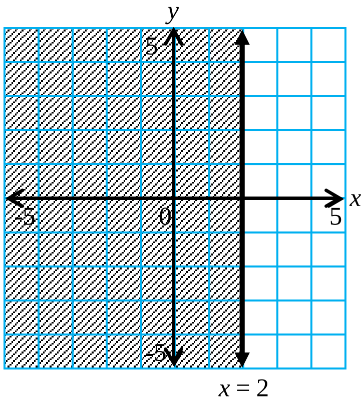


<u>Step2</u> Shade the region

$x <$ (value) or $x \leq$ (value) …to the left

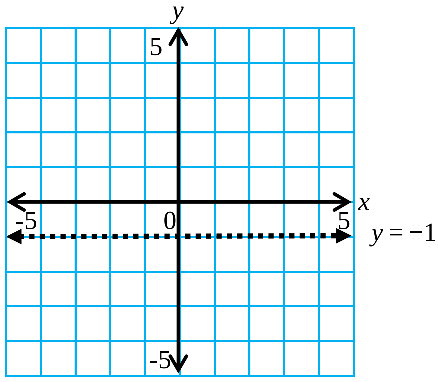
$x >$ (value) or $x \geq$ (value) …to the right

$x \leq 2$ means x less than equal



(b) <u>Step1</u> Draw a line $y = -1$

$y > -1$ has $>$ sign so the line is dotted.

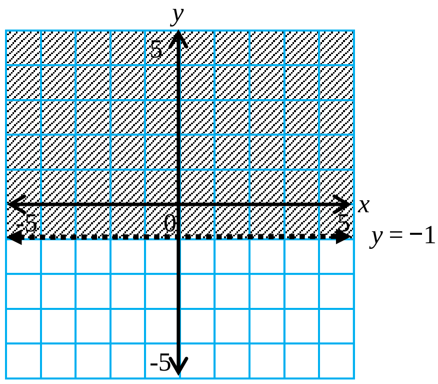


<u>Step2</u> Shade the region

$y <$ (value) or $y \leq$ (value) …below

$y >$ (value) or $y \geq$ (value) …above

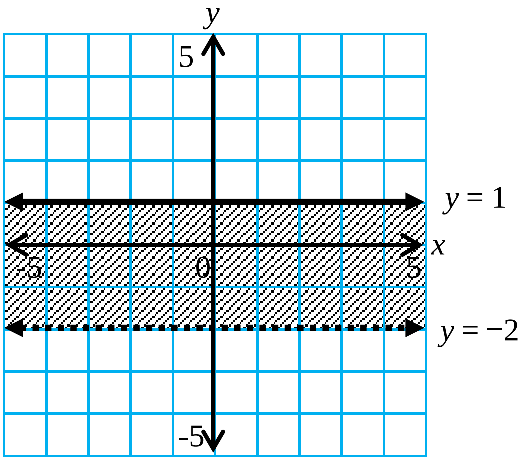
$y > -1$ means y less than equal



Example 4

Draw the graph of the function $\{(x, y)\text{: -2} < y \leq 1\}$

[Answer]



Exercise 3

On a Cartesian plane, draw the graph of the following region.

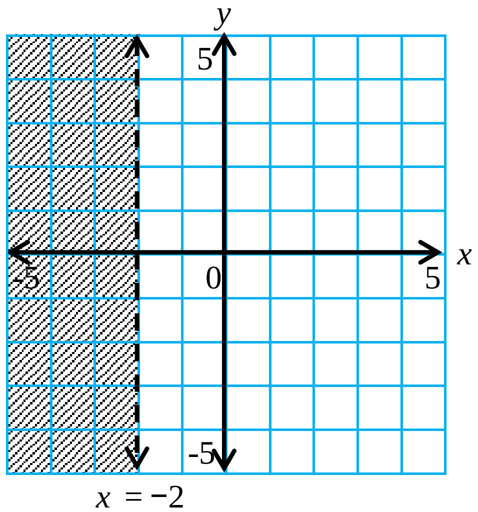
(a) $\{(x, y) : x > 1\}$

(b) $\{(x, y) : y \leq -2\}$

(c) $\{(x, y) : -2 < x \leq 4\}$

Example 5

Express the graph shown below in a set builder notation.



[Answer]

The line drawn is a **dotted line** , so either $<$ *or* $>$ is used. The area shaded is below -2 which means that $<$ is the inequality used to describe the shaded area.

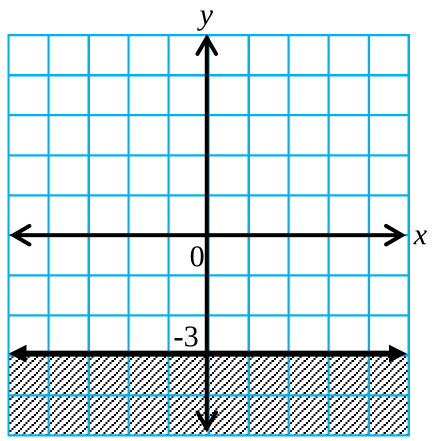
Therefore the answer is given as

$$\{\,(x, y): x < -2\,\}$$

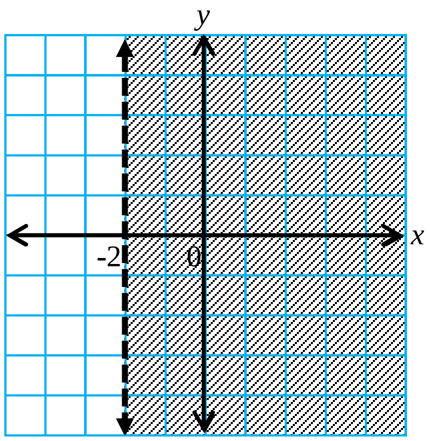
Exercise 4

Express each of the following in set builder notation.

(a)



(b)



Review Exercise

1. Sketch the graph of the following functions.

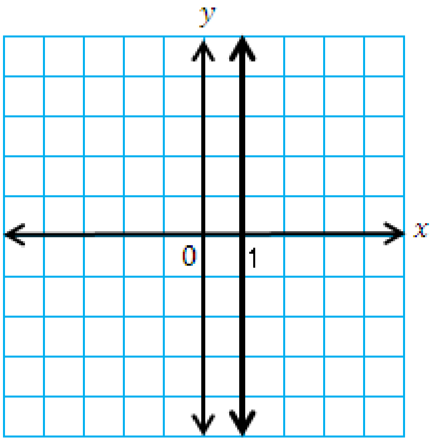
   (a) $x = 2$ (b) $x = -2$ (c) $x = 0$

2. Sketch the graph of the following functions.

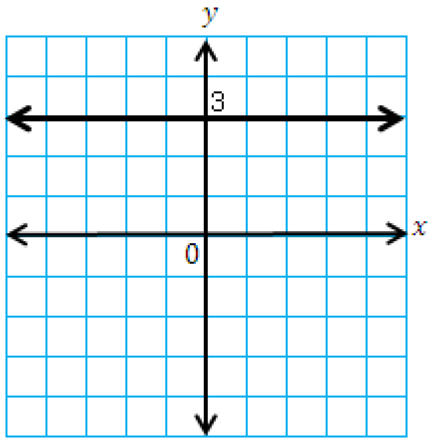
   (a) $y = -2$ (b) $y = 3$ (c) $y = 5$

3. Find the equation of each of the following graphs.

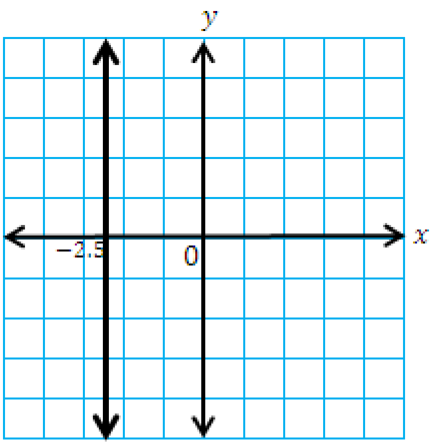
(a)



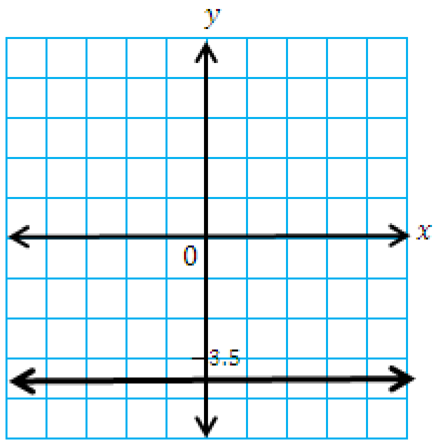
(b)



(c)



(d)



4. Answer the following questions

(a) Sketch the graph of the line parallel to the x – axis and passes through the point (0, -1)

(b) Sketch the graph of the line parallel to the x – axis and passes through the point (0, 4.5)

(c) Sketch the graph of the line parallel to the y – axis and passes through the point (-3, 0)

(d) Sketch the graph of the line parallel to the y – axis and passes through the point (0.5, 0)

5. Sketch the graphs of the following functions on Cartesian plane.

(a) $\{ (x, y): y \geq 2 \}$

(b) $\{ (x, y): x \leq -1 \}$

(c) $\{ (x, y): y > -3 \}$

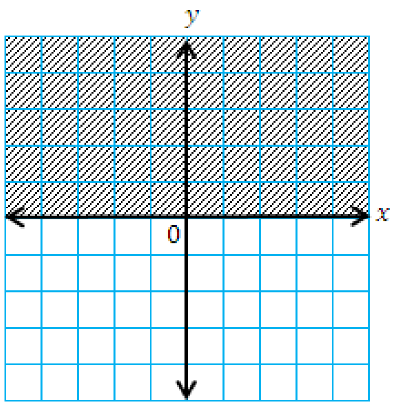
(d) $\{ (x, y): x \leq 3 \}$

(e) $\{ (x, y): -3 < x \leq 2 \}$

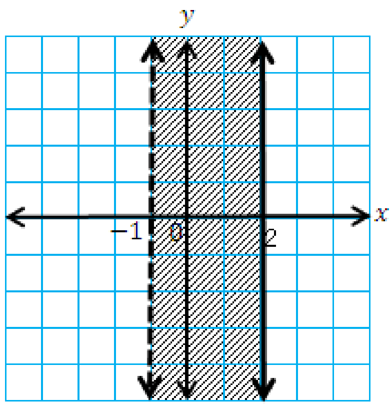
(f) $\{ (x, y): -2 \leq y < 3 \}$

6. Express each of the following in set builder notation

(a)



(b)



UNIT-5

SOCIAL MATHS

UNIT 5 **SOCIAL MATHEMATICS**

MONEY CALCULATIONS IN PERCENTAGES

Percentage (%) means per hundredth or items are measured out of a hundred.

e.g. 3% means the ratio $\frac{3}{100}$ or 0.03.

$$\boldsymbol{Perentage = \frac{Quantity\ being\ compared}{Total\ quantity} \times 100}$$

Example 1

(a) Express 13% as a fraction.

[Answer] $\boldsymbol{\frac{13}{100}}$

(b) Express $\frac{3}{4}$ as a percentage.

[Answer] $\frac{3}{4} \times 100 = \mathbf{75}\%$

(c) Express 37% as a decimal.

[Answer] Just divide 37 by 100 $\frac{37}{100}$ = **0.37**

(d) Express 0.47 as a percentage.

[Answer] **47%**

Example 2

(a) Work out 45% of \$200.00.

(b) What percentage of \$50.00 is \$20.00?

[Answer]

(a) $\frac{45}{100} \times \$200.00 = \$\mathbf{90.00}$

(b) $\frac{\$20.00}{\$50.00} \times 100 = \mathbf{40}\%$

Exercise 1

1. Express the following as percentages.

   (a) $\frac{1}{4}$ (b) $\frac{3}{10}$ (c) 0.2 (d) 0.12

2. Express the following percentages as decimals.

   (a) 4% (b) 12% (c) 41.6%

3. Express the following percentages as fractions.

   (a) 54% (b) 40% (c) 8%

4. Work out the following.

   (a) 8% of \$200.00 (b) 10% of \$50.00 (c) 2 % of \$ 400.00

5. Work out the percentage for each of the following.

   (a) \$35.00 of \$100.00 (b) \$30.00 of \$150.00 (c) \$18.00 of \$45.00

Example 3

(a) Increase \$20.00 by 10%.

(b) Decrease \$400.00 by 25%.

[Answer]

(a)

Method 1

First work out 10% of \$20.00.

$$\frac{10}{100} \times \$20.00 = \$2.00$$

Adding 10% of \$20.00 to the \$20.00 will give the answer required.

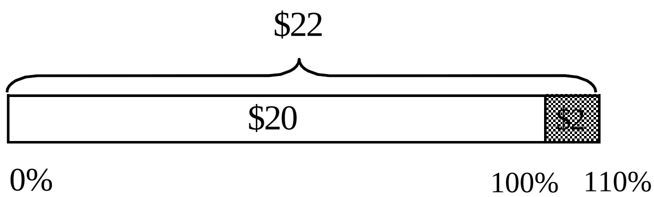
\$2.00 + \$20.00 = **\$22.00**

Method 2

\$20.00 is equivalent to 100% and an increase by 10% will give us a total of 110%.

$$110\% \text{ of } \$20.00 = \frac{110}{100} \times \$20.00$$

$$= \mathbf{\$22.00}$$



(b)

Method 1

Work out 25% of \$400.00.

$$\frac{25}{100} \times \$400.00 = \$100.00$$

Subtract 25% of \$400.00 from \$400.00.

\$400.00 − \$100.00 = **\$300.00**

Method 2

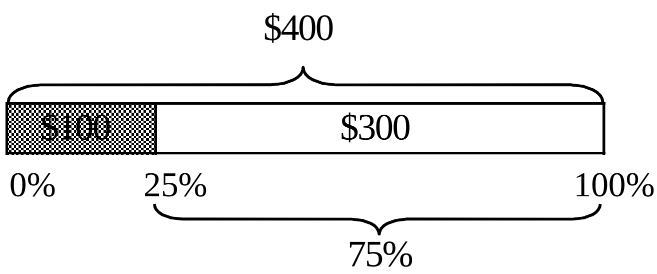
If the amount is decreased by 25%, the new amount should be 75%.

100% − 25% = 75%

To decrease \$400 by 25% simply means to work out 75% of \$400.00.

$$\therefore\ 75\% \text{ of } \$400 = \frac{75}{100} \times \$400$$

$$= \mathbf{\$300.00}$$



Exercise 2

Work out the following.

(a) Increase \$150.00 by 10%.

(b) Increase \$500.00 by 30%.

(c) Decrease \$100.00 by 18%.

(d) Decrease \$50.00 by 20%.

Example 4

The cost of sugar increased from \$1.00 to \$1.05 a kilogram.

Work out the increase cost as a percentage.

[Answer]

The increase cost (new price – original price) is

$$\$1.05 - \$1.00 = \$0.05$$

$$\text{The increase cost as a percentage} = \frac{\text{Increase cost}}{\text{Original price}} \times 100$$

$$= \frac{\$0.05}{\$1.00} \times 100$$

$$= 5$$

The increase cost as a percentage = 5%.

Exercise 3

The cost of an apple has decreased from \$0.50 to \$0.40. Work out the decrease cost of an apple as a percentage.

Example 5

A house that was bought two years ago for \$80, 000.00 was sold at a profit of 25%. Work out the selling price of the house.

[Answer]

The selling price = profit + original price

Method 1

Work out the profit

25% of \$80,000 = $\frac{25\%}{100\%} \times$ \$80, 000.00

= \$20,000.00

The selling price

= profit + original price

= \$20, 000.00 + \$80, 000.00

= \$100, 000.00

Method 2

If the house is sold at a profit of 25%, the selling price should be 125% of original price.

100% + 25% = 125%

$$\frac{125\%}{100\%} \times \$80{,}000.00 = \$\mathbf{100,000.00}$$

Exercise 4

A house was sold at a profit of 40%. If the original cost of the house was \$50,000.00, work out the following

(a) 40% of \$50,000.00

(b) The selling price of the house

Example 6

A new car is sold for \$50,000.00 and its deposit is \$20,000.00. Work out the deposit of the car as a percentage.

[Answer]

$$\boldsymbol{Perentage = \frac{Quantity\ being\ compared}{Basic\ quantity} \times 100}$$

$$\frac{\$20{,}000.00}{\$50{,}000.00} \times 100 = \mathbf{40}\%$$

Practice 1

1. Express the following as percentages.

   (a) $\frac{3}{5}$ (b) $\frac{9}{10}$ (c) 0.009 (d) 1.35

2. Express the following percentages as decimals.

   (a) 87.2% (b) 0.6% (c) 123.76%

3. Express the following percentages as fractions.

   (a) 48% (b) 0.1% (c) 72%

4. Work out the following.

   (a) 25% of $60.00 (b) 12% of $300.00 (c) 23% of $800.00

5. Express as percentages.

   (a) $3 of $10 (b) $24 of $40 (c) $90 of $225

6. Work out the following.

   (a) Increase $40.00 by 50% (b) Increase $90.00 by 25%

   (c) Decrease $800.00 by 18% (d) Decrease $10.00 by 22%

7. A car that was bought for $12, 000.00 in 2006 was sold for $8, 000.00 in 2010.

   (a) How much did the car owner lose?

   (b) Work out the percentage loss.

8. In an advertisement, any customer who wishes to purchase a brand new DVD player from their company would be given a 12 % discount. The cost of the DVD player is $300.00.

   (a) Work out 12% of $300.00.

   (b) How much would a customer pay for the DVD player?

9. Vimlesh bought the following items from a supermarket near his home:

Flour	$12.00
Rice	$4.00
Eggs	$6.00
Washing soap	$2.00

(a) Work out the total cost of items bought by Vimlesh.

