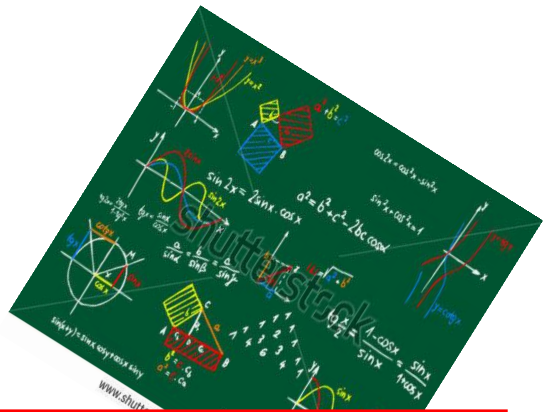
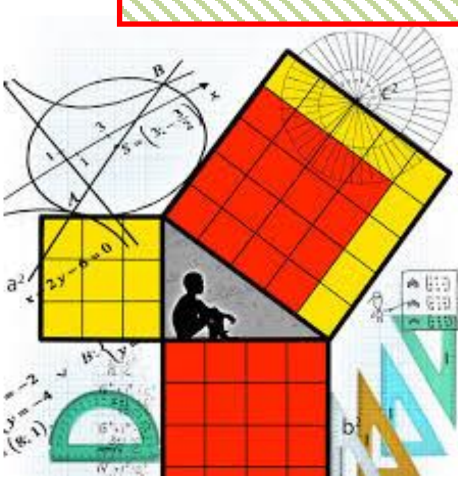
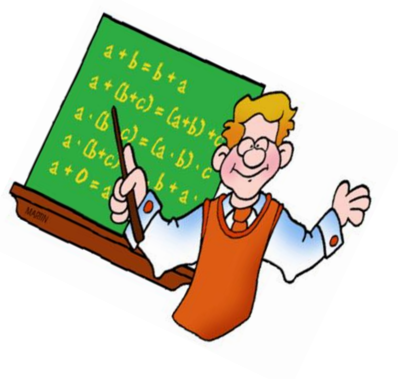
Is a theorem that gives the relationship between the sides of a right - angled triangle.



MATHEMATICS





YEAR 10

TEXTBOOK

MINISTRY OF EDUCATION

MATHEMATICS TEXTBOOK

YEAR 10

CURRICULUM DEVELOPMENT UNIT

FIJI, 2015

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PREFACE

The revised Year 10 Syllabus has been presented to Year 10 teachers in all secondary schools throughout the nation. The development of this Textbook was entirely based on this revised Year 10 Syllabus. It has a total of six strands: Functions, Algebra, Numbers, Geometry, Measurement and Chance and Data and these are further divided into Sub - Strands.

The contents of this book have been simplified so that it can be used by all students of different capabilities.

It is confidently believed that it will furnish Year 10 students with the necessary number and variety of exercises essential to successful instructions in mathematics.

The book's step – by – step instructions in the methods and examples will make it suitable for both direct one – on - one tutoring and as well as regular classroom use. Moreover, there are a spectrum of exercises and illustrations that significantly enrich students understanding of mathematical concepts.

All examples that have been introduced can even be attempted by an average pupil without assistance. They have been carefully graded to suit the slow learners as well while there are some problems that are provided for advance learners.

ACKNOWLEDGEMENT

Throughout the process in writing this textbook, a number of people have sacrificed their valuable time to assist the Ministry of Education. They must be acknowledged for their active participation and without their insights, guidance and continued support, this book may not have been possible. The Ministry of Education, therefore, hereby acknowledges the following people for their valuable contributions to this book:

Mr Seru Ramakita	Dilkusha Methodist High School
Ms Amrita Devi	Marist Brothers High School
Mrs Amelia Siga	Higher Education Commission
Mr Timoci Vosailagi	DAV Girls College
Mrs Evia Turaganivalu	Suva Grammar School
Mrs Asenaca Tuinabua	Suva Grammar School
Mr Emosi Lutunaika	Curriculum Development Unit

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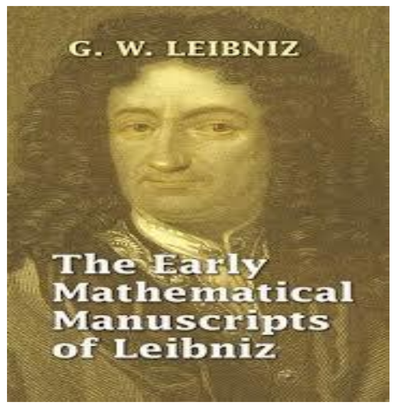
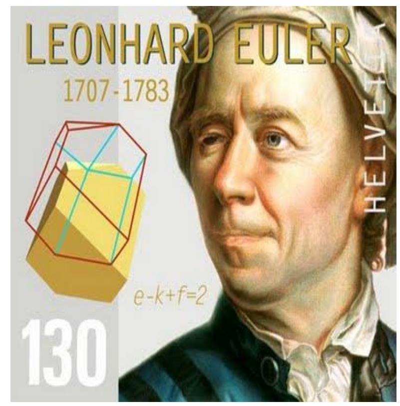
STRAND 1 → FUNCTIONS

HISTORY OF FUNCTIONS

The idea of a function was developed in the seventeenth century. During this time, Rene Descartes (1596-1650), in his book Geometry (1637), used the concept to describe many mathematical relationships. The term "function" was introduced by Gottfried Wilhelm Leibniz (1646-1716) almost fifty years after the publication of Geometry. The idea of a function was further formalized by Leonhard Euler (pronounced "oiler" 1707-1783) who introduced the notation for a function, y = f(x).





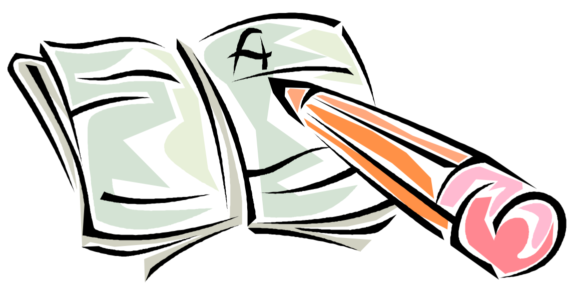
Source: http://science.irank.org/pages/2881/Function-History-functions.html

1.1 Linear and Quadratic Functions

LEARNING OUTCOMES

Students should be able to:

- Describing linear and quadratic functions
- Identifying and describing domain and range of functions
- Calculating functions using function notations
- Generating domain and range of functions as ordered



A RELATION is a set of ordered pairs

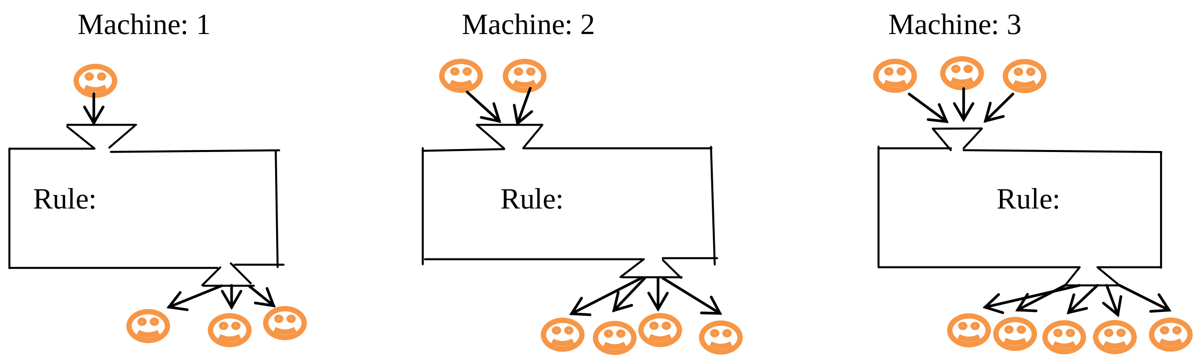
A FUNCTION is a set of information or data that has a clear output for each input. a function is a set of ordered pairs in which each x-element has only one y-element associated with it.

Types Of Function

I. Linear Function

ACTIVITY

Three number machines are given. For each number machine, state the rule being followed by the machines.