(b) What percentage of the total cost was spent on flour?

(c) What percentage of the total cost was spent on rice, flour and washing soap?

10. A house was sold at a profit of 35%. If the original cost of the house was $60,000.00, work out the following.

(a) 35% of $60,000.00

(b) The selling price of the house

RATIOS, PROPORTIONS AND RATES

Ratio: It is the division of two quantities with the same unit.

It is often expressed as a fraction or may also be expressed with a colon which is in the form of ***a : b*** .
The ration must be in its simplest form.

Example 7

In a class of 40 students, there are 16 boys and 24 girls. Write the ratio of boys to girls.

[Answer]

16 boys and 24 girls = 16 : 24

$= \frac{16}{8} : \frac{24}{8}$ (Divide by 8, the highest common factor)

$= \mathbf{2 : 3}$ **or** $\mathbf{\frac{2}{3}}$

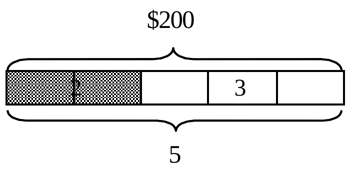
Exercise 5

Express the following in their simplest ratios.

(a) 20 : 12 (b) 25: 15 (c) 10: 20

Example 8

Two brothers, Tom and Paul, share $200.00 in the ratio 2 : 3. How much does each receive?



[Answer]

Tom : Paul 2: 3

Tom receives $\frac{2}{2+3} \times \$200.00 = \frac{2}{5} \times \$200.00$

$= \mathbf{\$80.00}$

Paul receives $\frac{3}{2+3} \times \$200.00 = \frac{3}{5} \times \$200.00$

$= \mathbf{\$120.00}$

Example 9

The ratio of Physics students to Geography students at Vuniniu Secondary School is 3: 5. If 20 students are taking Geography, how many students are taking Physics?

[Answer]

Represent physics students as *p*

Physics : Geography

3 : 5 = p : 20

Converting these into fractions will give us

$$\frac{3}{5} = \frac{P}{20}$$

$$3 \times 20 = p \times 5$$

$$60 = 5p$$

$$12 = p \qquad \text{(divide both sides by 5)}$$

∴ **12 students** are taking Physics.

Exercise 6

1. Divide \$300.00 in the following ratios

   (a) 1 : 2 (b) 2: 3 (c) 1 : 5

2. Divide the following in the ratio 2 : 3.

   (a) \$50.00 (b) 80 coconuts (c) 75 litres of petrol

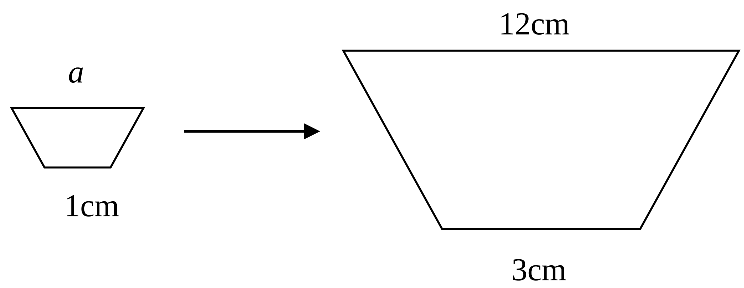
Proportion: It is the comparing and equating of two ratios

Example 10

Figure *1* is enlarged to Figure *2*. Work out the length of the side marked *a*.

[Answer]

Figure 1 Figure 2



$$\frac{1}{a} = \frac{3}{12}$$

$$a \times 3 = 1 \times 12$$

$$3a = 12$$

$$a = \mathbf{4} \qquad \text{(divide both sides by 3)}$$

Example 11

Two men made an investment in a business. Mr. Peters contributed \$10,000.00 and Mr. Thomas contributed \$6,000.00.

(a) Express the two men's contributions as ratio in its simplest form.

(b) If they were going to share the profit using the above ratio, how much would each man receive if their profit was \$ 9,000.00?

[Answer]

(a) Mr. Peters : Mr. Thomas = 10000.00 : 6000.00

= 10 : 6 (Divide by 1000)

= **5 : 3** (Divide by 2)

(b) Mr. Peter's share is $\frac{5}{8} \times$ \$9000.00 = **\$5625**

Mr. Thomas' share is $\frac{3}{8} \times$ \$9000.00 = **\$3375**

Exercise 7

1. A square of length 6 cm was enlarged to 15cm. By what ratio was it increased?

2. Two business men Mr. A and Mr. B invested a total of $10, 000.00 in a business. The ratio of the amount of their contributions was 3: 1. How much did Mr. B invest?

Rate:

It is comparing two quantities with different units in which the denominator is **1 unit.**

e.g. Speed (kilometers per hour), Price (dollars per kilogram), Wage (dollars per day)

Example 12

A car travels a distance of 600m in 10 seconds. Work out its average speed.

[Answer]

Average speed = $\frac{\text{Distance}}{\text{Time}}$

$= \frac{600\text{m}}{10\text{s}}$

$= \frac{60m}{1s}$

= **60m/s**

Example 13

An employee of a security firm receives $3 in an hour. He receives a weekly wages of $120.00. How many hours does he work?

[Answer]

Represent his working hours as *h*.

$$\frac{120}{h} = \frac{3}{1}$$

$$120 = 3h$$

$$h = 40$$

∴ **He works for 40 hours a week.**

Practice 2

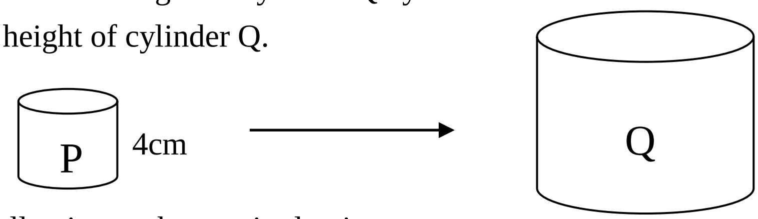
1. Express the following in their simplest ratios.
   (a) 6: 18 (b) 45 : 20 (c) 3 : 9

2. Divide the following in the ratio 3 : 5.
   (a) 160 pineapples (b) 72 cars (c) 400 boys

3. Increase $32.00 in the ratio 2 : 5.

4. Decrease 54 litres in the ratio 5 : 1.

5. Cylinder P has been enlarged to cylinder Q by scale factor 3.
   Work out the height of cylinder Q.



6. Convert the following to the required unit.
   (a) 40 m to cm (b) 3km to m
   (c) 5 hrs to mins (d) 6000secs to hrs
   (e) 3.5 hrs to secs (f) 4km/hr to m/s

7. A car travels at a constant speed of 20m/s. Work out the distance travelled [in metres], by the car after
   (a) 4 seconds (b) 12 seconds
   (c) 3 minutes (d) 2 hours

8. If Mr. Bula bought 12 kg of sugar for $13.80, what would be the cost of 3 kg sugar?

9. A long distance runner ran 10, 000 metres in 28 minutes. Work out his speed in
   (a) metres / min (b) metres /sec

10. Mr. Toro's starting wages was $116.00 weekly for a period of 40hours.
    (a) Work out Mr. Toro's hourly rate.

    (b) What would be Mr. Toro's weekly wages for the 40 working hours if he received a pay rise of $3.25 per hour?

Review Exercise

1. Express the following as percentages.

   (a) $\frac{7}{10}$ (b) $\frac{3}{4}$ (c) 0.004 (d) 2.47

2. Express the following percentages as decimals.

   (a) 13.5% (b) 0.3% (c) 129.8%

3. Express the following percentages as fractions.

   (a) 50% (b) 0.9% (c) 84%

4. Work out the following.

   (a) 30% of $50.00 (b) 29% of $400.00 (c) 145% of $450.00

5. Express the following in their simplest ratios.

   (a) 6 : 8 (b) 27 : 18 (c) 85 : 34

6. Divide $480.00 in the following ratios.

   (a) 3 : 1 (b) 3 : 7 (c) 11 : 1

7. A businessman bought 130 bundles of dalo for $1,040.00. He made a profit of $ 2.00 for each bundle of dalo that he sold.

   (a) How much would be the cost of each bundle of dalo bought by the businessman?

   (b) How much would be the cost of each bundle of dalo sold by the businessman?

   (c) How much profit would the businessman gain from the selling of 130 bundles?

   (d) Work out his profit as a percentage.

8. Peni bought a laptop for $1, 500.00 and a few months later, he decided to sell it for $1, 200.00.

   (a) How much money did Peni lose when selling the laptop for $1,200.00?

   (b) Work out the percentage loss.

9. Mr. Richard invested $10,000.00 in a bank for 1 year. The interest rate for that year was 6%.
   (a) How much interest would Mr. Richard collect at the end of that year?
   (b) What would be the total amount collected by Mr. Richard at the end of that particular year?

10. The following marks were achieved by 6 students in a Mathematics test which was out of 80 marks. Convert their marks to percentages.

(a) Jone – 60	(b) Mary – 65	(c) Vikash – 45
(d) Seru – 70	(e) Sami – 50	(f) Tomasi – 32

11. In an examination, a student scored 50% in Mathematics. What was the student's mark if the examination paper was out of 80 marks?

12. Convert the following to the required unit.

(a) 7600m to km	(b) 240secs to mins
(c) 12km/hr to m/s	(d) 36m/s to km/hr

13. An object moving horizontally covers a distance of 450 metres in 6 seconds. What is the speed of the object in m/ s?

14. A boat leaves Beqa at 9.30am and arrives in Suva at 1.30pm.
   (a) Beqa is 40km from Suva. How long does it take for the boat to travel from Beqa to Suva?

   (b) Work out the speed of the boat in km/hr.

   (c) Work out the speed of the boat in metres /sec.

UNIT 6

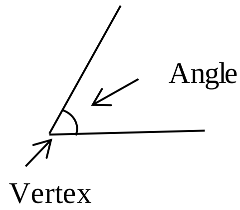
BASIC GEOMETRY 1

UNIT 6 **BASIC GEOMETRY 1**

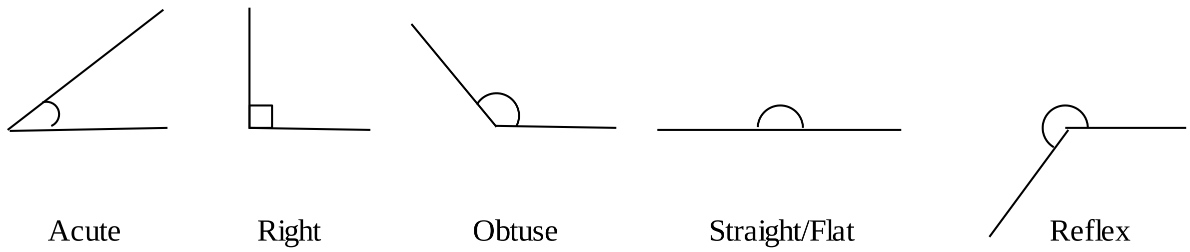
ANGLE

An **angle** is measured between two straight lines that meet at a point.

The point where two lines meet is known as the **vertex**.



TYPES OF ANGLES



Acute	Right	Obtuse	Straight/Flat	Reflex
less than 90°	equal to 90°	between 90° and 180°	equal to 180°	between 180° and 360°

An **acute angle** is less than 90°.

A **right angle** is equal to 90°.

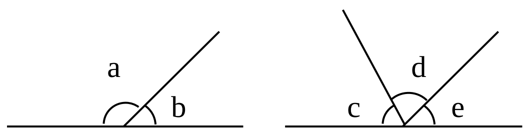
An **obtuse angle** is between 90°and 180°.

A **straight (flat) angle** is equal to 180°.

A **reflex angle** is between 180°and 360°.

PAIRS OF ANGLES

Angles that add up to 180^o are called **supplementary** angles.



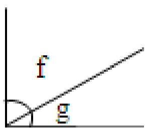
Angles a and b are supplementary

$a + b = 180°$

Angles c, d and e are supplementary

$c + d + e = 180^o$

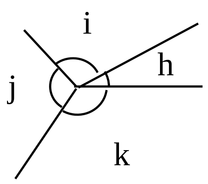
Angles that add up to 90^o are called **complementary** angles.



Angles f and g are complementary

$f + g = 90^o$

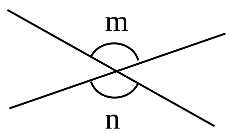
The sum of the angles around a **point** is 360^o.



h, i, j and k are angles around a point

$h + i + j + k = 360^o$

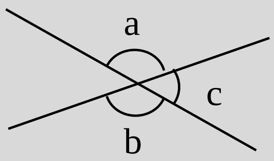
Vertically opposite angles are equal.



Angles m and n are vertically opposite

$m = n$

< Proof >



$a + c = 180°$ (Straight angle)

$a = 180° - c$

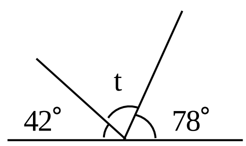
$b + c = 180°$ (Straight angle)

$b = 180° - c$

$\therefore$ **a = b**

Example 1

Work out the angle marked t.



[Answer]

Angles on a straight line are supplementary. Therefore their sum is 180^o.

$42^o + t + 78^o = 180^o$

$42^o + 78^o + t = 180^o$ (put the like terms on LHS together)

$120^o + t = 180^o$

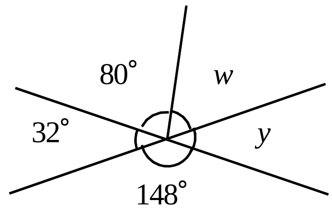
$120^o - 120^o + t = 180^0 - 120^o$ (subtract 120^o on both sides)

$0^o + t = 60^o$

$\mathbf{t = 60^o}$

Example 2

Work out the angle marked *w* and *y*.



[Answer]

Before the above problem is attempted, it is very important to study the diagram very carefully and know what rules to apply.

Method 1

The figure above shows angles around a point. As stated earlier, the sum of angles around a point is equal to 360^o.

$$32^o + 80^o + w + y + 148^o = 360^o$$

$$260^o + w + y = 360^o$$

32^o and y lie opposite each other on two lines intersecting at a point. These angles are called vertically opposite angles and are always equal.

So $\boldsymbol{y = 32^o}$

$260^o + w + 32^o = 360^o$ (substitute the value of *y*)

$260^o + 32^o + w = 360^o$

$292^o + w = 360^o$ (subtract 292^o on both sides)

$292^o - 292^o + w = 360^o - 292^o$

$\boldsymbol{w = 68^o}$

Method 2

Another way of working out the value of *w* is by adding the angles on one of the straight lines where *w* is located.

One straight line will be

$32^o + 80^o + w = 180^o$

$112^o + w = 180^o$

$112^o - 112^o + w = 180^o - 112^o$ (subtract 112^o on both sides)

$\boldsymbol{w = 68^\circ}$

The other straight line will be

$80^o + w + y = 180^o$

$80^o + 68^o + y = 180^o$ (substitute the value of *w*)

$148^o + y = 180^o$ (add the two known angles on LHS)

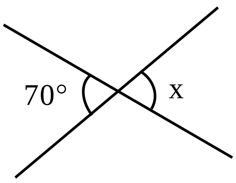
$148^o - 148^o + y = 180^o - 148^o$ (subtract 148^o on both sides)

$\boldsymbol{y = 32^o}$

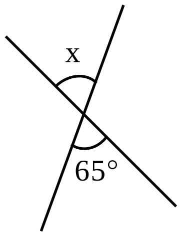
Exercise 1

1. Work out the angles marked with letters.

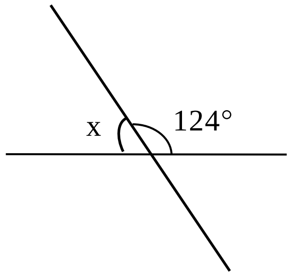
(a)



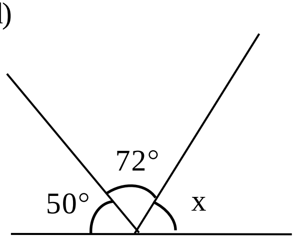
(b)



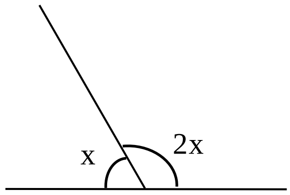
(c)



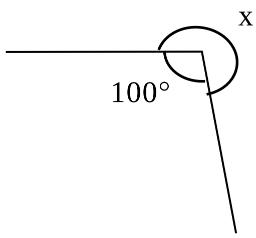
(d)



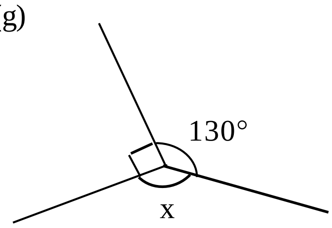
(e)



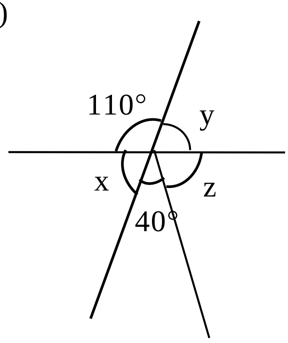
(f)



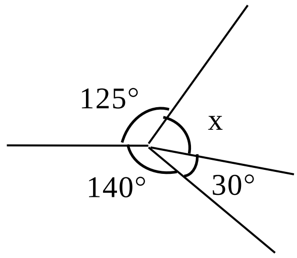
(g)



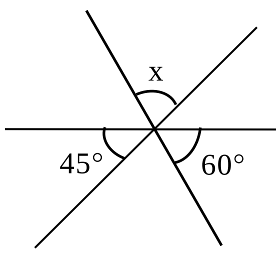
(h)



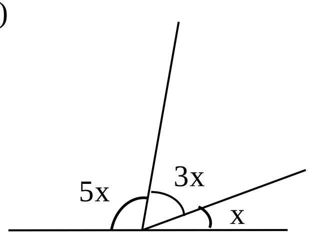
(i)



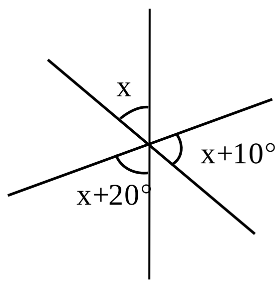
(j)



(k)



(l)



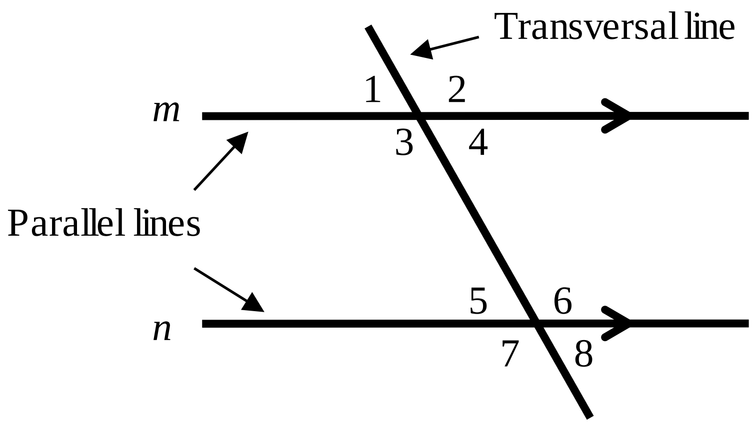
AN ANGLE IN PARALLEL LINE

Two lines that are in the same plane and do not intersect at any point are known as **parallel lines.** Parallel lines will never intersect one another.