INPUT	OUTPUT
1	3
2	4
3	5

If x is to represent the input and y is to represent the output, the common rule derived from the three number machines in terms of x and y would be: $y = x + 2$

LINEAR FUNCTION: A function whereby the degree or index on the input variable is 1 e.g. $y = x + 2$ has the degree 1 i.e. the index on x the input variable is 1

Example 1.1

For the linear function, $y = x + 2$ where $x \in \{$ -2, -1, 0, 1, 2$\}$ list the function as:

(i) a set of ordered pairs

(ii) an arrow diagram

(iii) a Cartesian graph

(iv) a table of values

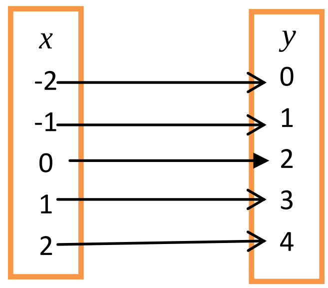
Answer

Linear function	x value	Substitute x	y value
y = x + 2	x = -2	y = (-2) + 2	y = 0
	x = -1	y = (-1) + 2	y = 1
	x = 0	y = (0) + 2	y = 2
	x = 1	y = (1) + 2	y = 3
	x = 2	y = (2) + 2	y = 4

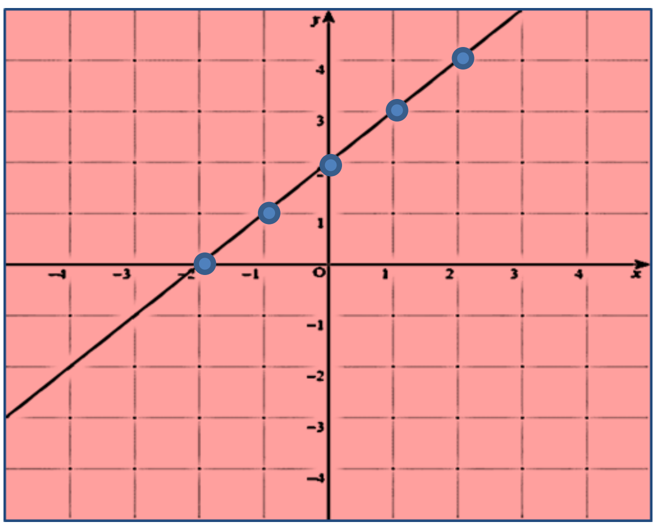
(i) Set of ordered pairs

R={(-2,0)(-1,1)(0,2)(1,3)(2,4)}

(ii) Arrow diagram



(iii) Cartesian graph



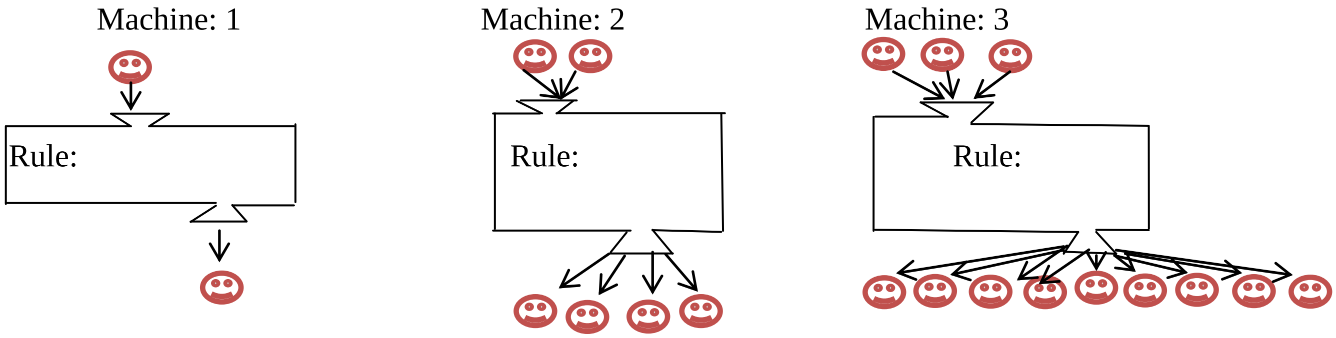
(iv) Table of values

x	-2	-1	0	1	2
y	0	1	2	3	4

II. Quadratic Function

ACTIVITY

Three number machines are given. For each number machine, state the rule being followed by the machines.



INPUT	OUTPUT
1	1
2	4
3	9

If x is to represent the input and y is to represent the output, the common rule derived from the three number machines in terms of x and y would be: $y = x^2$

QUADRATIC FUNCTION: A function whereby the degree, power or index on the input variable x is equal to 2 e.g. $y = x^2$ has the degree of 2 i.e. the index on x the input variable is 2

Example 1.2

For the quadratic function, $y = x^2$ where $x \in \{-2, -1, 0, 1, 2\}$ list the function as:

(i) a set of ordered pairs

(ii) an arrow diagram

(iii) a Cartesian graph

(iv) a table of values

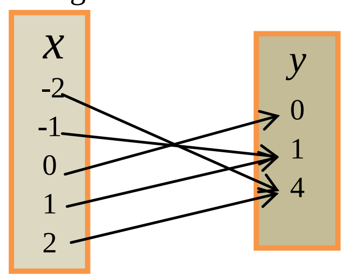
Answer

Linear function	x value	Substitute x	y value
$y = x^2$	x = -2	$y = (-2)^2$	y = 4
	x = -1	$y = (-1)^2$	y = 1
	x = 0	$y = (0)^2$	y = 0
	x = 1	$y = (1)^2$	y = 1
	x = 2	$y = (2)^2$	y = 4

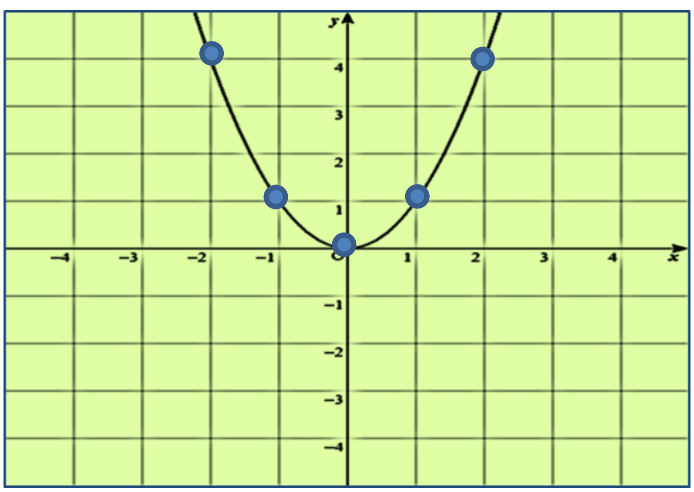
(i) Set of ordered pairs

R={(-2,4)(-1,1)(0,0)(1,1)(2,4)}

(ii) Arrow diagram



(iii) Cartesian Graph



(iv) Table of values

x	-2	-1	0	1	2
y	4	1	0	1	4

Example 1.3

For the linear function y = 3x – 1 where $x \in \{-2, -1, 0, 1, 2\}$, list the fuction as:

(i) a set of ordered pairs

(ii) an arrow diagram

(iii) a Cartesian graph

(iv) a table of values

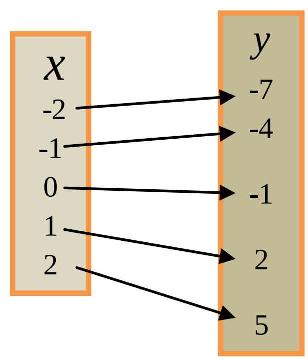
Answer

Linear function	x value	Substitute x	y value
y = 3x- 1	x = -2	y = 3(-2) - 1	y = -7
	x = -1	y = 3(-1) - 1	y = -4
	x = 0	y = 3(0) - 1	y = -1
	x = 1	y = 3(1) - 1	y = 2
	x = 2	y = 3(2) - 1	y = 5

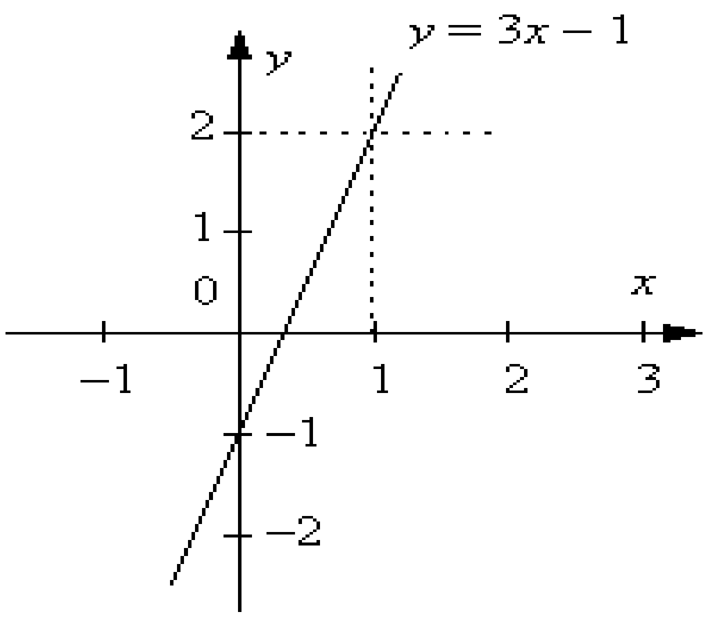
(i) Set of ordered pair

R={(-2,-7)(-1,-4)(0,-1)(1,2)(2,5)}

(ii) Arrow diagram



(iii) Cartesian graph



(iv) Table of values

x	-2	-1	0	1	2
y	-7	-4	-1	2	5

Exercise 1.1

1. For the function $y = x - 1$ where $x \in$ {-2, -1, 1, 0, 1, 2} list the function as:

   i. A set of ordered pairs

   ii. An arrow diagram

   iii. A Cartesian graph

2. For the function $y = 2x^2$ where $x \in R$ list the function as:

   i. A set of ordered pairs

   ii. An arrow diagram

   iii. A Cartesian graph

3. For the function $y = -2x + 1$ where $x \in$ {-5,-3,-1, 0, 1, 3, 5} list the function as:

   i. A set of ordered pairs

   ii. An arrow diagram

   iii. A Cartesian graph

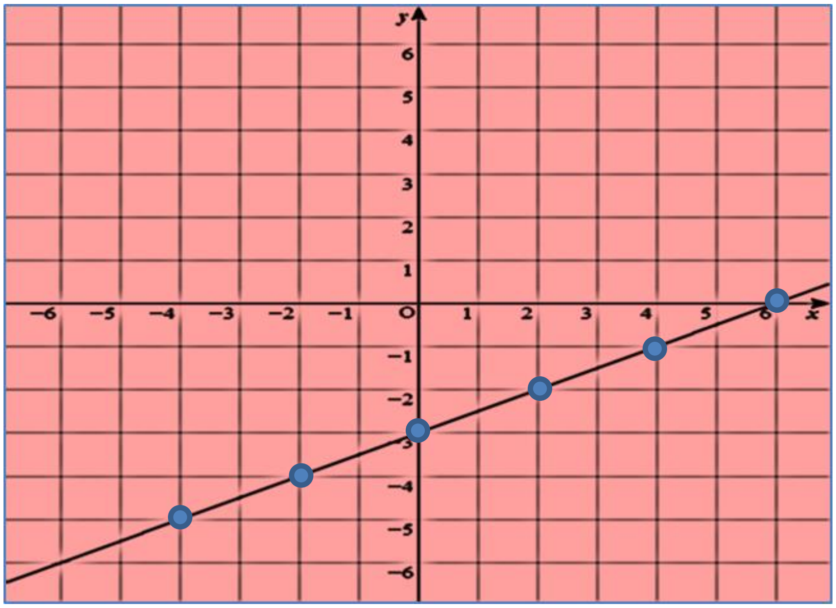
4. For the function $y = -2x^2$ where $x \in$ {-4, -2, 0, 2, 4} list the function as:

   i. A set of ordered pairs

   ii. An arrow diagram

   iii. A Cartesian graph

5. The diagram shows a function given as a Cartesian graph.



Using the points indicated on the line graph, show the function as:

i. An ordered pair

ii. An arrow diagram

iii. A rule

III. Function Notation

A way to indicate that an equation is a function.

$f(x)$ Or $g(x)$ etc. is read as f of x or g of x

⇨

Example 1.4

Given $f(x) = x + 2$ where $x \in \{-2, -1, 0, 1, 2\}$

$f(-2) = -2 + 2 = 0$, $f(-1) = -1 + 2 = 1$,

$f(0) = 0 + 2 = 2$, $f(1) = 1 + 2 = 3$,

$f(2) = 2 + 2 = 4$

Exercise 1.2

1. If $f(x) = 3x - 4$ and $g(x) = x^2 - 2$, find:

i. $f(2)$ ii. $f(-3)$ iii. $g(3)$ iv. $g(-4)$

2. Two functions are given as $h(x) = -\frac{1}{2}x + 3$ and $k(x) = -3x^2$. Find:

i. $k(3)$ ii. $h(4)$ iii. $k(-6)$ iv. $h(-8)$

3. $f(x) = x + 4$ and $g(x) = \frac{x-4}{3}$. Evaluate

i. $f(-2)$ ii. $f(\frac{2}{3})$ iii. $g(13)$ iv. $g(-14)$

v. For what value of x is $f(x) = -5$

vi. For what value of x is $g(x) = -3$

IV. Domain (D_x) And Range (R_y)

***Domain** (D_x): The set of Input Values or set of x values that can be put in an equation.*

***Range** (R_y): The set of output values or the set of possible $y-$values.*

Example 1.5

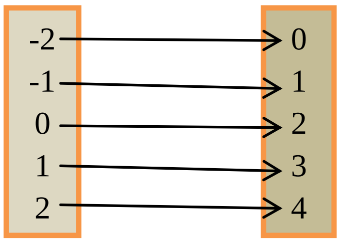
Give the domain and range for the following function.

(i) R= {(-2,0)(-1, 1)(0,2)(1,3)(2,4)}

Domain, $(D_x) = \{-2, -1, 0, 1, 2\}$

Range, $(R_y) = \{0, 1, 2, 3, 4\}$

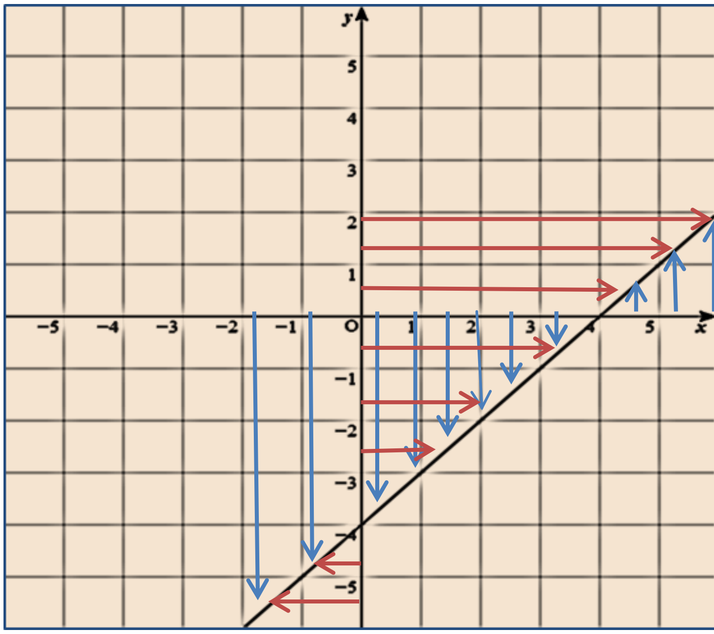
(ii)



Domain, $(D_x) = \{-2, -1, 0, 1, 2\}$

Range, $(R_y) = \{0, 1, 2, 3, 4\}$

(iii) $f(x) = x - 4$ where $x \in R$



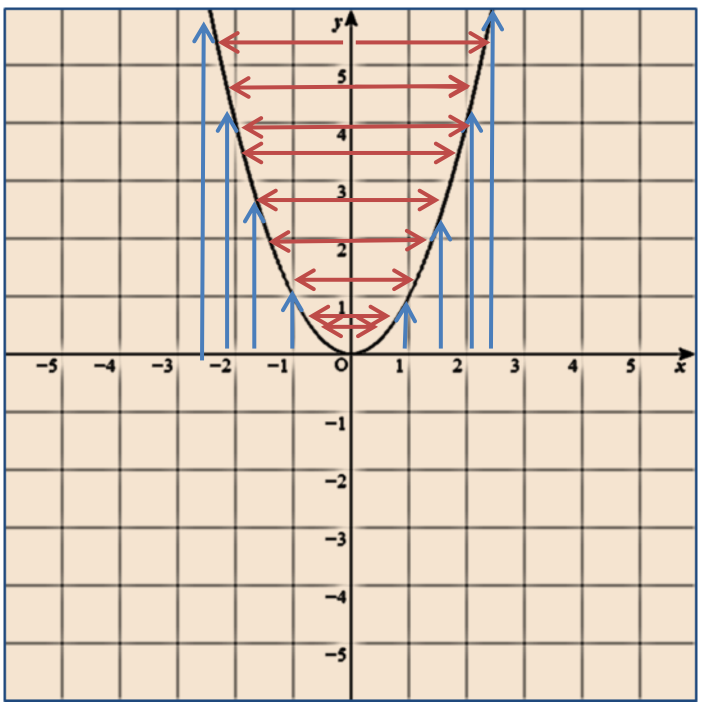
Domain $D_x = X \in R$

Why? All values of x on the $x-$axis would be mapped on the line as shown on the blue arrow (↑↓)

Range $R_y = y \in R$

Why? All values of y on the $y-$axis would be mapped on the line as shown by the maroon arrow (⇄)

(iv) $y = x^2$ where $x \in R$



Domain $D_x = X \in R$

Why? All values of x on the $x-$axis would be mapped on the line as shown on the blue arrow (↑↓)

Range $y \geq 0,\ y \in R$

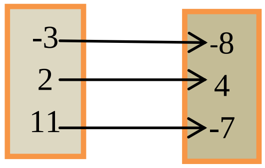
Why? Only values of y that is greater than and equal to zero on the y axis would be mapped on the line as shown on the maroon arrow (⇔)

Exercise 1.3

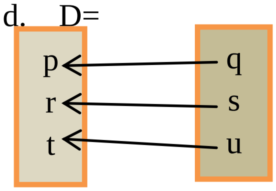
1. List the domain and range for each of the following functions

a. A = {(1, 6), (2, 8), (3, 10), (4, 12)} b. B ={-4, 13), (-3, 6), (-2, 1), (-1, -2), (0, -3)}

c. D =



d. D=



2. For the functions given as a rule, list the domain and range.

   a. $y = -2x - 3,\ x \in \{0, 1, 2, 3, 4\}$

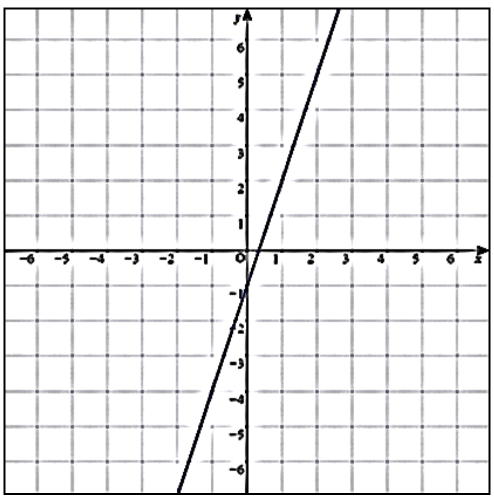
   b. $y = x^2 + 3,\ x \in \{-4, -3, -2, -1, 0\}$

   c. $f(x) = -3x^2,\ x \in \{-2, -1, 0, 1, 2\}$

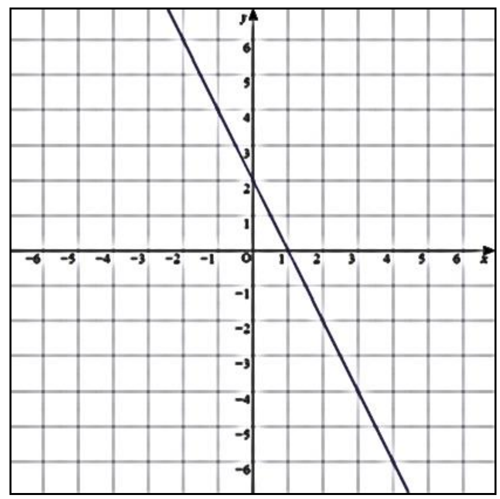
   d. $g(x) = \frac{2}{3}x - 3,\ x \in \{-9, -6, -3, 3, 6, 9\}$