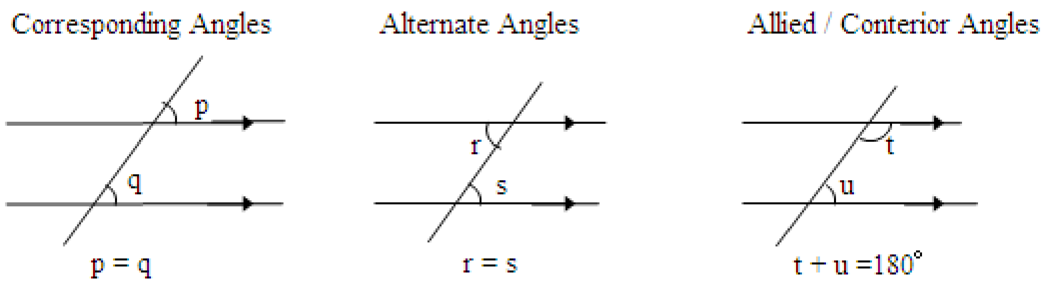
For example, lines *m* and *n* below are parallel lines.

The arrows are placed on the lines to indicate that they are parallel.

A line that intersects two or more parallel lines is known as **transversal line**. Each of the two parallel lines intersected by the transversal line has four angles surrounding each point of intersection.

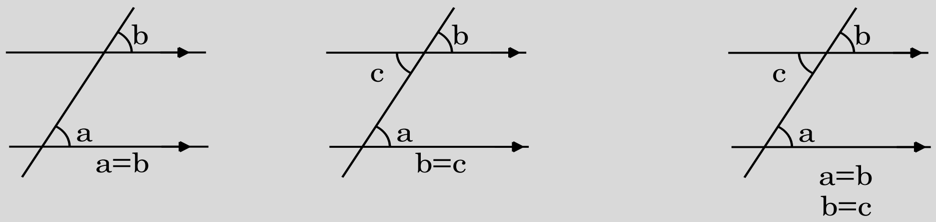


Corresponding angles are always equal.
Alternate angles are always equal.
Allied or cointerior angles are supplementary and that is they add up to 180^o.



< Proof >

Given that corresponding angles and vertical angles are equal, show that alternate angles are equal.



(Given) (Vertical angles are equal) (Substitute c for b)

Exercise 2

In the diagram below:

(a) Find two vertical angles.

(b) Find a pair of corresponding angles.

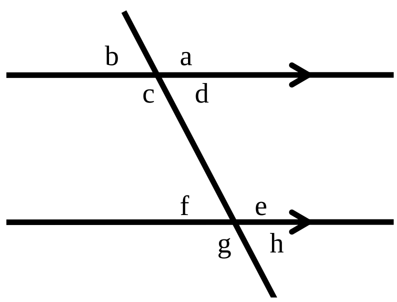
(c) Find a pair of alternate angles.

(d) Find a pair of allied angles.

(e) Name three angles equal to c.

(f) Name two angles that are supplementary to h.

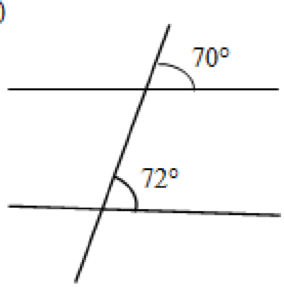
(g) Is h equal to b? Why or why not?



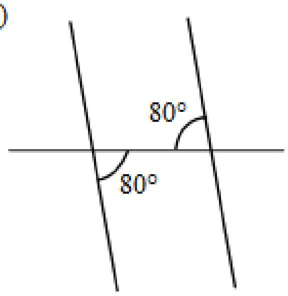
Exercise 3

Tell whether the lines are parallel or not. Give reasons for your answers:

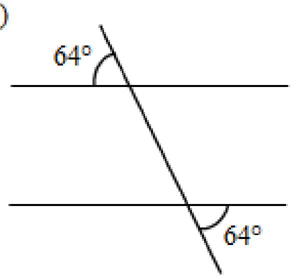
(a)



(b)



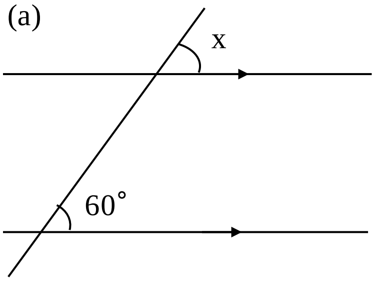
(c)



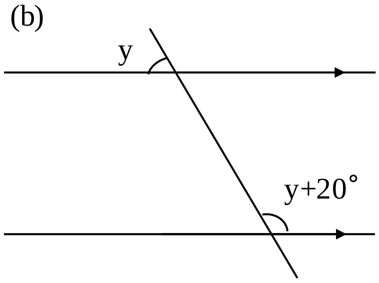
Exercise 4

Work out the sizes of the angles marked with letters.

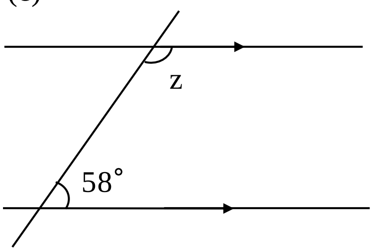
(a)



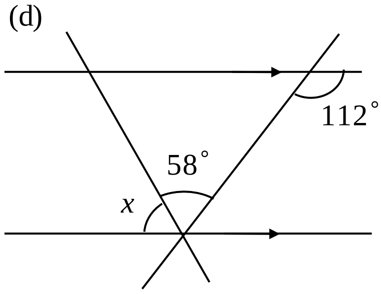
(b)



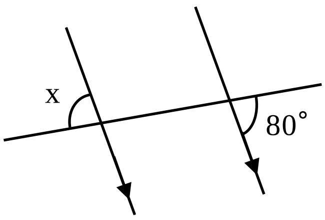
(c)



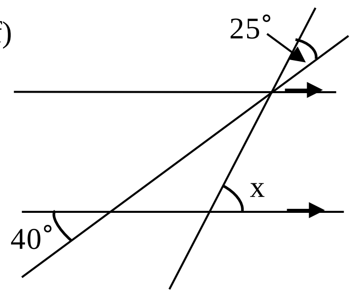
(d)



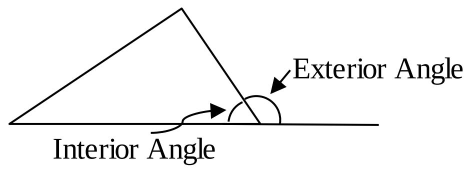
(e)



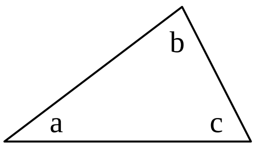
(f)



ANGLES IN A TRIANGLE

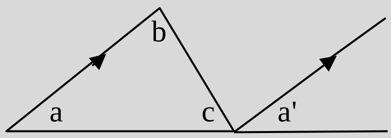


The sum of the interior angles in a triangle is 180°

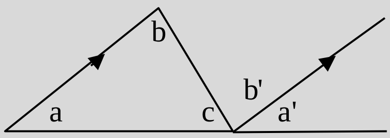


$a+b+c=180^\circ$

< Proof >



a = a'
Corresponding Angle



b = b'
Alternate Angle

$a'+b'+c=180^\circ$

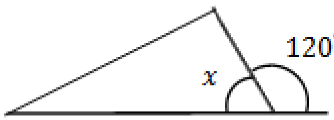
$a = a' \quad b = b'$

$\boldsymbol{a + b + c = 180^\circ}$

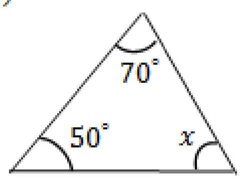
Exercise 5

Work out the angles marked with letters.

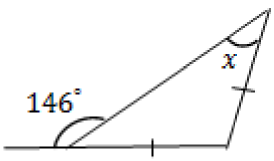
(a)



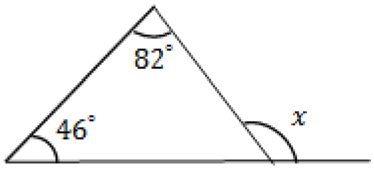
(b)



(c)



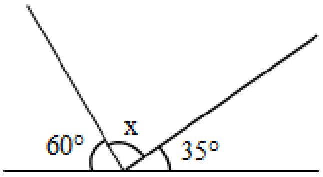
(d)



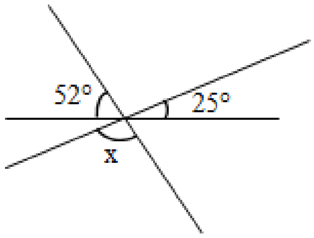
Practice 1

Work out the sizes of the angles marked with letters.

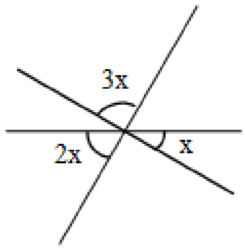
(a)



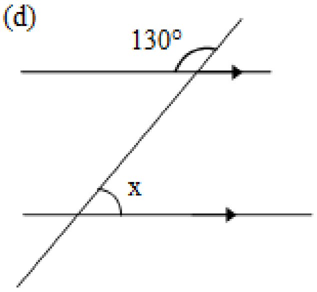
(b)



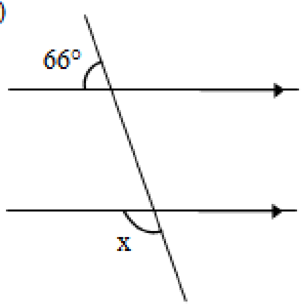
(c)



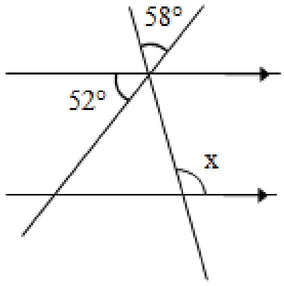
(d)



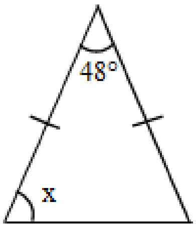
(e)



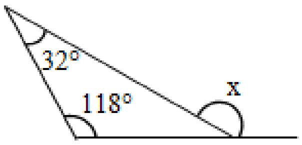
(f)



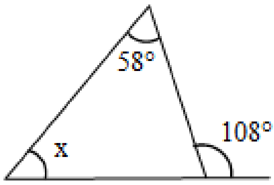
(g)



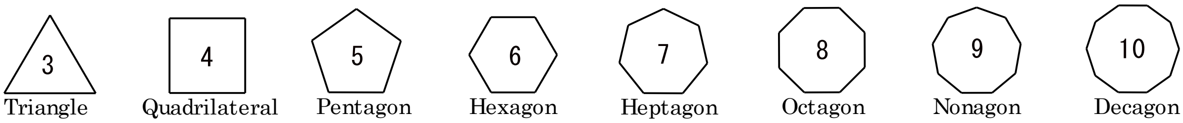
(h)



(i)

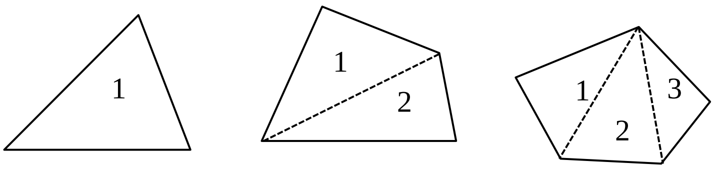


ANGLES IN A POLYGON



The sum of the interior angles in an n-sided polygon is $180^{o} \times (n - 2)$

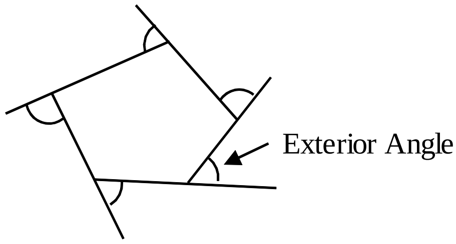
Polygon



Sides	3	4	5	n
Angle	180^{o}	$180^{o} \times 2$	$180^{o} \times 3$	$180^{o} \times (n-2)$

We can divide a polygon into triangles.
The number of triangles is always 2 less than the number of sides.

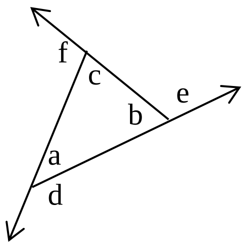
In a polygon, an **exterior angle** is formed by a side and extension of an adjacent side.



The sum of the exterior angles in an n-sided polygon is always equal to 360^{o}

Example 3

Triangle



★ $a + b + c = 180^o$ [Triangle]

☆ $(a + d) + (b + e) + (c + f)$

$= 180^o + 180^o + 180^o = 540^o$

[Straight angles]

$d + e + f =$ ☆ − ★ $= 540^o - 180^o = 360^o$

< Proof >

[The sum of exterior angles] $= 180^o \times n$ – [The sum of interior angles]

$= 180^o \times n - 180^o \times (n - 2)$

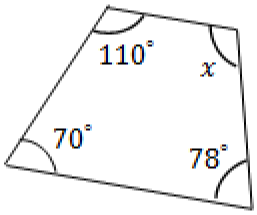
$= 180^o \times n - 180^o \times n + 180^o \times 2$

$= 360^o$

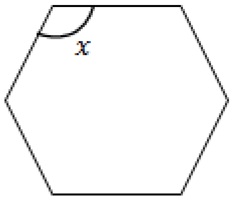
Exercise 6

Work out the angle marked with letters.

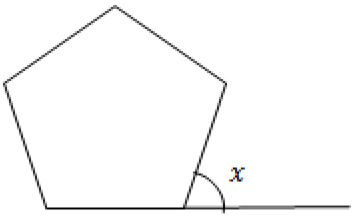
(a)



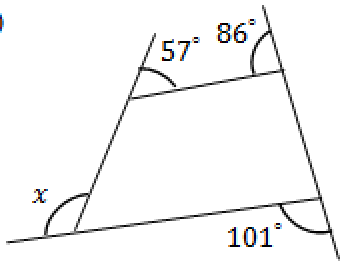
(b) regular hexagon



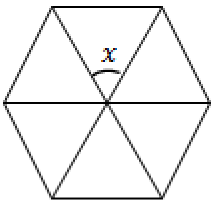
(c) regular pentagon



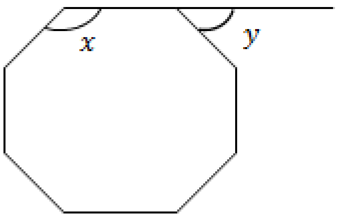
(d)



(e) regular hexagon

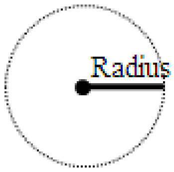


(f) regular octagon

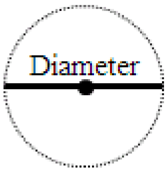


ANGLES IN A CIRCLE

A **circle** is a closed curve with every point at fixed distance from a point called the **centre.**



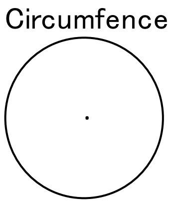
The distance from the centre of the circle to any point on the circumference is called **radius.**



The distance of a line from a point to another point on the circumference passing through the centre of the circle is called **diameter**

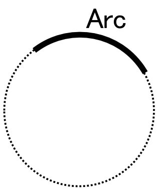
The diameter = 2 × radius

d = 2r

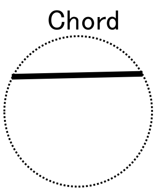


The **circumference or perimeter** of a circle is the distance around the circle.

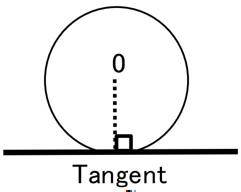
Circumference = π x diameter = π x 2 x radius **c = πd** or **c = 2πr**



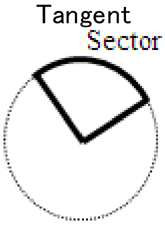
An **arc** is part of the circumference of the circle.



A **chord** is a straight line that joins two points on the circumference of a circle. The longest chord of any circle is the diameter of the circle. A circle has an infinite set of chords.