3. List the domain and range for the functions given as a graph.

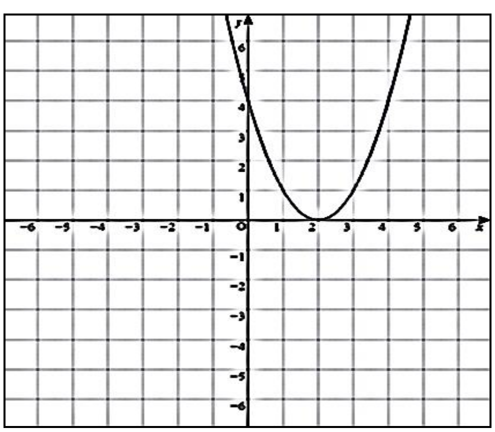
A.



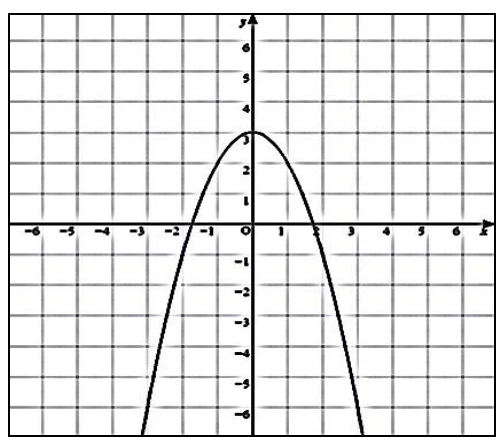
B.



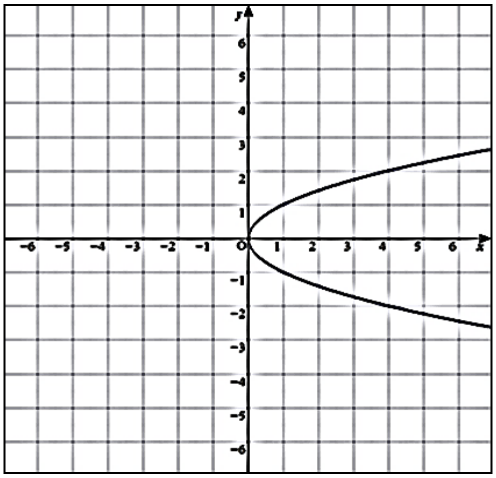
C.



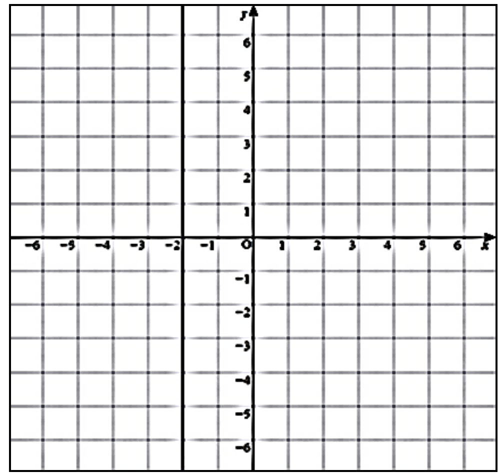
D.



E.



F.



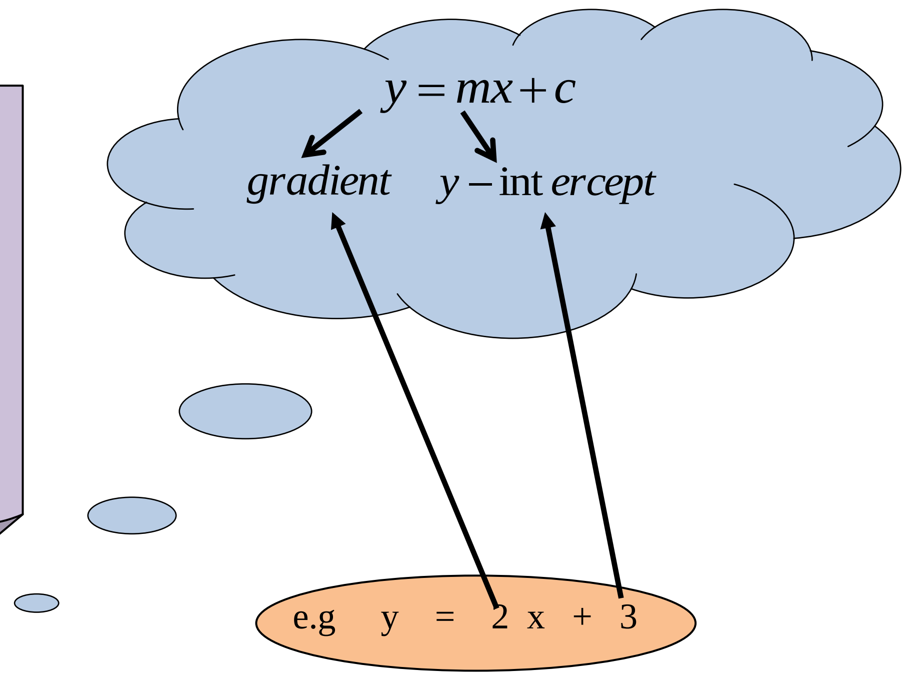
1.2 Graphing Equations and Inequations

LEARNING OUTCOMES

Students should be able to:

- Calculating intercepts and gradient of the linear equation in the form $y = mx + c$
- Draw graphs of linear equation
- Identifying intercepts from the graph of linear equation
- Determine and shade regions indicated by inequations.

Linear Equation: $y = mx + c$



I. Gradient

Definition: the slope of a line or how steep a line is.

Positive Gradient (m)

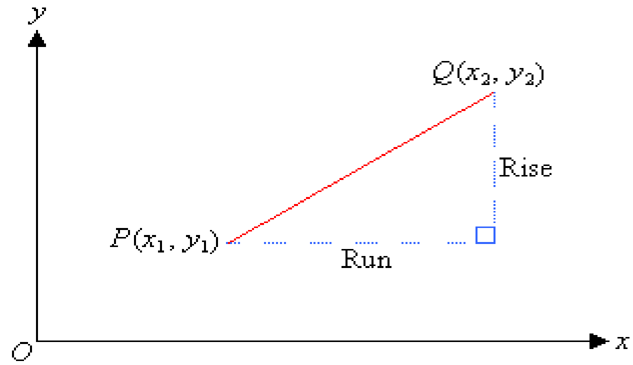
Negative Gradient (m)

II. Calculating Gradient

Gradient (m) is equal to distance along the $y-axis$ divided by the distance along the $x-axis$.

The **gradient** of a straight line is the rate at which the line rises (or falls) vertically for every unit across to the right. That is:

$$\begin{aligned}\text{Gradient} &= \frac{\text{Rise}}{\text{Run}} \\ &= \frac{\text{Change in } y}{\text{Change in } x} \\ &= \frac{y_2 - y_1}{x_2 - x_1}\end{aligned}$$



Note:

The gradient of a straight line is denoted by *m* where:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 1.6

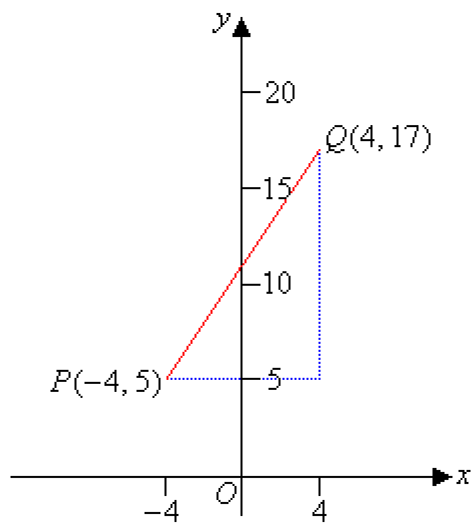
Find the gradient of the straight line joining the points *P*(– 4, 5) and *Q*(4, 17).

Solution:

Let $(x_1, y_1) = (-4, 5)$ and $(x_2, y_2) = (4, 17)$.

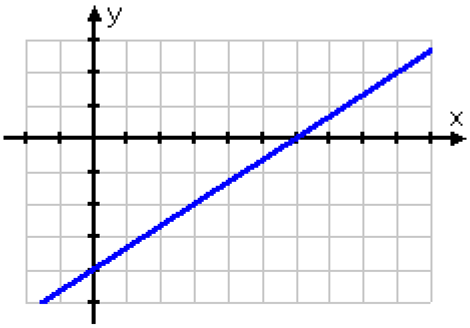
$$\begin{aligned}m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{17 - 5}{4 - (-4)} \\ &= \frac{12}{8} \\ &= 1.5\end{aligned}$$

So, the gradient of the line *PQ* is 1.5.