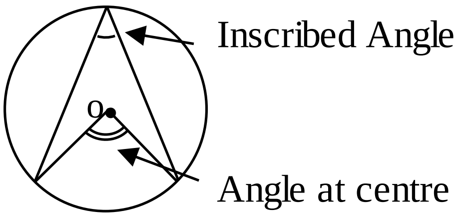


A straight line which touches the circumference at only one point is known as the **tangent** to the line.

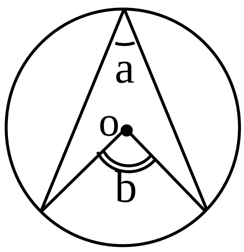


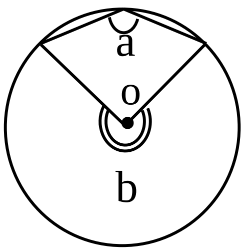
Part of a circle formed by an **arc** and two **radii**. The smaller part of the circle is called the **minor sector**, and the larger part is called the **major sector**.

INSCRIBED ANGLE



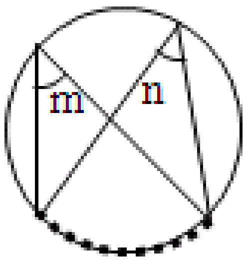
1. The inscribed angle is half of the angle at the centre of the circle that intercepts the same arc.





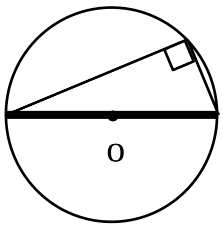
b = 2a

2. Angles on the same arc are always equal.



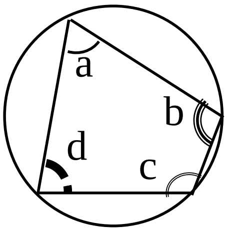
m = n

3. Any angle subtended by a diameter is a right angle.



4. A **cyclic quadrilateral** is a quadrilateral inside a circle with all its 4 vertices touching the circumference of the circle .

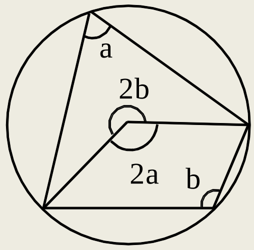
(i) The interior opposite angles are supplementary.



a + c = 180°

b + d = 180°

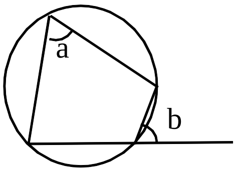
< Proof >



$2a + 2b = 360^\circ$

$\therefore a + b = 180^\circ$

(ii) An exterior angle is equal to the opposite interior angle.



Exterior angle = opposite interior angle

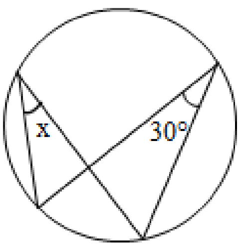
a = b

In the above diagram, the opposite of the exterior angle, **b**, is angle **a** so therefore the two angles are equal.

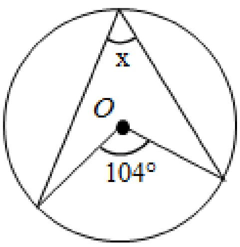
Exercise 7

1. For some circles given below, *O* is the centre. Work out the angles marked with letters:

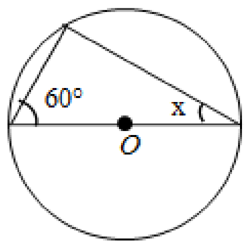
(a)



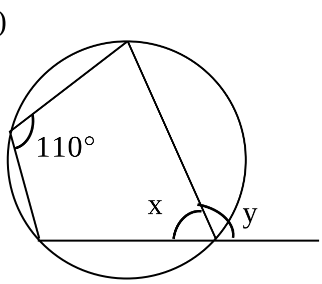
(b)



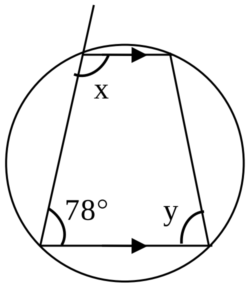
(c)



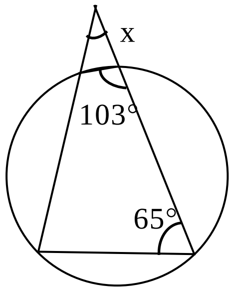
(d)



(e)



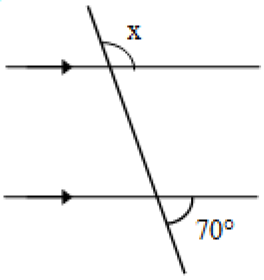
(f)



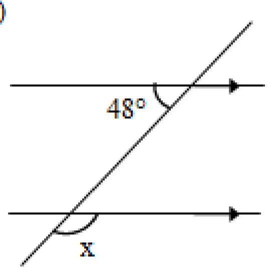
Review Exercise

1. Work out the angles marked with letters:

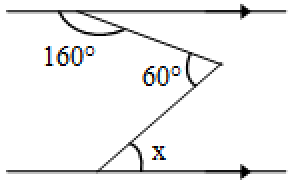
(a)



(b)

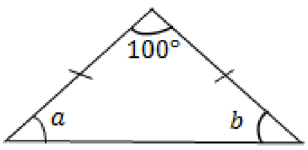


(c)

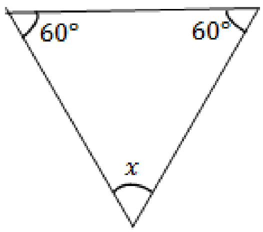


2. Work out the angles marked with letters:

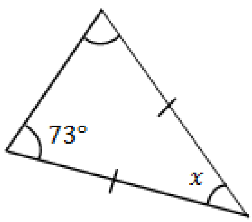
(a)



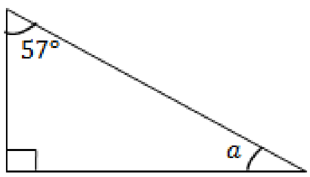
(b)



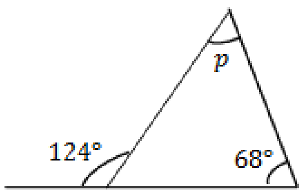
(c)



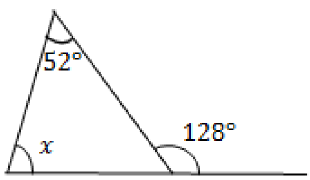
(d)



(e)

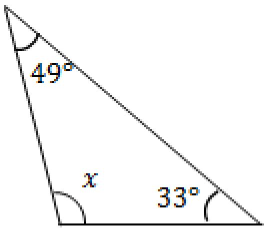


(f)

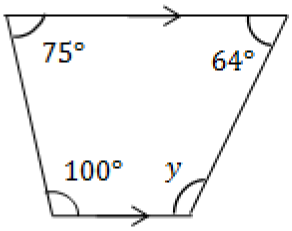


3. Work out the angles marked with letters:

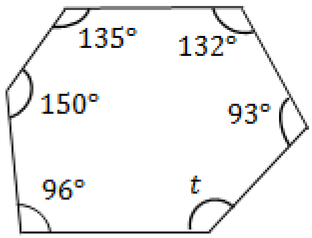
(a)



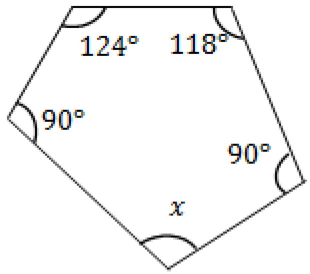
(b)



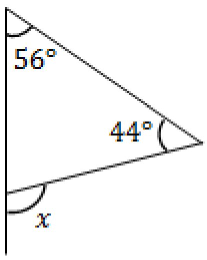
(c)



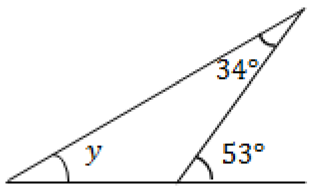
(d)



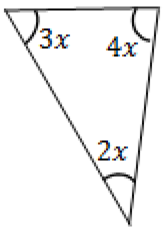
(e)



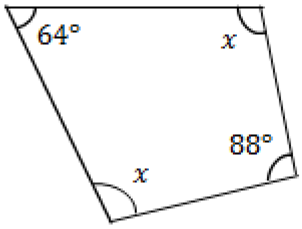
(f)



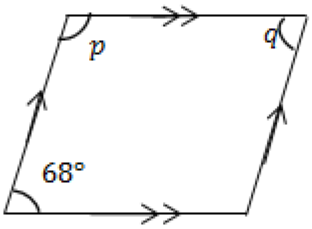
(g)



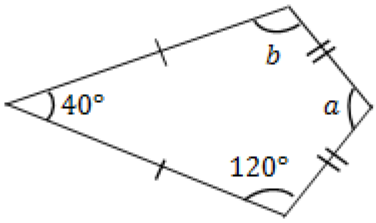
(h)



(i)

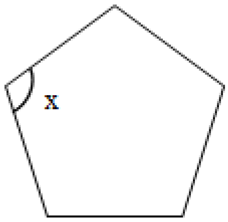


(j)

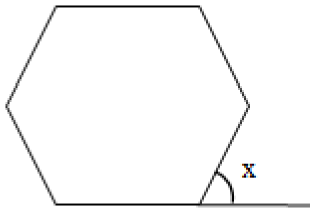


4. Work out the angles marked with letters:

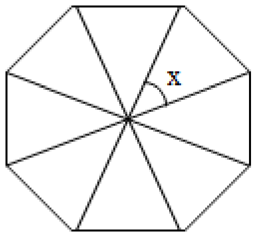
(a) regular pentagon



(b) regular hexagon

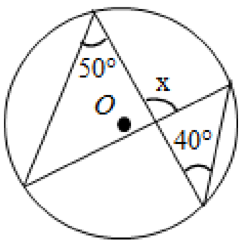


(c) regular octagon

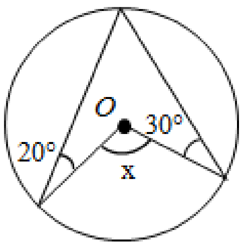


5. Work out the angles marked with letters:

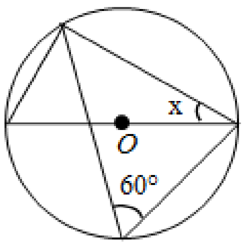
(a)



(b)

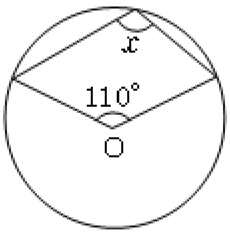


(c)

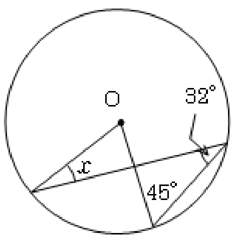


6. Work out the angles marked with letters:

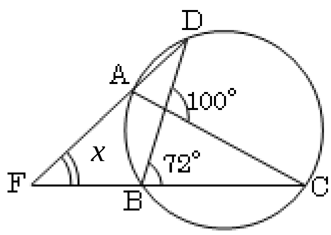
(a)



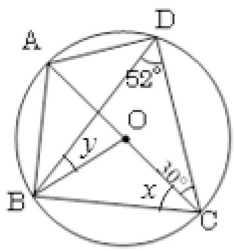
(b)



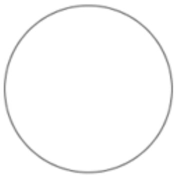
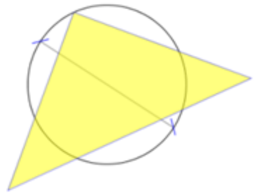
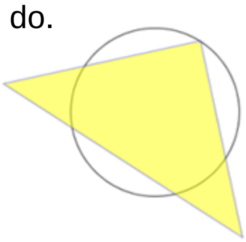
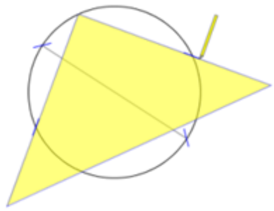
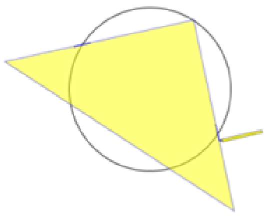
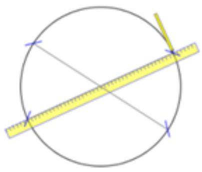
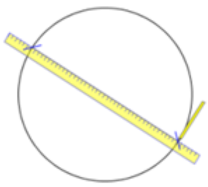
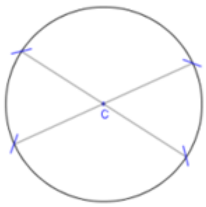
(c)



(d)



Let's try!!

How to find the centre of a circle	
We start with a given circle. 	**4.** Place the right-angle corner of the object at any other point on the circle. Any point will do, but for greatest accuracy, make it about a quarter the way round the circle from the first point. 
1. Place the right-angle corner of any object at any point on the circle. Any point will do. 	**5.** Make a mark where the two sides of the right-angle cross the circle. 
2. Make a mark where the two sides of the right-angle cross the circle. 	**6.** Connect these two points with a straight line. This is the second diameter. 
3. Draw a line between these two marks. This is a diameter of the circle. 	**7.** Done. The point where the two diameters intersect is the center of the circle. 

[Why can you find the centre of a circle with this method?]

UNIT 7

TRANSFORMATION

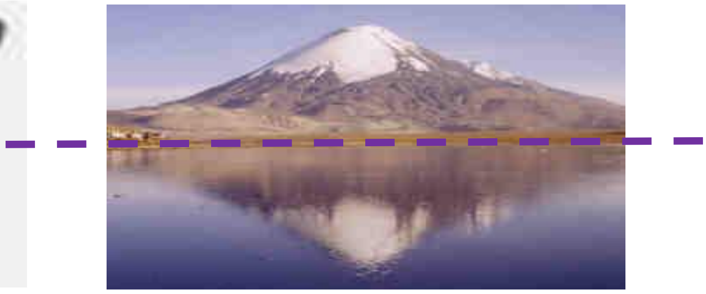
UNIT 7 TRANSFORMATION

LINE SYMMETRY

A shape is **symmetrical** when it can be folded into exactly two halves. The line in which the paper is folded is known as the **axis of symmetry** or the **mirror line**. The two halves are of the same size and shape so they are identical.

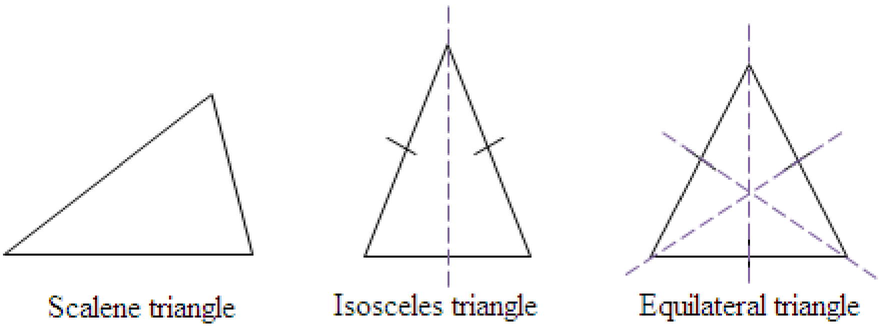
Example 1

The diagrams shown below have **one** axis of symmetry.

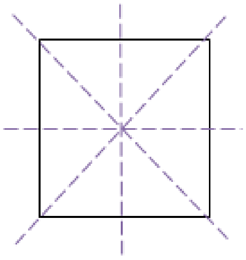
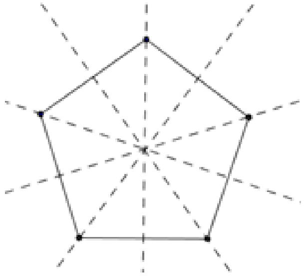
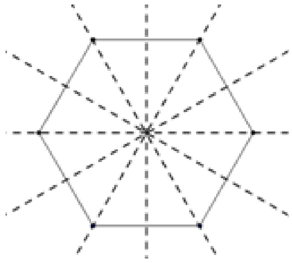


Example 2

Scalene triangle has no axis of symmetry since all the sides are different whereas.
Isosceles triangle has only one axis of symmetry since two sides are equal.
Equilateral triangle has three axes of symmetry since all the three sides are equal.



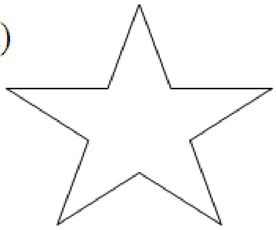
THE AXES OF SYMMETRY OF SOME REGULAR POLYGONS

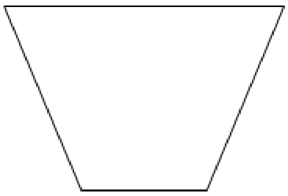




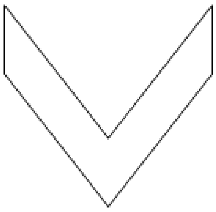
	Square	Regular Pentagon	Regular Hexagon
The number of the axes of symmetry	4	5	6

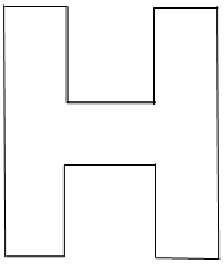
Exercise 1

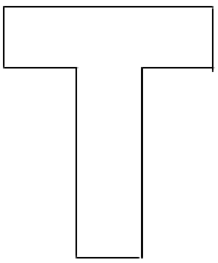
1. Sketch the axes of symmetry of the following figures.