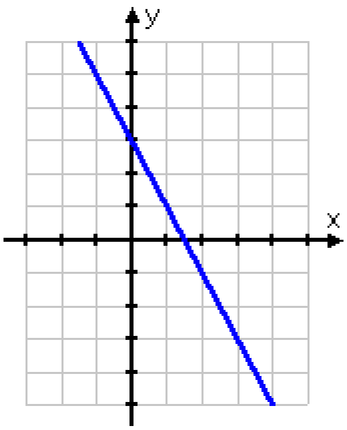


Note:

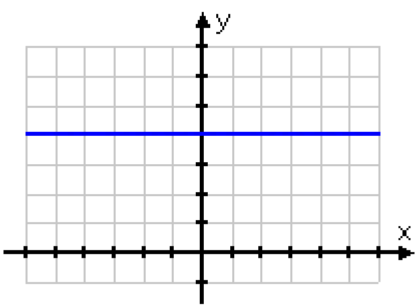
1. If the gradient of a line is **positive**, then the line **slopes upward** as the value of x increases



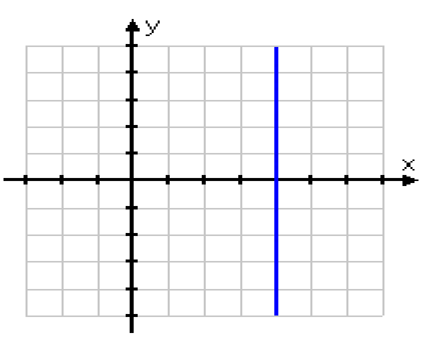
2. If the gradient of a line is **negative**, then the line **slopes downward** as the value of x increases



3. If the gradient of a line is **zero**, then the line is horizontal



4. If the gradient of a line is **undefined**, then the line is vertical



Example 1.7

Draw the graph for the linear equation: $y = 2x + 3$

Solution:

Method I

STEP: 1 Identify the gradient (m) and $y-intercept$

$\therefore$ $m = 2$ and $y-intercept$ =3 or (0,3)

STEP: 2 Define the gradient (m) and its movement on the Cartesian plane

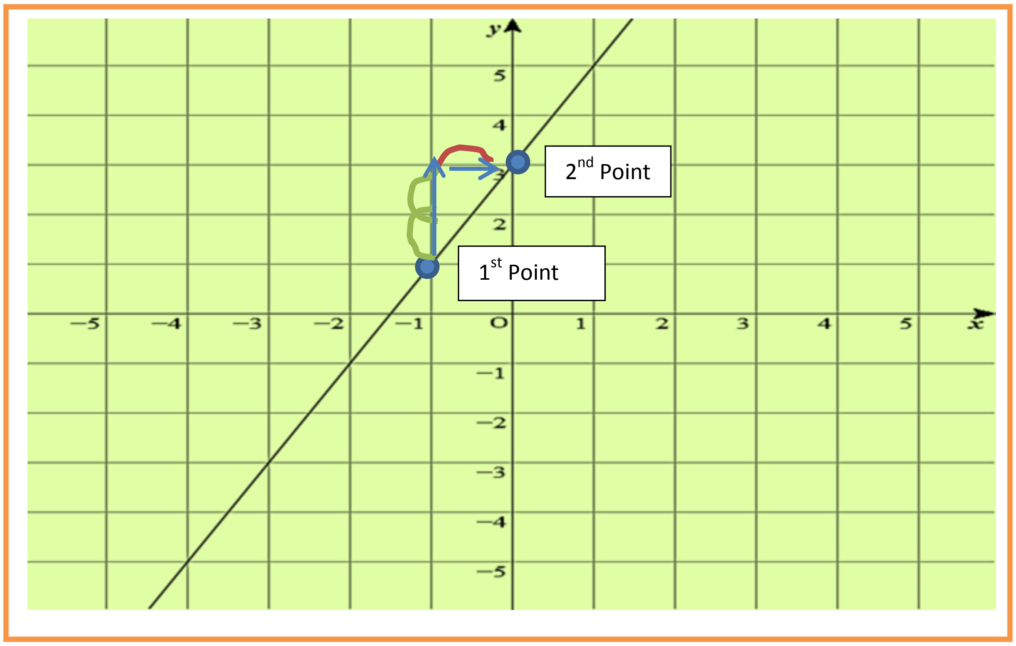
$\therefore$ m =2 as a fraction would read $\frac{2}{1}$ meaning 2 steps along the $y-axis$ (upward movement) and 1 movement along the $x-axis$ (movement to the right).

STEP: 3

Plot the $y-intercept$ on the Cartesian plane to get the first point

STEP: 4

From the $y-intercept$, using the gradient plot the second point, ie, 2 movement upwards and 1 movement to the right



Method II

Using table of values

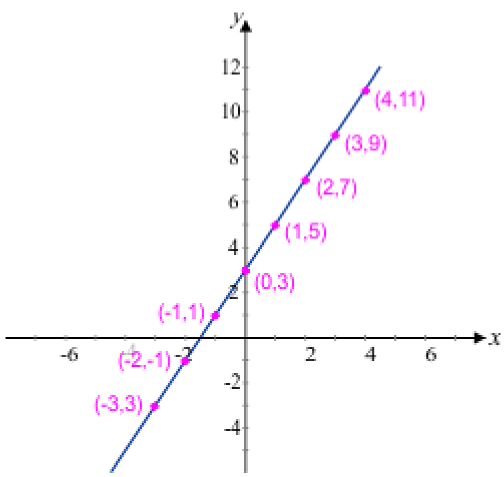
Step: 1 Choose some x values (negative, zero and positive values)

x	-2	-1	0	1	2

Step: 2 Substitute x values into the equation to get the y values

x	-2	-1	0	1	2
y	-1	1	3	5	7

Step: 3 Plot points on Cartesian plane join them with a line



Method III

Intercept method

Step: 1 Workout the x intercept by substituting y with 0, ie, y = 0

$$y = 2x + 3$$

$$0 = 2x + 3$$

$$-3 = 2x$$

$\frac{-3}{2}$ = x or ($\frac{-3}{2}$, 0)

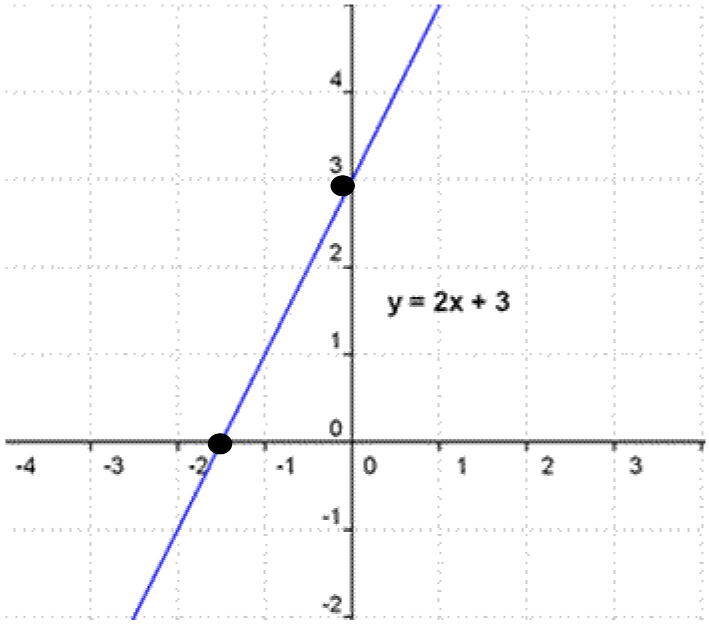
Step: 2 Workout the y intercept by substituting x with 0, ie, x = 0

$y = 2x + 3$

$y = 2(0) + 3$

$y = 3$ or $(0, 3)$

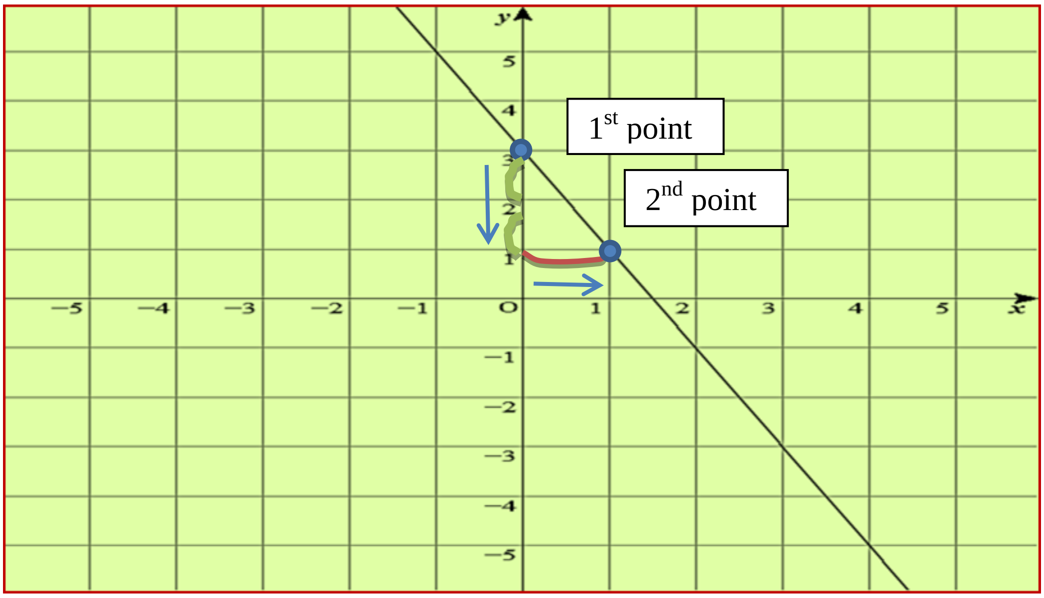
Step: 3 Plot the x and y intercepts on Cartesian plane and them with a straight line