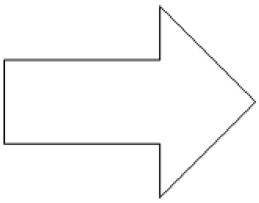
(a)


(b)


(c)


(d)


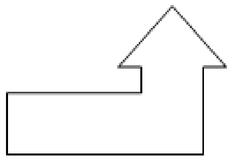
(e)


(f)


2. Which of these figures are symmetrical?

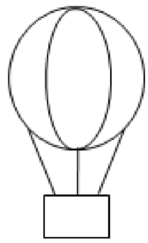
(a)

(b)



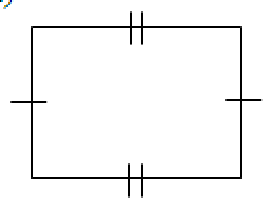
(c)

(d)

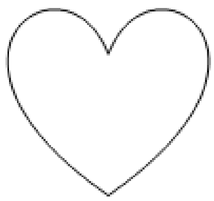


3. How many lines of symmetry could you draw on each diagram?

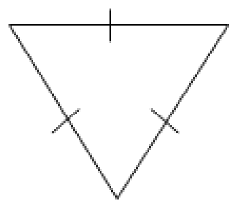
(a)



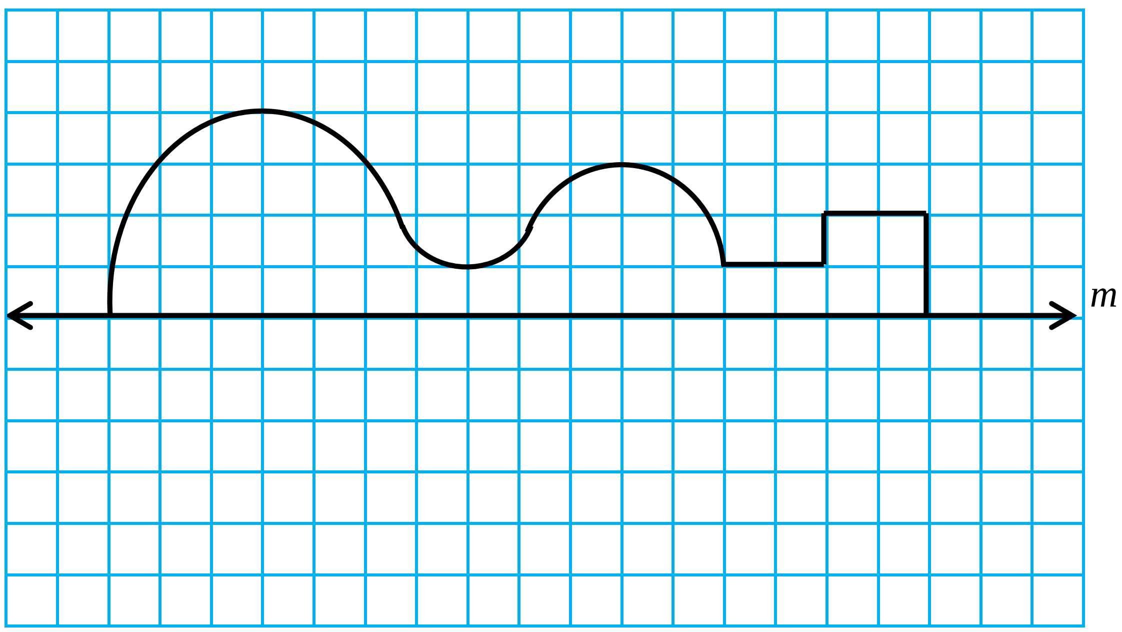
(b)



(c)



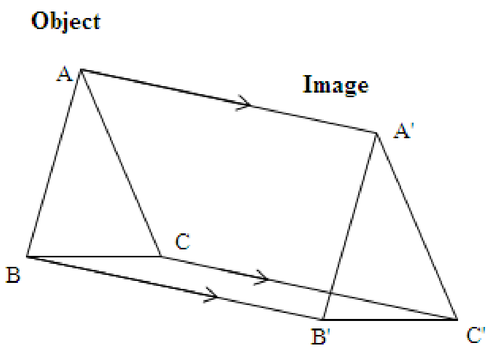
<u>Let's try!!</u> Draw symmetrical figure.



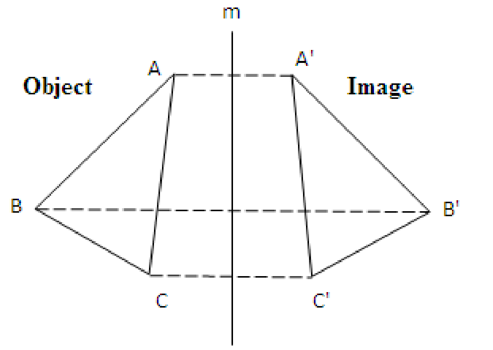
<u>**TRANSLATION, REFLECTION AND ROTATION**</u>

Transformation involves the turning over, turning around, moving without turning and enlarging of a given figure. The common transformations we normally use are: **translation**, **reflection**, **rotation** and **enlargement**.

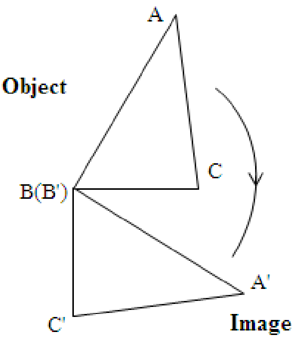
When a figure is translated, the original figure is called an **object**, and the translated object is called an **image**.



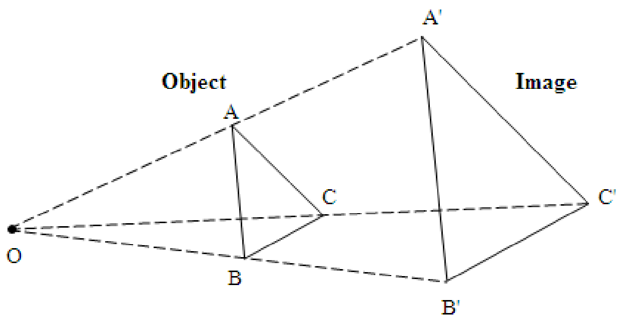
Translation



Reflection



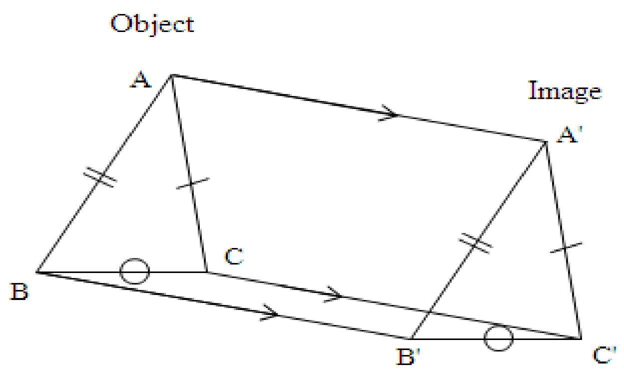
Rotation



Enlargement

TRANSLATION

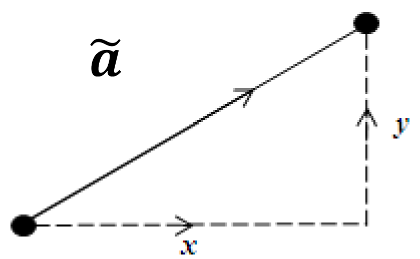
Translation is the shifting of an object from its original position to its new position. In other words translation is moving without turning.



Under translation, the following do not change **(invariant):** the shape, the size and the orientation or its direction. The only change that occurs is the position. Under any translation, the points on the object correspond to the points on the image.

VECTOR

When we are asked to fully describe a translation, we must indicate the distance and direction in which the object moves, that is, how many units to the right or left and how many units upwards or downwards. Normally, the direction taken are represented by a **vector.**



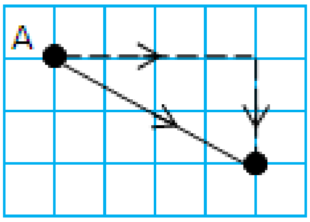
A **vector** is a set of ordered numbers written in the form below

$$\tilde{a} = \begin{pmatrix} x \\ y \end{pmatrix}$$

x represents the *x* value or the horizontal shift and *y* represents the *y* value or the vertical shift. A **vector** involves the **size** of movement (magnitude) and the **direction** from the starting point.

Example 3

A point initially at A shifts 4 to the right and then shifts 2 downwards to B.



In the diagram given, the length of the vector represents the size of movement while the arrow indicates the direction in which the object is moving from the starting point.

Horizontal translation (*x*) is done first then followed by the vertical translation (*y*).

If *x* is **positive**, the figure is shifted *x* units to the **right**.

If *x* is **negative**, the figure is shifted *x* units to the **left**.

If *y* is **positive**, the figure is shifted *y* units **upwards**.

If *y* is **negative**, the figure is shifted *y* units **downwards**.

According to the magnitude and direction of the point,

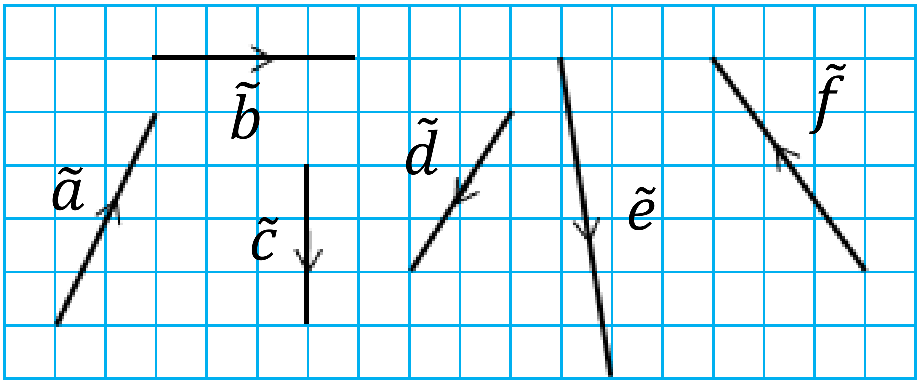
the value of *x* is 4 and

the value of *y* is -2

In vector form it is written as $\binom{4}{-2}$.

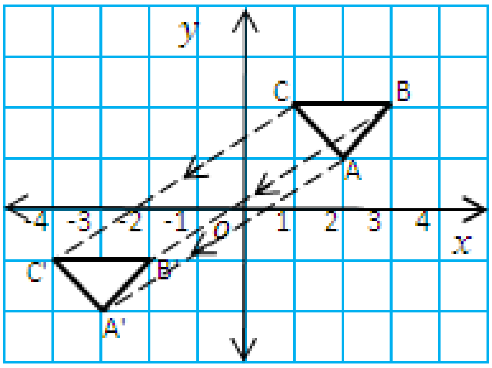
Exercise 2

Express each of the following as a column vector, i.e. in the form $\binom{a}{b}$.



Example 4

Shift ΔABC 5 units to the left and then 3 units down.



In a vector form it is written as $\begin{pmatrix}-5\\-3\end{pmatrix}$.

Exercise 3

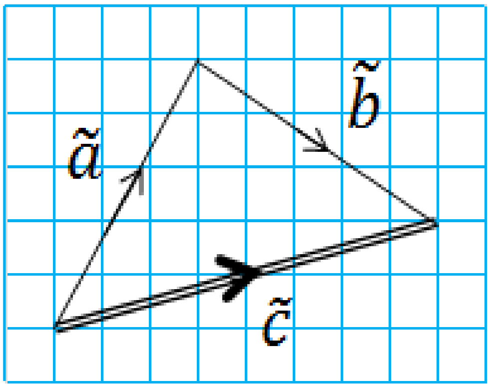
1. Translate the following points and find the coordinates of the images.
   - (a) A (2, 1) 5 units to the right
   - (b) B (−1, 4) 6 units down
   - (c) C (−5, −6) 3 units to left and 4 units up
   - (d) D (1, −7) 1 unit to the left and 1 unit down

2. In a Cartesian plane, the following points are translated by vector $\begin{pmatrix}1\\-2\end{pmatrix}$. Work out the coordinates of the image points.
   - (a) A (3, −2) (b) B (−1, 2) (c) C (0, 4)
   - (d) D (4, −2) (e) E (−3, 4) (f) F (−1, −5)

3. Work out the translation vector that maps the object to its image.
   - (a) A (−1, 4) → A’ (2, 3) (b) P (−3, 1) → P’ (0, 4)
   - (c) M (1, 5) → M’ (2, 7) (d) F (2, 4) → F’ (2, 1)
   - (e) G (6, −1) → G’ (4, −3) (f) V (3, −3) → V’ (5, 6)

ADDING VECTORS

Example 5



$\tilde{a} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ $\tilde{b} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$ $\tilde{c} = \begin{pmatrix} 8 \\ 2 \end{pmatrix}$

$$\tilde{a} + \tilde{b} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} + \begin{pmatrix} 5 \\ -3 \end{pmatrix} = \begin{pmatrix} 3+5 \\ 5-3 \end{pmatrix} = \begin{pmatrix} 8 \\ 2 \end{pmatrix} = \tilde{c}$$

$$\tilde{a} + \tilde{b} = \tilde{c}$$

The sum of two column vectors is the sum of the corresponding numbers. The sum of two vectors is called the **resultant** vector.

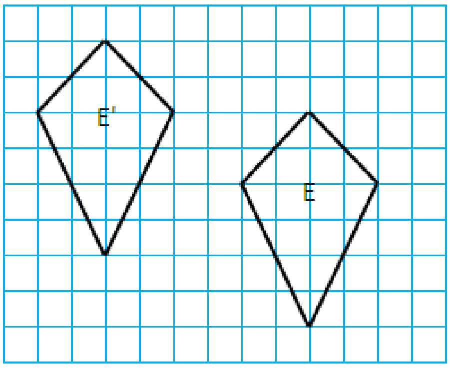
Exercise 4

Work out the sum of following vectors.

(a) $\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} -2 \\ 4 \end{pmatrix}$ (b) $\begin{pmatrix} -1 \\ -3 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ (c) $\begin{pmatrix} 5 \\ 6 \end{pmatrix} + \begin{pmatrix} -1 \\ 4 \end{pmatrix}$ (d) $\begin{pmatrix} 2 \\ 8 \end{pmatrix} + \begin{pmatrix} 5 \\ 1 \end{pmatrix}$

Exercise 5

In the diagram below, Figure E has been translated to Figure E′.



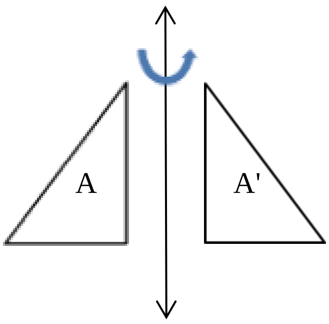
(a) Work out the vector that has translated Figure E to Figure E′.

(b) Work out the vector that would translate Figure E′ back to Figure E.

(c) Add vectors (a) and (b) above.

REFLECTION

Reflection is the movement of a figure when it is flipped over a line called mirror line.



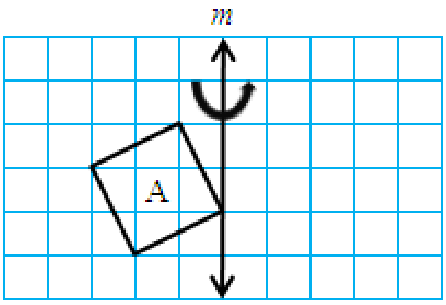
When an object is reflected in a mirror, both the object and the image make symmetrical shapes**.**

Under reflection, the shape, corresponding angle, length and the size do not change **(invariant)** but its orientation changes.

When we are asked to fully describe a transformation under reflection, we must also clearly indicate the equation of the axis of symmetry or mirror line.

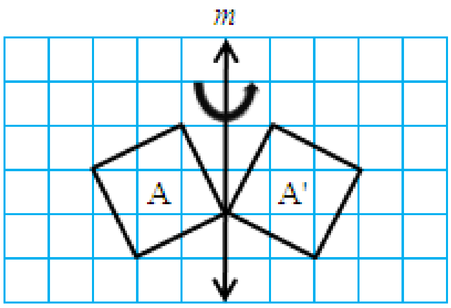
Example 6

In the diagram given below, reflect figure A in line *m* and label it A′.



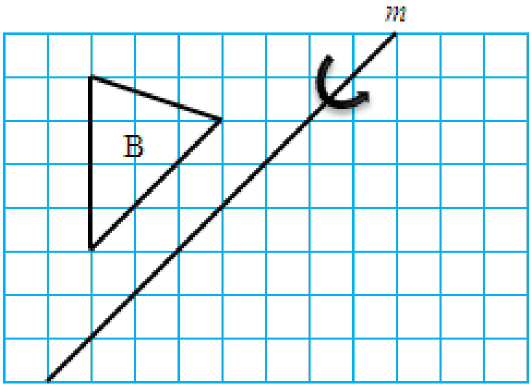
[Answer]

The distance of each point on the object from line *m* must be the same as the corresponding points on the image from line *m.*

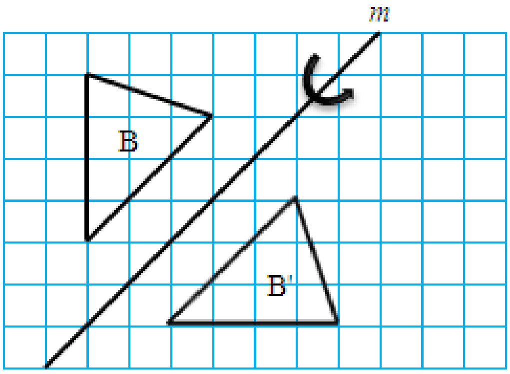


Example 7

In the diagram given below, reflect figure B in line *m* and label it B′.