Example 1.8

Draw the graph for the linear equation $y = -2x + 3$

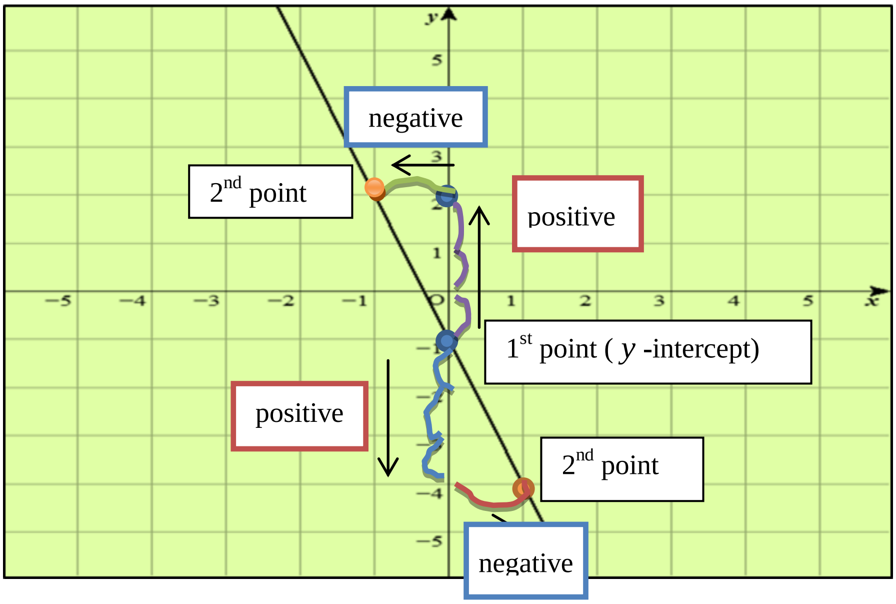
$\therefore m = -2$ and y-intercept = 3 or (0,3)



III Working out Equation of the Line from the given Linear Graph: $y = mx + c$

Example 1.9

For the graph given below, work out the equation of the line:



STEP: 1 Identify the y-intercept i.e the point at which the linear graph crosses on the y - axis $\therefore$ y-intercept = (0, -1)

STEP:2 Choose 2 points on the linear graph with the y-intercept as the 1st point

STEP:3 Use the 1st point as the starting point, move to the 2nd point by first moving along the y-axis and then along the x-axis.

NOTE

1. ***Positive y-axis movement:-upward movement***
2. ***Negative y-axis movement:- downward movement***
3. ***Positive x-axis movement:- movement to the right***
4. ***Negative x-axis movement:- movement to the left***

STEP:4 Calculate the gradient(m) using the formula i.e gradient is equal to movement along the y-axis divided by movement along the x-axis

$\therefore m = \frac{3}{-1}$ or $m = \frac{-3}{1}$

$\therefore m = -3$ $m = -3$

STEP:5 Substitute the value of the y-intercept and the gradient (m) in the equation

$$y = mx + c$$

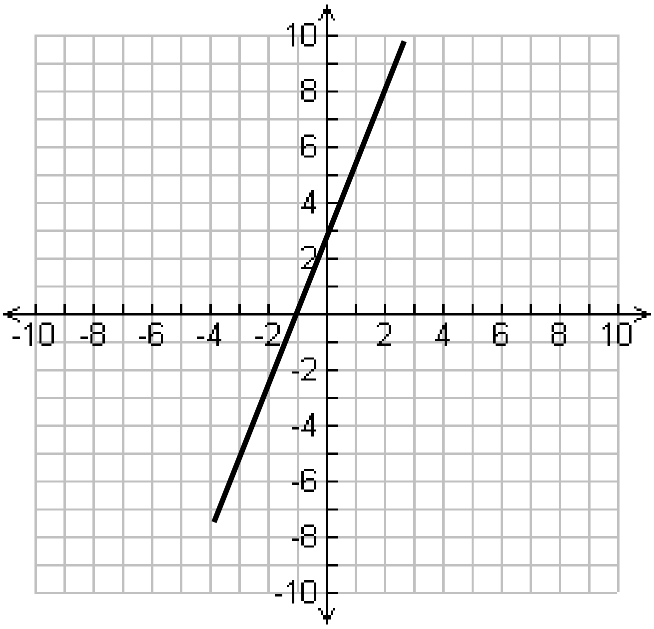
$\therefore$ Equation of the line: $y = -3x - 1$

-3 → m

-1 → $y-$intercept

Example 1.10

Work out the equation of the line given below



1. y intercept, y = 3 or (0, 3)

2. gradient, m = $\frac{rise}{run} = \frac{1}{1} = 1$

3. y = mx + c

   y = 1x + 3 or y = x + 3

"This one shows the time and money spent on making graphs."

Exercise 1.4

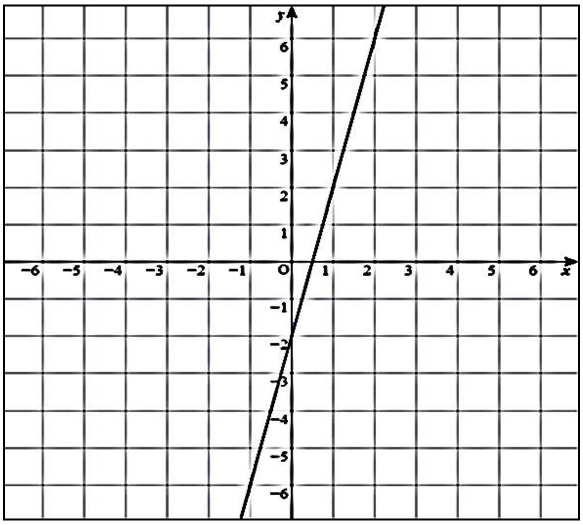
1. For the following equations, identify the gradient and the $y-intercept$.

   a. $y = x + 3$ b. $y = -3x - 1$ c. $y = \frac{2}{3}x + 4$ d. $y = 2 - \frac{1}{3}x$

2. Rearrange the equations to the form $y = mx + c$ and state the gradient and $y-intercept$

   a. $y - 3x = 2$ b. $y + 3 = 4x$ c. $y - 2x + 1 = 0$ d. $6x + 3y = -9$

3. A function is given by the graph below.



   (a) Find the gradient of the graph

   (b) Find the $y-intercept$

   (c) Write the equation of the function in the form $y = mx + c$

4. Draw graphs of the following linear functions using intercept method

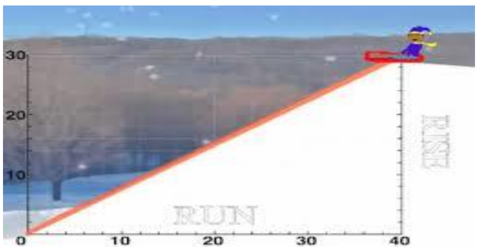
   a. y = x + 3 b. y = -x – 2 c. y = -2x + 1

   d. y + 3 – 3x = 0

5. Draw graphs of the following linear functions using table of values method.

   a. $y = -x - 5$ b. $y = 2x + 1$ c. $y - 2 = 2x$

   d. $y + 4x - 1 = 0$

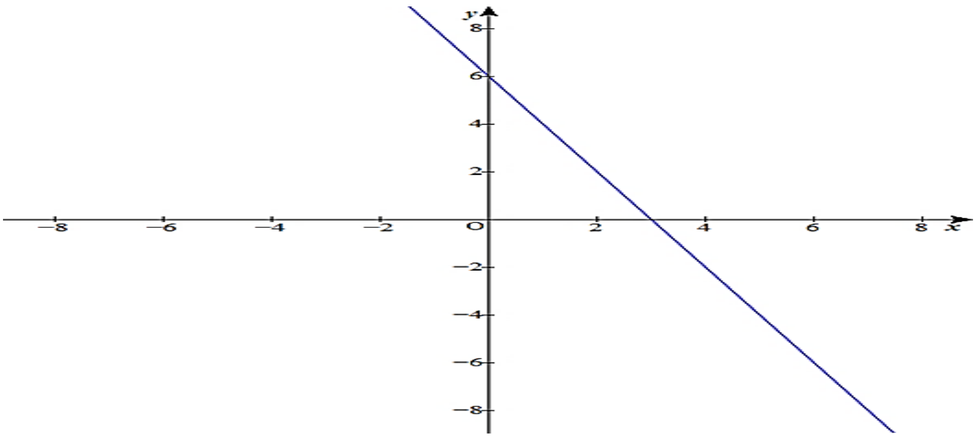


Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+cartoon+gradient+and+y+intercepts

6. Draw graphs of the following linear functions using the gradient, y intercept method.

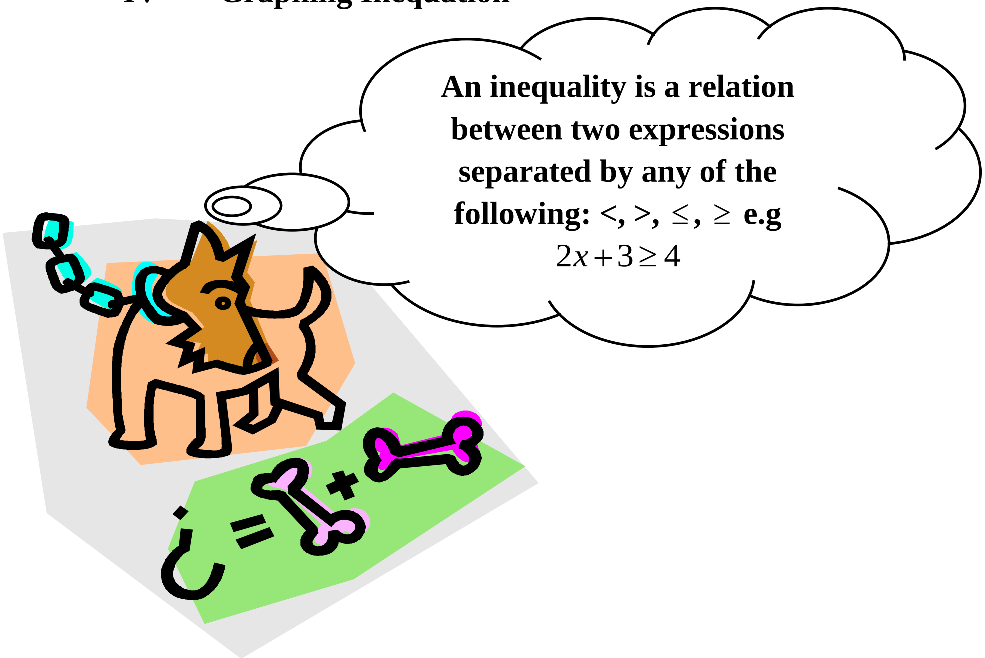
a. $y = 2x - 1$ b. $y = \frac{2}{5}x + 1$ c. $y - 6x + 4 = -4$

7. A straight line graph is shown below.



(a) Write the coordinates of the $y-intercept$.
(b) Calculate the gradient, m, slope of the line.
(c) Write the equation of the straight line shown in the form $y = mx + c$.
(d) If another line $x = 2$ is drawn on the same axes, what are the coordinates of the point of intersection.

IV Graphing Inequation



An inequality is a relation between two expressions separated by any of the following: <, >, ≤, ≥ e.g $2x + 3 \geq 4$

Example 1.11

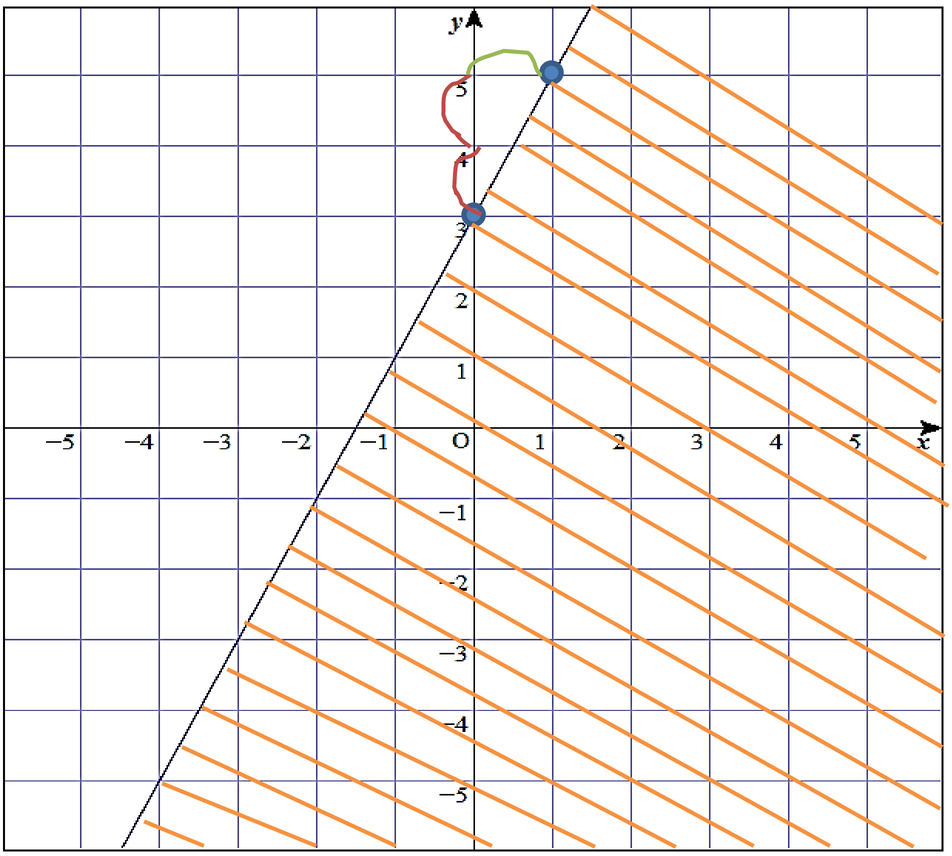
Graph the inequation $y \leq 2x+3$

<u>STEP: 1</u> Find the equal part i.e graph $y = 2x+3$

<u>STEP: 2</u> Shade the region indicated by the inequality sign

<u>NOTE:</u>

1. ***If the inequality sign is < or >, the line of the graph would have a broken line i.e*** - - - - - - - - - - - - - - - - -
2. ***If the inequality sign is ≤ or ≥, the line of the graph would be bold i.e*** ________________
3. ***If the inequality sign is ≤ or <, the shading would be below the linear graph***
4. ***If the inequality sign is ≥ or >, the shading would be above the graph***

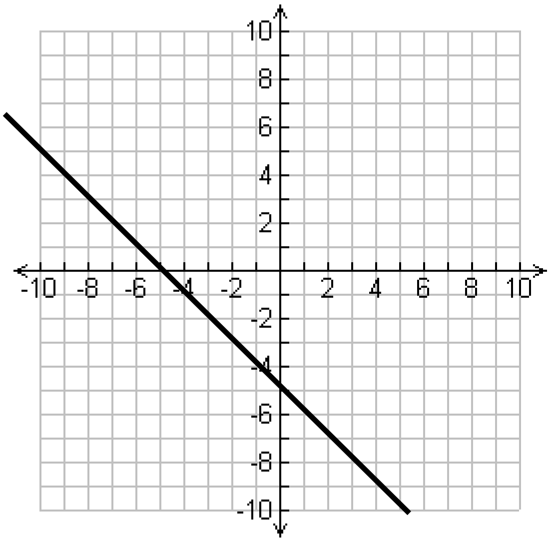


Example 1.12

Graph the inequation $y > -x - 5$

Step: 1 Graph $y = -x - 5$

y intercept = (0, - 5) x intercept = (-5, 0) gradient, m = -1

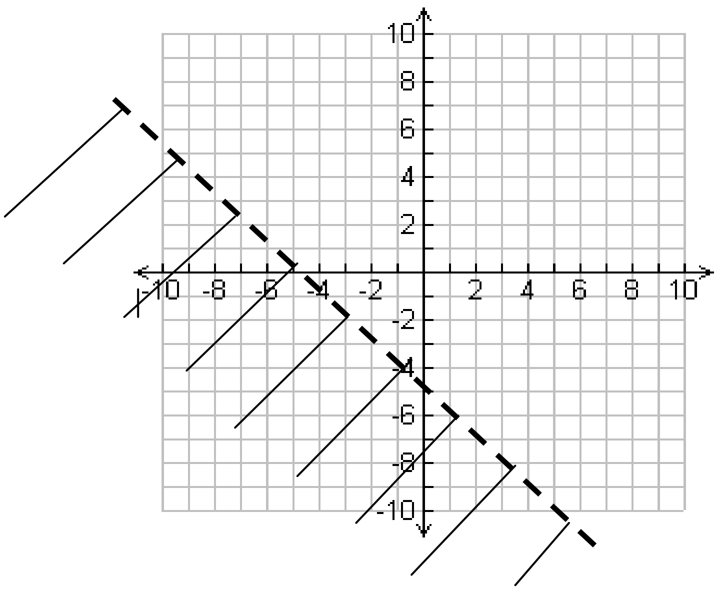


Step: 2 Look at inequality sign and draw graph accordingly

$y > -x - 5$

For less than - broken line is used.