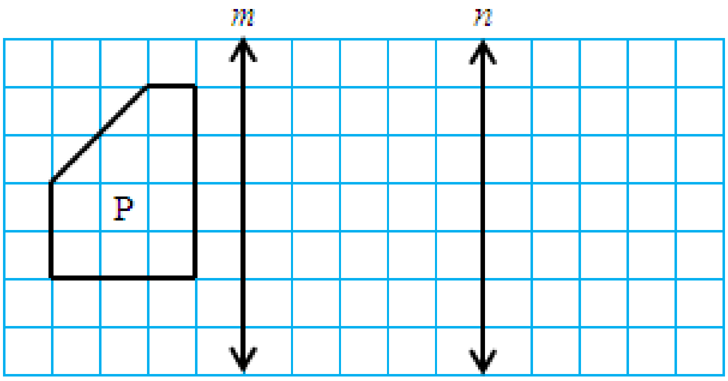


[Answer]



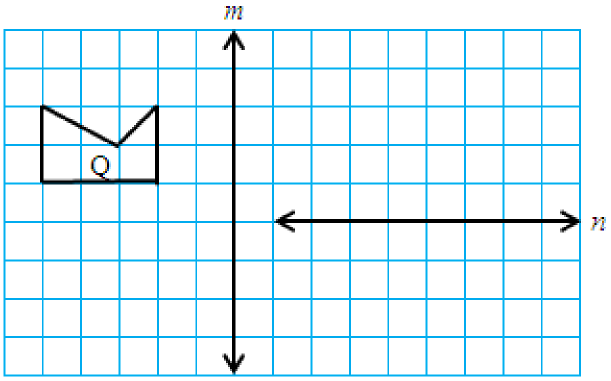
Exercise 6

Reflect figure P as shown below in line *m* and label it P′. Reflect P′ in line *n* and label it P′′.



Exercise 7

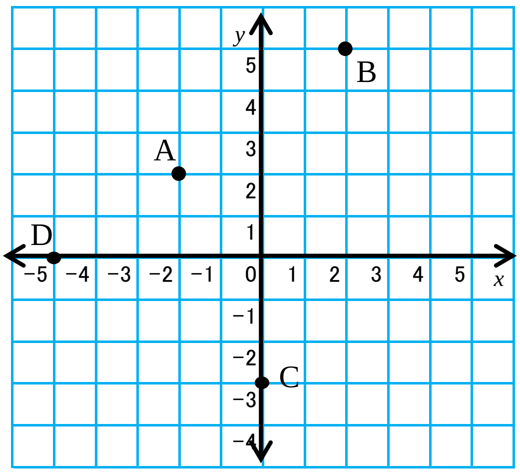
1. Reflect figure Q in line *m* and label its image as Q′. Reflect Q′ line *n* and label the image as Q′′.



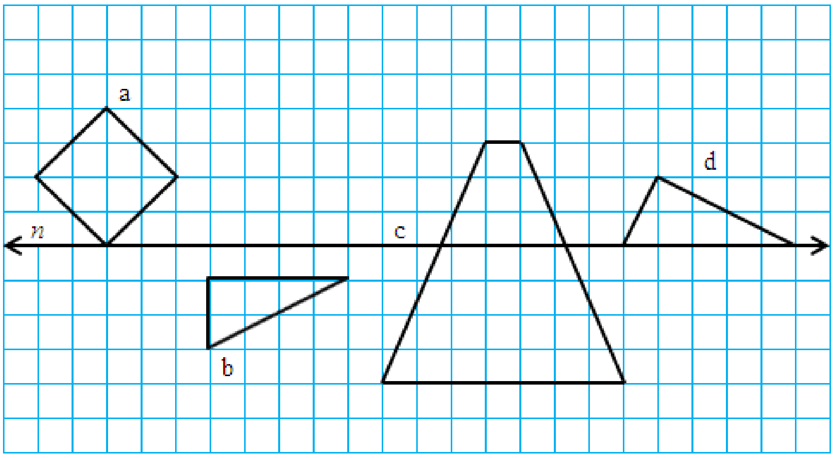
Practice 1

1. In the Cartesian plane given below, copy and plot the images of points shown under reflection in the *y* axis.
   (a) Label the image of point A as A′, B as B′,C as C′ and D as D′.

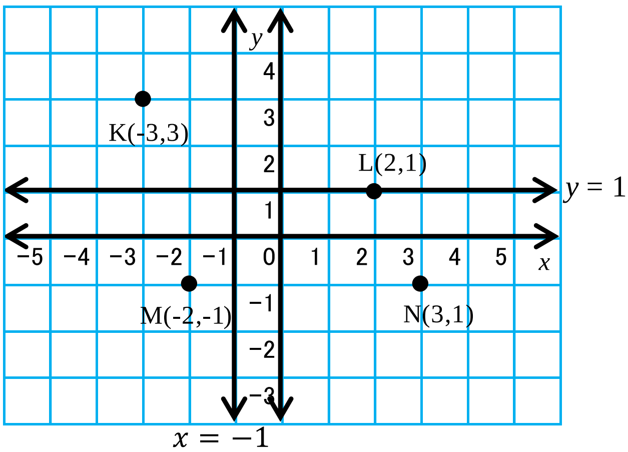
   (b) Work out the coordinates of A′, B′, C′ and D′.



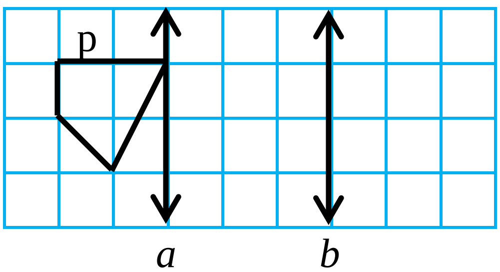
2. In the diagram given below, copy and then complete the figure drawn and its image under reflection in line *n*.



3. The Cartesian plane below shows the graphs of $x = -1$ and $y = 1$. Reflect points K, L, M and N first in the line $x = -1$ and then again reflect them in $y = 1$. Label the images as K′, L′, M′, and N′.



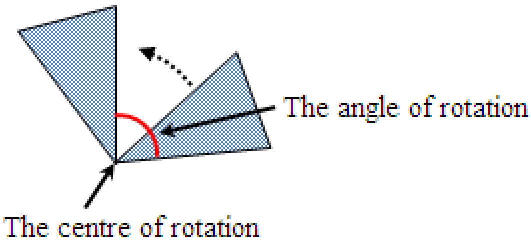
4. In the diagram below, reflect figure P in line *a* and label the image P′. P′ is then reflected in line *b*. Label its image P′′.



ROTATION

We are familiar with objects which rotate, such as car wheels, Ferris wheel, water irrigation.

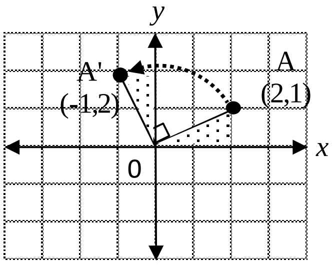
Rotation is a transformation that has to do with object **turning** or rotating around a **point** known as the **centre of rotation**.
It is either rotated in a **clockwise** or **anticlockwise** direction depending on the instruction given.



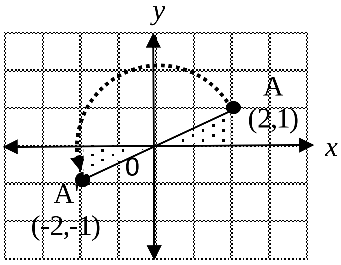
The angle in which we want to turn the object is called **angle of rotation**. If the angle of rotation is **positive** then we rotate the object in the anticlockwise direction and if the angle is **negative**, we turn the object in the clockwise direction.
Under rotation, the shape, the size, orientation and the centre of rotation are **invariant**.
The angle between the lines joining the centre of rotation to a point on the object and centre of rotation to the corresponding point on the image must be equal to the angle of rotation.
When we are asked to fully describe a transformation under rotation, we must clearly indicate the centre of rotation and the angle of rotation with direction.

ROTATION ABOUT THE ORIGIN

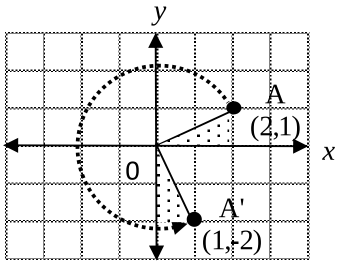
The origin is the place where the two axis meet i.e. (0, 0)



Rotation by 90°
at the origin
A(a,b)→A'(-b,a)

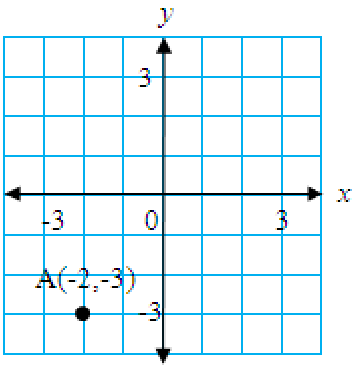


Rotation by 180°
at the origin
A(a,b)→A'(-a,-b)



Rotation by 270°
at the origin
A(a,b)→A'(b,-a)

Exercise 8

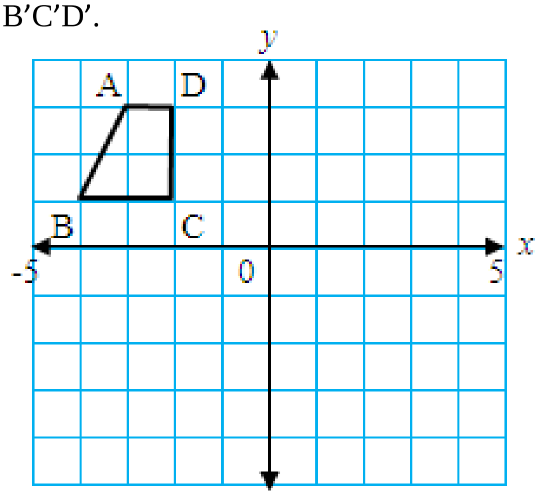


(a) Work out the coordinates of A(−2, −3) under a rotation by 90° about the origin and label it A′.

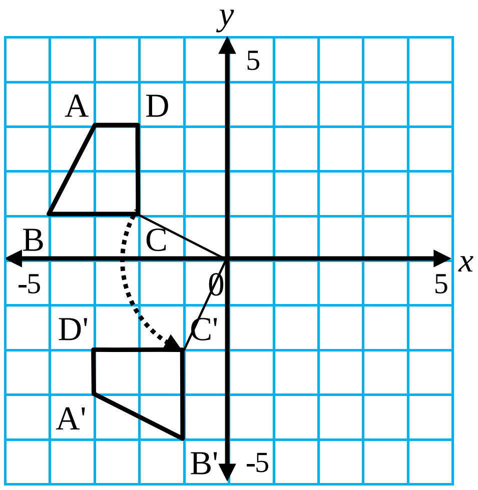
(b) Work out the coordinates of A(−2, −3) under a rotation by 180°about the origin and label it A″.

Example 8

Rotate figure ABCD by 90° about the origin. On the same Cartesian plane, draw its image and label it as A′B′C′D′.



[Answer]



A(-3,3)→A'(-3,-3)
B(-4,1)→B'(-1,-4)
C(-2,1)→C'(-1,-2)
D(-2,3)→D'(-3,-2)

Exercise 9

1. Work out the coordinates of the image of each point if it is rotated by 90^{o} about the origin.

   (a) $(-1, 2)$ (b) $(3, 2)$ (c) $(5, -3)$

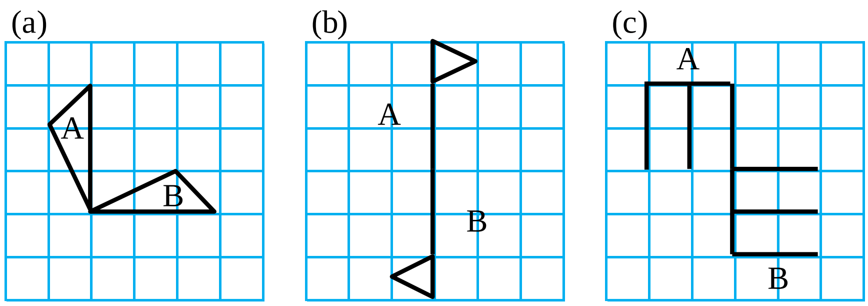
   (d) $(4, 1)$ (e) $(6, -2)$ (f) $(5, -4)$

2. Work out the coordinates of the image of $(3, -2)$ under the following angles of rotation.

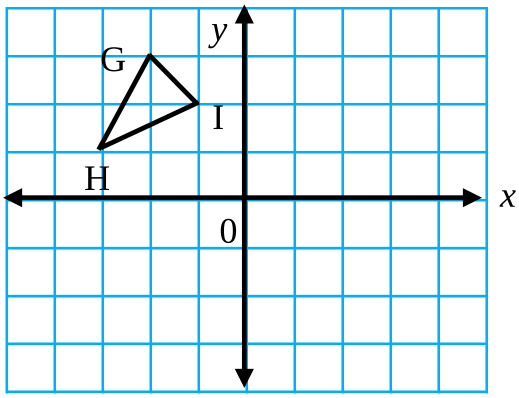
   (a) -90^{o} about the origin (b) 180^{o} about the origin

   (c) 90^{o} about the origin (d) 270^{o} about the origin

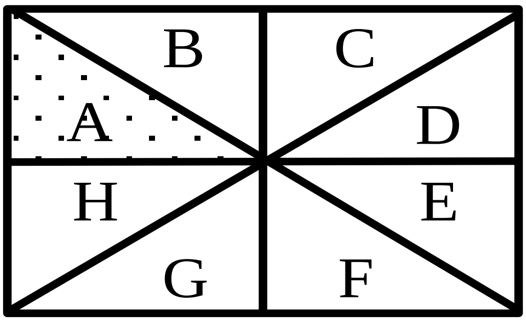
3. In the diagram given below, object A is rotated to give object B. Copy and locate the centre of rotation and label it O.



4. The figure shown below is rotated by 90^o about the origin. Sketch the image and label it G'H'I'. The image G'H'I' is again rotated by 180^o about the origin. Sketch and label the second image as G"H"I".



5. In the diagram given below, answer the following questions.



(a) Find the figure which is translated from triangle A.

(b) Find the two figures which are reflected from triangle A.

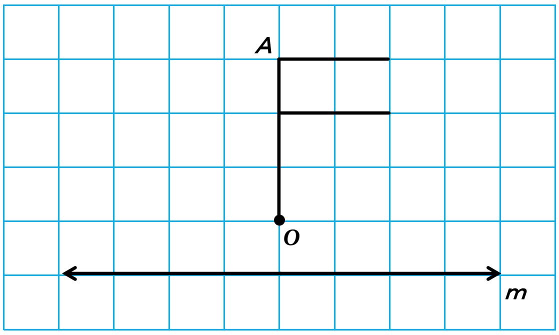
(c) Find the two figures which are rotated from triangle A.

COMBINED TRANSFORMATION

The combination of two or more transformations is known as **Combined Transformation.**

Example 9

Figure A is rotated by 90° about point O. Draw its image and label it A′. The image A′ is then reflected in line *m*. Draw the image and label it A″.



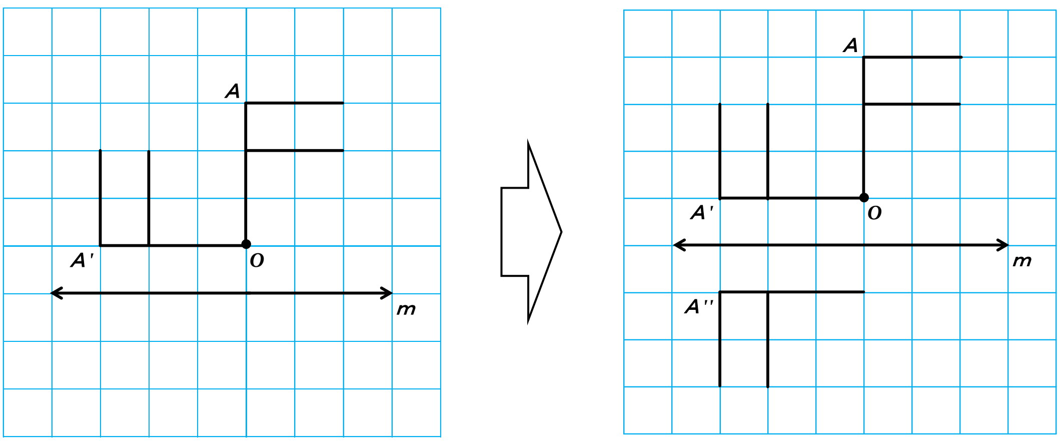
[Answer]

The First Transformation

Figure A is first rotated by 90° about O or 90° in the anticlockwise direction.

The Second Transformation

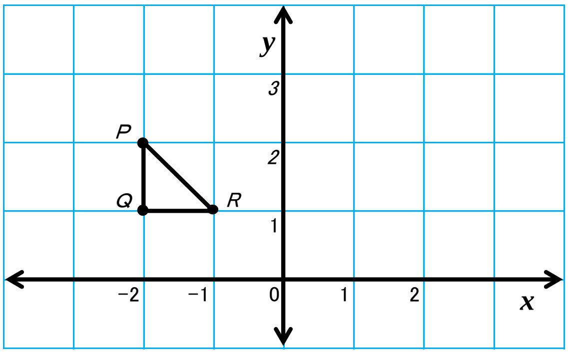
The second transformation is reflection in line *m*, that is, A′ is reflected in line *m*.



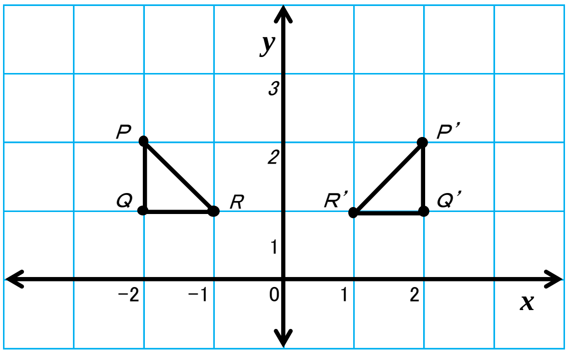
Example 10

P(−2, 2), Q (−2, 1), R(−1, 1) form a triangle. Reflect ∠PQR in the line $x = 0$ and label its image as ΔP’Q’R’ and then translate ΔP’Q’R’ by vector $\tilde{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and label its image as P’’Q’’R’’.

[Answer]

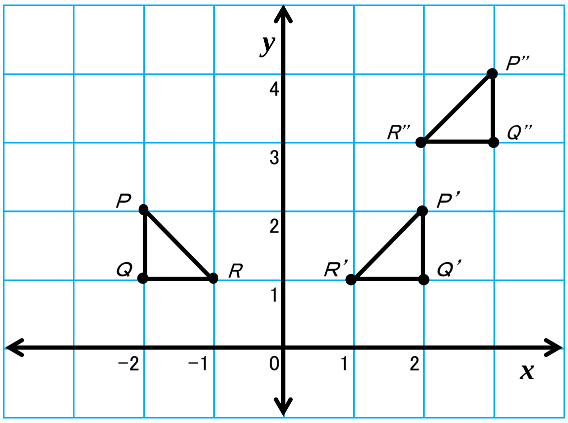


The mirror line is $x = 0$ or the y – axis. The distance of each point on the object from the mirror line should be the same with the distance of each corresponding point on the image from the mirror line.



P’Q’R’ is then translated vector *a*

where $\tilde{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.



Exercise 10

1. Translate the following points by vector $\begin{pmatrix}1\\2\end{pmatrix}$ and then reflect them in the *x* axis.

   (a) $(-1, 2)$ (b) $(3, 4)$ (c) $(-5, -3)$

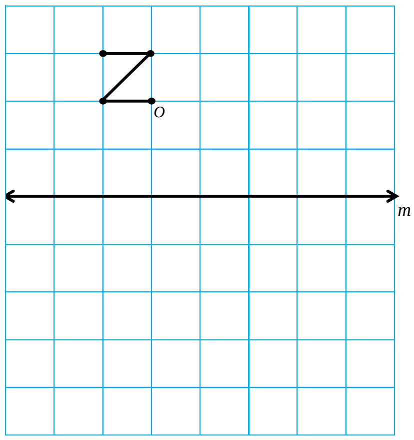
2. Reflect the following points in the *y*-axis and rotate them 90^o about the origin.

   (a) $(2, 3)$ (b) $(-1, -1)$ (c) $(-4, 1)$

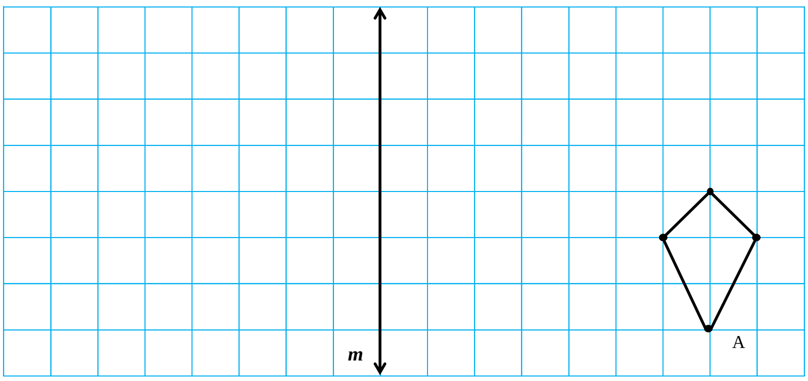
3. Translate the following points by vector $\begin{pmatrix}2\\-2\end{pmatrix}$ and then rotate them 180^o about $(0, 0)$.

   (a) $(2, 1)$ (b) $(3, -1)$ (c) $(-3, -2)$

4. Figure Z is rotated by -90^o about point O. Draw its image and label it Z′. The image Z′ is then reflected in line *m*. Draw the image and label it Z″.



5. Figure A is translated by vector $\begin{pmatrix}-2\\3\end{pmatrix}$. Draw its image and label it as A′. The image A′ is then reflected in line *m*. Draw the image and label it A″.



Review Exercise

1. In a Cartesian plane, the following points are translated by vector $\begin{pmatrix}-3\\2\end{pmatrix}$. Find the coordinates of the image points.

   (a) $A(4,2)$ (b) $B(-1,-1)$ (c) $C(1.5,-3)$

   (d) $D\left(-\frac{3}{2},0\right)$ (e) $E\left(0,\frac{1}{3}\right)$ (f) $F\left(\frac{5}{2},\frac{1}{4}\right)$

2. Find the translation vector that maps the object to its image.

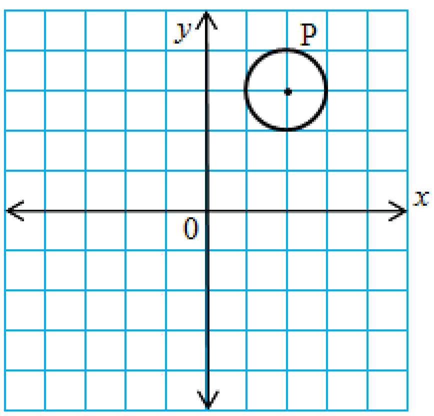
   (a) $A(-3,5) \rightarrow A'(2,3)$ (b) $B(4,1) \rightarrow B'(7,-2)$

   (c) $C\left(\frac{3}{2},-1\right) \rightarrow C'\left(-1,\frac{1}{2}\right)$ (d) $D(2.5,1.5) \rightarrow D'(-3,5)$

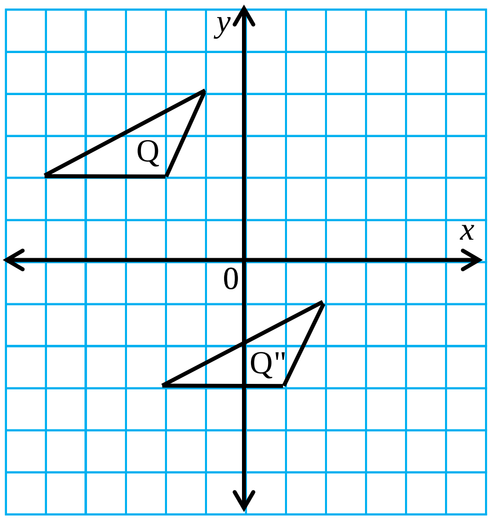
3. Work out the sum of following vectors.

   (a) $\begin{pmatrix}1\\-3\end{pmatrix}+\begin{pmatrix}4\\4\end{pmatrix}$ (b) $\begin{pmatrix}6\\-2\end{pmatrix}+\begin{pmatrix}-1\\-3\end{pmatrix}$ (c) $\begin{pmatrix}\frac{1}{3}\\2\end{pmatrix}+\begin{pmatrix}\frac{2}{3}\\-\frac{1}{3}\end{pmatrix}$ (d) $\begin{pmatrix}8\\\frac{1}{4}\end{pmatrix}+\begin{pmatrix}-\frac{1}{2}\\\frac{1}{2}\end{pmatrix}$

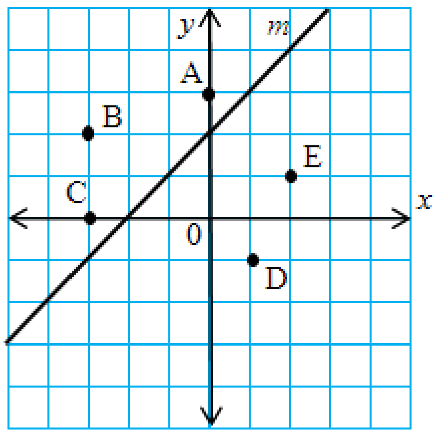
4. Translate circle P by $\tilde{a}=\begin{pmatrix}-5\\-4\end{pmatrix}$ and label it P



5. Figure Q is translated by $\tilde{a} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and label it Q'. Then find the vector that has translated figure Q' to figure Q" given. Find the vector that has translated figure Q" to figure Q.



6. In the Cartesian plane below, reflect points A, B, C, D and E in the line *m*.



7. Find the coordinates of the image of each point if it is rotated about the origin.

(1) By $-90°$

(a) $(2, -1)$ (b) $(0, 3)$ (c) $(-2, -3)$

(2) By $270°$

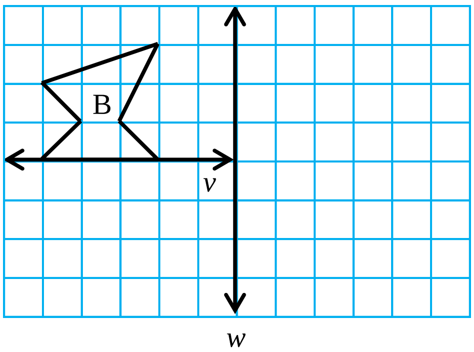
(a) $(2, -1)$ (b) $(0, 3)$ (c) $(-2, -3)$

8. For each point given below, work out how many degrees it rotated.

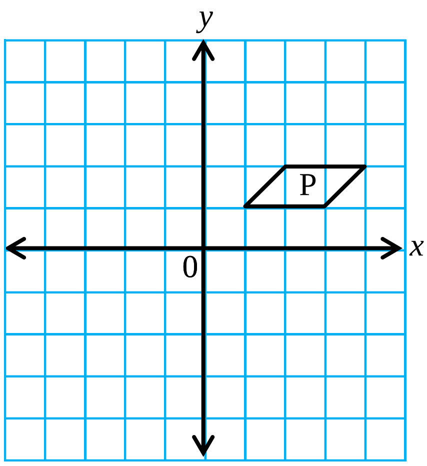
(a) $A(2, 1) \rightarrow A'(1, -2)$ (b) $B(-1, 3) \rightarrow B'(1, -3)$

(c) $C(-3, -2) \rightarrow C'(3, 2)$

9. In the diagram below, reflect figure B in line *v* and label the image B'. Then reflect figure B' in line *w* and label the image B".



10. Figure P is rotated by 180° about point O. Draw its image and label it P'. Then the image P' is reflected in *x* – axis. Draw the image and label it P". Finally, the figure P" is translated by $\tilde{a} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ and label it P"'



UNIT 8

ENLARGEMENT

AND

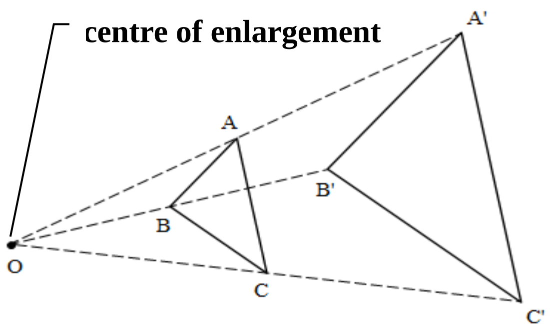
SIMILARITY

UNIT 8 ENLARGEMENT AND SIMILARITY

ENLARGEMENT

Enlargement is a transformation whereby the size of the object is changed to a bigger or a smaller one but the shape is not changed. It has a centre called the **centre of enlargement.** The size of the image is determined by the **scale factor**.

When we are asked to fully describe a transformation under enlargement, we must clearly indicate the **centre of enlargement** and its **scale factor**.



$OA' = 2 \times OA$ $\quad$ $A'B' = 2 \times AB$

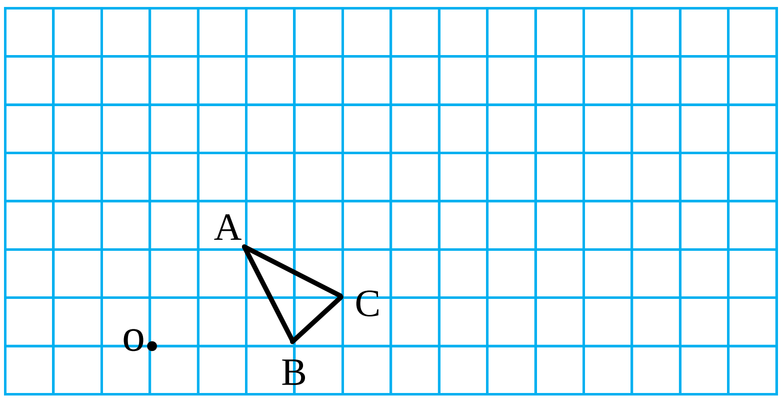
$OB' = 2 \times OB$ $\quad$ $B'C' = 2 \times BC$

$OC' = 2 \times OC$ $\quad$ $C'A' = 2 \times CA$

So the scale factor of this enlargement is 2.

Example 1

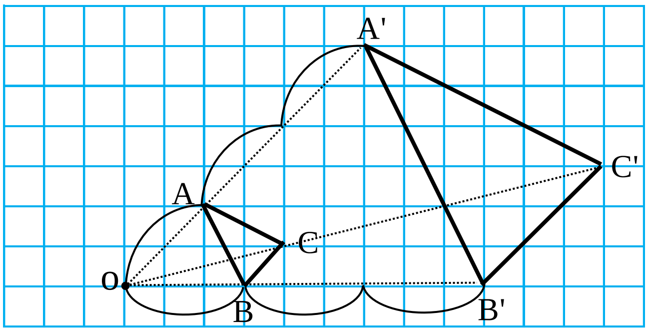
Enlarge $\triangle ABC$ by a scale factor 3 with centre of enlargement O and label it $\triangle A'B'C'$.



[Answer]

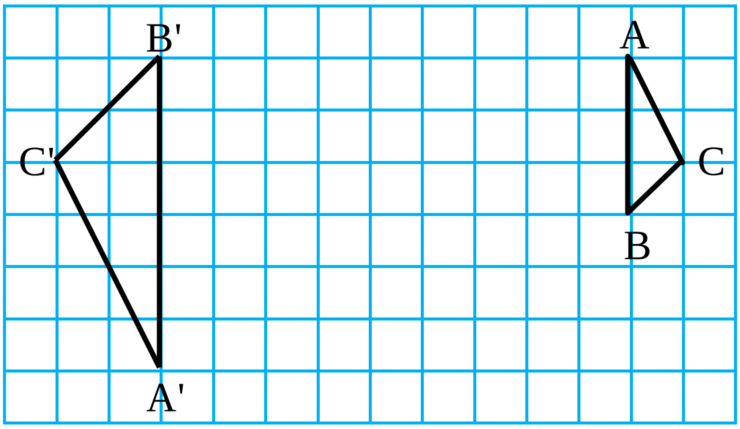
In this example, we use the scale factor and the centre of enlargement to enlarge $\triangle ABC$.
Since the scale factor is 3, we will triple the size of the object and triple the distance from the centre of enlargement.

Draw straight lines through the vertices of the object and the centre of enlargement. Mark the image points and sketch the image.



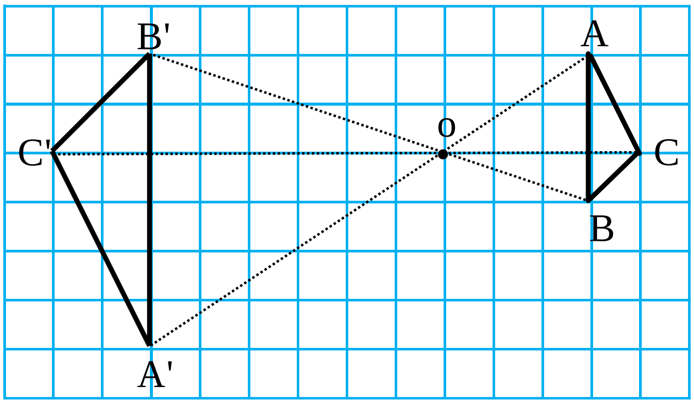
Example 2

For the diagram given below, ΔABC is enlarged to $\Delta A'B'C'$. Work out the centre of enlargement and label it O.



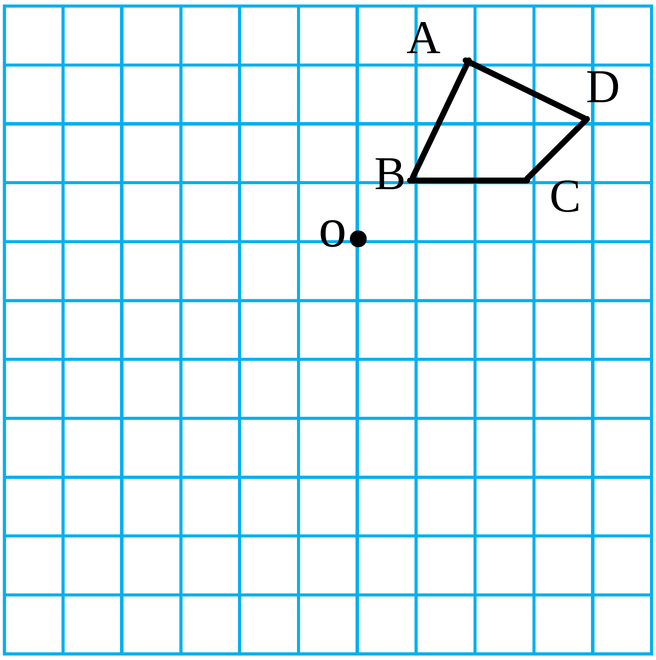
[Answer]

First, identify corresponding points and then draw a straight line through each pair of corresponding points.

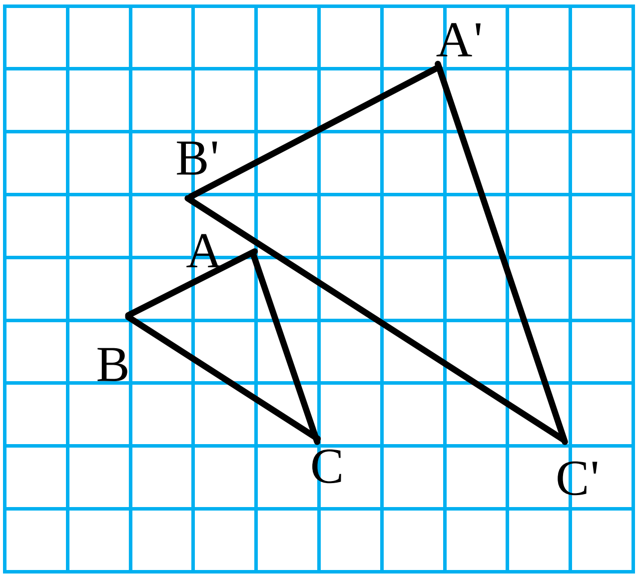


Exercise 1

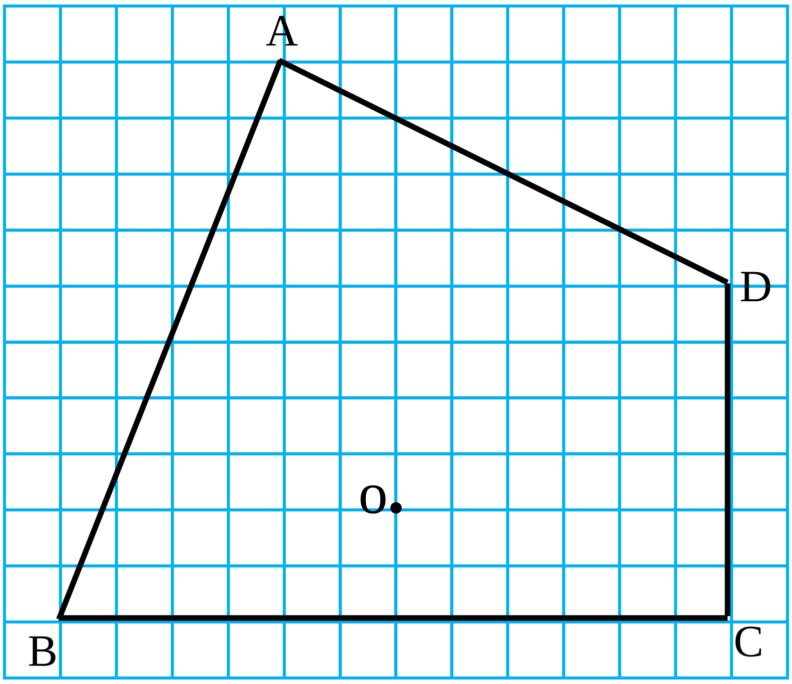
1. Enlarge figure $ABCD$ by a scale factor 2 with centre of enlargement O and label it $A'B'C'D'$.



2. For the diagram given below, ΔABC is enlarged to $\Delta A'B'C'$. Find the centre of enlargement and label it O.



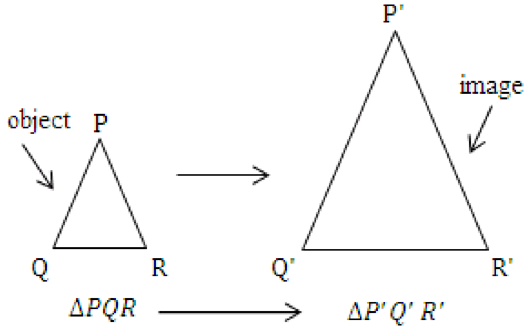
3. Enlarge figure $ABCD$ by a scale factor $\frac{1}{2}$ with centre of enlargement O.



SIMILAR FIGURES

When we enlarge an object, the object and the image are called **similar figures**. Similar figures have the **same shape** but not necessarily the same size. They have the following properties.

- Corresponding angles are equal.
- The ratios of corresponding sides are equal.



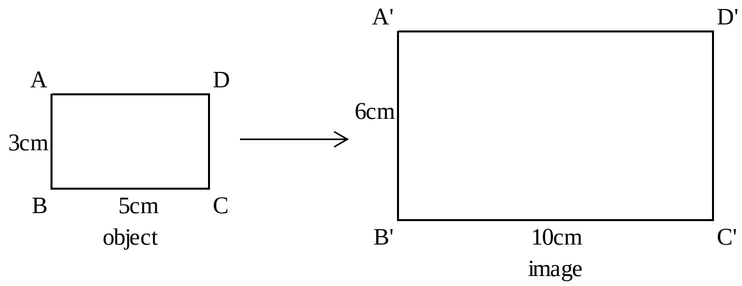
$$\frac{P'Q'}{PQ} = \frac{Q'R'}{QR} = \frac{R'P'}{RP} = scale\ factor$$

$$\angle P = \angle P', \angle Q = \angle Q', \angle R = \angle R'$$

$$\text{scale factor} = \frac{\text{length of the image}}{\text{length of the object}}$$

Example 3

In the diagram given below, rectangle $ABCD$ is enlarged to rectangle $A'B'C'D'$. Work out the length scale factor.



[Answer]

We equate the ratios of two corresponding sides.

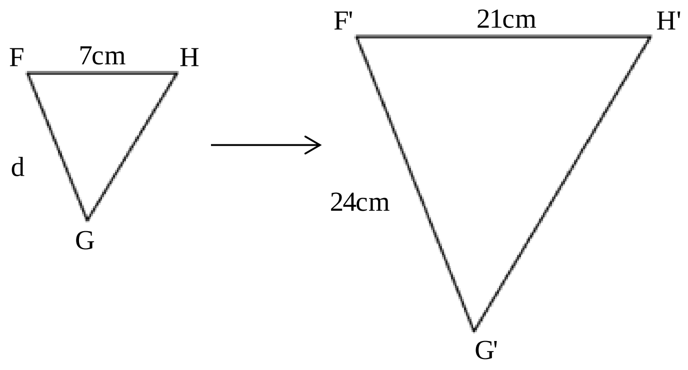
$$\frac{A'B'}{AB} = \frac{6}{3} = 2$$

$$\frac{B'C'}{BC} = \frac{10}{5} = 2$$

$\therefore$ length scale factor is **2**.

Example 4

In the diagram given below, $\triangle$ FGH is enlarged to $\triangle$ F′G′H′.



(a) Work out the length scale factor of this enlargement.

(b) Work out the value of d.

[Answer]

(a) Length scale factor $= \frac{\text{length of the image}}{\text{length of the object}}$

$$\frac{F'H'}{FH} = \frac{21}{7} = 3$$

$\therefore$ Length scale factor is **3**.

(b) Length scale factor $= \frac{\text{length of the image}}{\text{length of the object}}$

$$3 = \frac{F'G'}{FG}$$

$$3 = \frac{24}{d}$$

$$3 \times d = \frac{24}{d} \times d$$

$$3d = 24$$

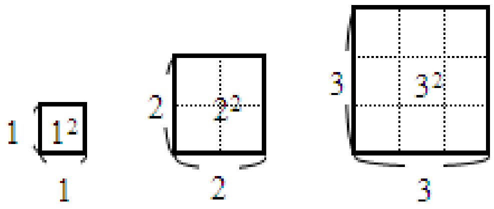
$$\frac{3d}{3} = \frac{24}{3}$$

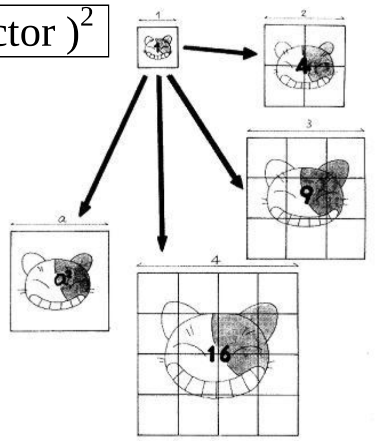
$$\mathbf{d = 8}$$

THE AREA SCALE FACTOR

The area **scale factor** is the square of the length scale factor and is used to determine the area of either the object or image whichever is required to be found.

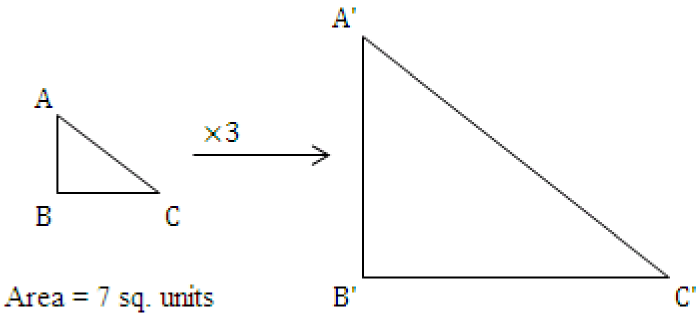
$$\text{Area scale factor} = (\text{length scale factor})^2$$





Example 5

The area of $\triangle ABC$ shown below is 7 square units.
Work out the area of $\triangle A'B'C'$ that is the image of $\triangle ABC$ with enlargement scale factor 3.



[Answer]

Length scale factor is 3.

Area scale factor = (length scale factor) ^{2}

$= (3)^2$

$= 9$

$\therefore$ The area of $\triangle A'B'C' =$ the area of $\triangle ABC \times$ area scale factor

$= 7 \text{ sq. units} \times 9$

$= \mathbf{63\ sq.\ units}$

Example 6

In the diagram below, $\triangle$ STU is enlarged to $\triangle$ SVW

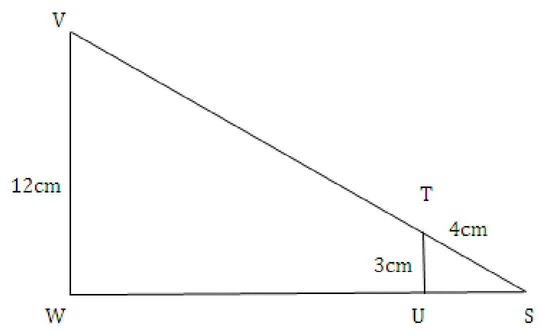
(a) Find the centre of enlargement.

(b) Work out the length scale factor.

(c) Work out the length of $\overline{VS}$.

(d) Work out the length of $\overline{VT}$.

(e) Work out the area scale factor.



[Answer]

(a) To find the centre of enlargement, draw straight lines joining any two points on the object with their corresponding points on the image. The point where the straight lines meet is the centre of enlargement.
According to the diagram given, a straight line through points V and T (a point on the object and its corresponding point on the image) and a straight line through points U and W (again a point and its image) meet at point S. For this example, the centre of enlargement is point S.

(b)

$$\text{Length scale factor} = \frac{\text{length of the image}}{\text{length of the object}}$$

$$= \frac{12 \text{ cm}}{3\text{cm}}$$

$$= \mathbf{4}$$

(c)

Equate the ratios of two corresponding sides

$$\frac{\overline{VS}}{\overline{TS}} = \frac{\overline{VW}}{\overline{TU}}$$

$$\frac{\overline{VS}}{4} = \frac{12}{3}$$

$$\frac{\overline{VS}}{4} \times 4 = \frac{12}{3} \times 4$$

$$\boldsymbol{\overline{VS} = 16}$$

(d)

$$\overline{VT} = \overline{VS} - \overline{TS}$$

$$= 16 - 4$$

$$= \mathbf{12}$$

(e)

Area scale factor = (length scale factor) ^{2}

$= (4)^2$

$=$ **16**

Exercise 2

1. In the diagram shown below,

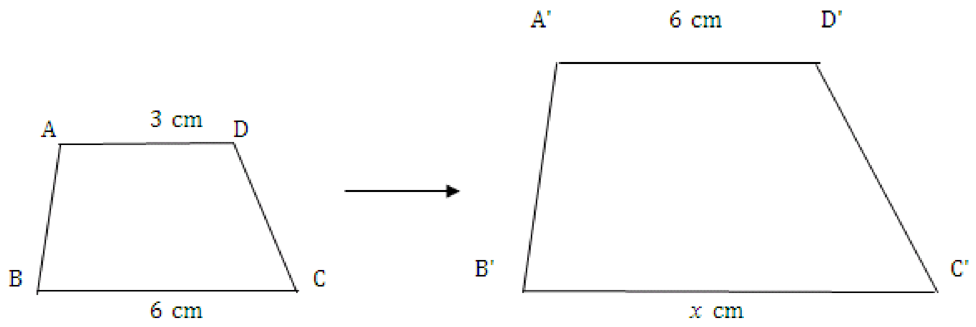
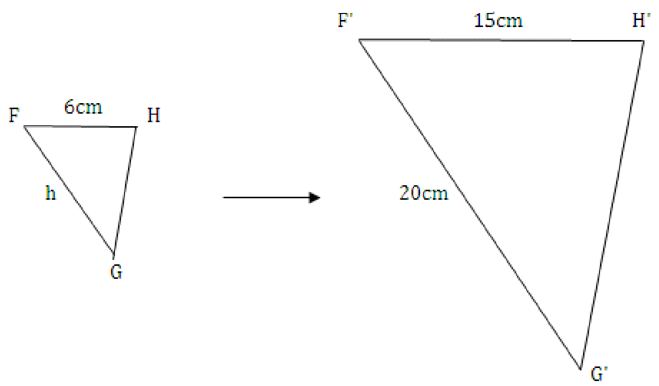


figure $ABCD$ is enlarged to figure $A'B'C'D'$.

(a) Work out the length scale factor.

(b) Work out the value of x.

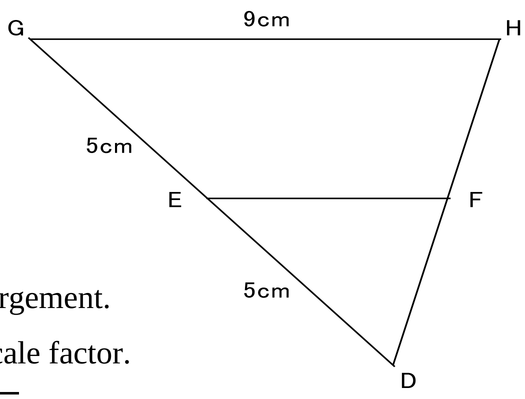
2. In the diagram given below, $\triangle FGH$ is enlarged to $\triangle F'G'H'$.



(a) Work out the length scale factor of this enlargement.

(b) Work out the value of h.

3. In the diagram shown below, $\triangle DEF$ is enlarged to $\triangle DGH$.



(a) Find the centre of enlargement.

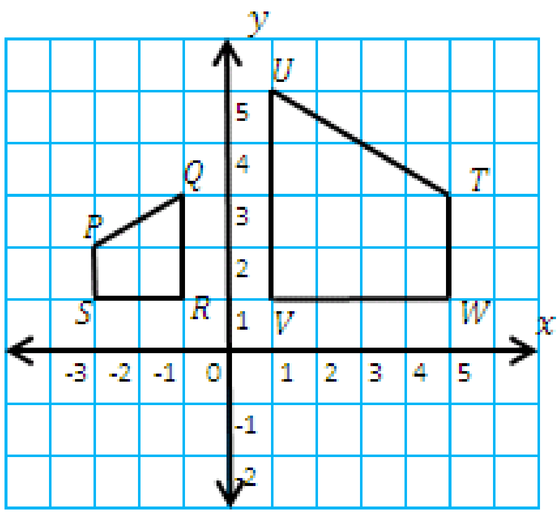
(b) Work out the length scale factor.

(c) Work out the length $\overline{EF}$.

(d) Work out the area scale factor.

(e) If the area of $\triangle DEF$ is 12 cm^2, what is the area of $\triangle DGH$?

4. Figure PQRS is transformed to figure under a combined transformation.



(a) Which point is corresponding to point P ?

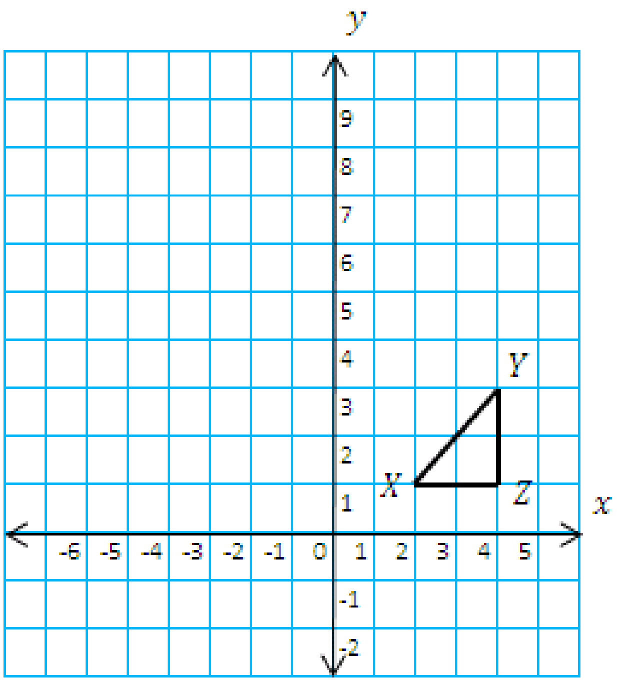
(b) Work out its length scale factor.

(c) Work out the area scale factor.

(d) If the area of figure TUVW is 12 sq units, what is the area of Figure PQRS?

(e) Identify the two types of transformations.

5. Transform $\triangle XYZ$ under a rotation by 90^{o} about the origin, label the image $\triangle X'Y'Z'$ and then followed by enlargement centre (0, 0) with scale factor 2. Label the second image $\triangle X"Y"Z"$.



6. Enlarge $\triangle TUV$ by a scale factor 2 with centre of enlargement T and label its image $\triangle T'U'V'$, and reflect it in line $T'U'$.