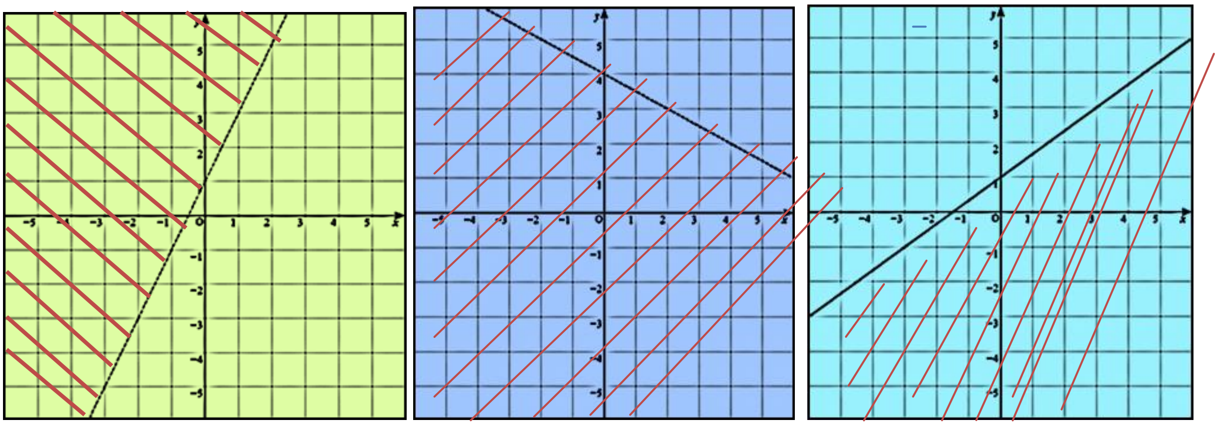
- area below the broken line is shaded



Exercise 1.5

1. For the given graphs, describe the inequalities.

a. b. c.



2. Draw the graphs of the following inequations.

a. $y \le x - 4$ b. $y > -2x + \frac{1}{2}$ c. $y - 5x \ge -2$ d. $y + 4x - 6 < -12$

Domain and range in real life situations

- *A local youth group is planning a trip to a local amusement park. They are taking their church bus which holds 32 people. It will cost $25 for parking and tickets to enter the park are $22.50 per person. The equation that models this situation is: c(n) = 22.5n + 25, where c represents the cost for the group to go to the park and n represents the number of people who go on this excursion.*

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+cartoon+youth+group

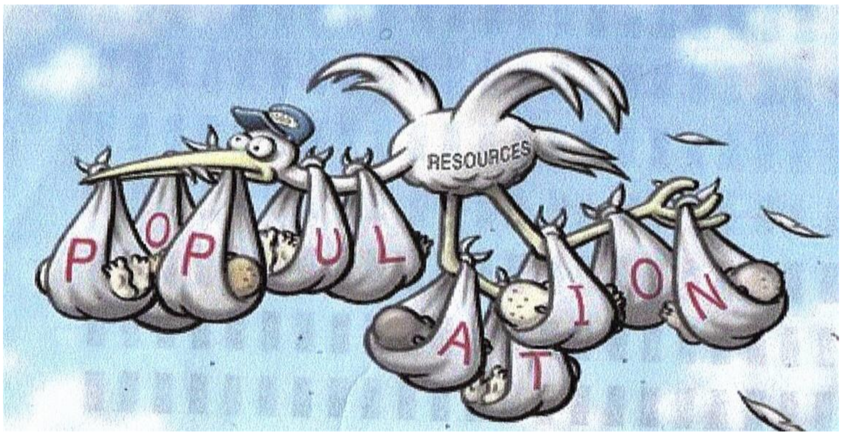
In this problem, for the domain, the problem says that the bus can only hold 32 people, so the domain has to be less than or equal to 32. However, since negative numbers are also less than 32 and impossible to have negative people (independent variable), It is a must to have a lower limit on domain of 0.To find the range values, simply use the limits set on the domain and substitute those values into the equation to find my limits on the range.

- ***A Function is given by a Table of Values below***

  The following table gives U.S. population in millions in the indicated year:

Year	*1960*	*1970*	*1980*	*1990*
U.S. Population (in millions)	*181*	*205*	*228*	*250*

Source: Statistical Abstracts of the United States,1993.



Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+cartoon+population

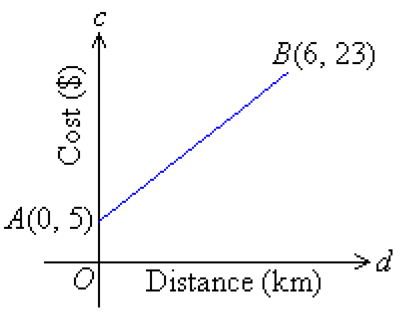
We can think of population as depending on time in years so the independent variable or input is the year and the dependent variable or output is the U. S. population. Since the table gives a unique population for each year, it represents a function. The domain is the set of years {1960, 1970, 1980, 1990} and the range is the set of populations in those years {181 million, 205 million, 228 million, 250 million}.

- *Peter needs to fill up his truck with gasoline to drive to and from school next week. If gas costs $2.79 per gallon, and his truck holds a maximum of 28 gallons, analyze the domain, range, and function values through the following questions.*

The domain is the number of gallons of gas purchased. On the graph, this is the possible x-values. The range is the costs of the gasoline. On the graph, this is the possible y-values.

Gradient in in real life situations

The cost of transporting documents by courier is given by the line segment drawn in the diagram. Find the gradient of the line segment; and describe its meaning.



Solution:

Let $(d_1, c_1) = (0, 5)$ and $(d_2, c_2) = (6, 23)$.

$$m = \frac{c_2 - c_1}{d_2 - d_1} = \frac{23 - 5}{6 - 0} = \frac{18}{6} = 3$$

So, the gradient of the line is 3. This means that the cost of transporting documents is $3 per km plus a fixed charge of $5, i.e. it costs $5 for the courier to arrive and $3 for every kilometre travelled to deliver the documents.

GLOSSARY		
12	Cartesian plane	A plane made up of an x axis (horizontal line)and y axis (vertical line)
11	Coordinates	Ordered set of numbers that define the position of a point
14	degree	The degree of a term is the exponent of the term
5	Domain	The set of x values
8	Equation	A written statement indicating the equality of two expressions
3	Function	A set of ordered pairs in which each x value has only one y value associated with it
16	Gradient (slope)	A measure of steepness of a line
15	In equation (inequality)	A relation that holds between two values when they are different A mathematical sentence built from expressions using one or more of the symbols $<$, $>$, $\geq, \leq$,
13	Index	A number to the right and above the base number
1	Intersection	The point where two lines intersect
4	Linear	Relating to a graph that is a linear, especially a straight line
9	Notations	Written symbols used to represent numbers
7	Quadratics	An equation where the highest exponent of the variable is a square (2)
6	Range	The set of y values
2	Relation	A set of ordered pairs
10	Variable	An element or feature that is liable to change
17	X intercept	It is where the graph cuts the x axis
18	Y intercept	It is where the graph cuts the y axis

STRAND 2 → ALGEBRA

HISTORY OF INEQUALITY SYMBOLS

Equal Sign

- *Before the equal sign came into popular use, equality was expressed in words. According to Lankham, Nachtergaele, and Schilling at University of California-Davis, the first use of the equal sign (=) came in 1557. Robert Recorde, 1510 to 1558, was the first to use the symbol in his work, "The Whetstone of Witte." Recorde, a Welsh physician and mathematician, used two parallel lines to represent equality because he believed they were the most equal things in existence.*

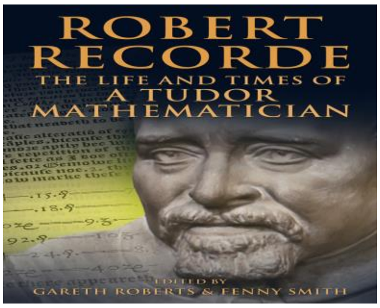
Inequalities

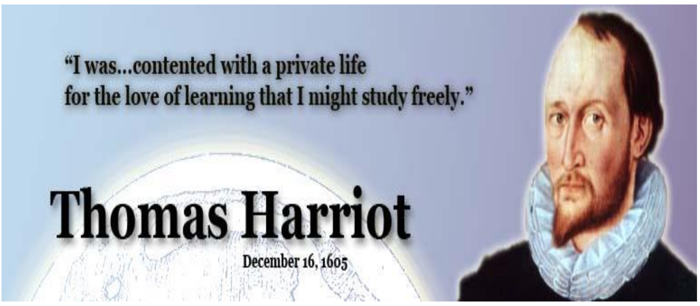
- *The signs for greater than (>) and less than (<) were introduced in 1631 in "Artis Analyticae Praxis ad Aequationes Algebraicas Resolvendas." The book was the work of British mathematician, Thomas Harriot, and was published 10 years after his death in 1621. The symbols actually were invented by the book's editor. Harriot initially used triangular symbols which the editor altered to resemble the modern less/greater than symbols. Interestingly, Harriot also used parallel lines to denote equality. However, Harriot's equal sign was vertical (II) rather than horizontal (=).*

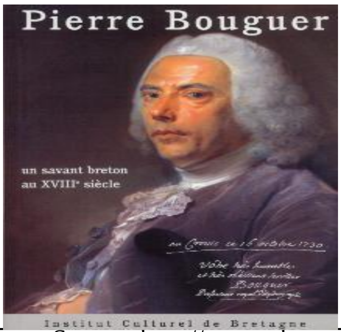
Less/Greater Than or Equal To

- *The symbols for less/greater than or equal to (< and >) with one line of an equal sign below them (≤ and ≥), were first used in 1734 by French mathematician, Pierre Bouguer.*

  John. Wallis, a British logician and mathematician, used similar symbols in 1670. Wallis used the greater than/less than symbols with a single horizontal line above them







Source: http://www.ehow.com/info_8143072_history-equality-symbols-math.html

2.1 Factorisation and Simplification of Algebraic Expressions

Factorisation

LEARNING OUTCOME

Students should be able to:

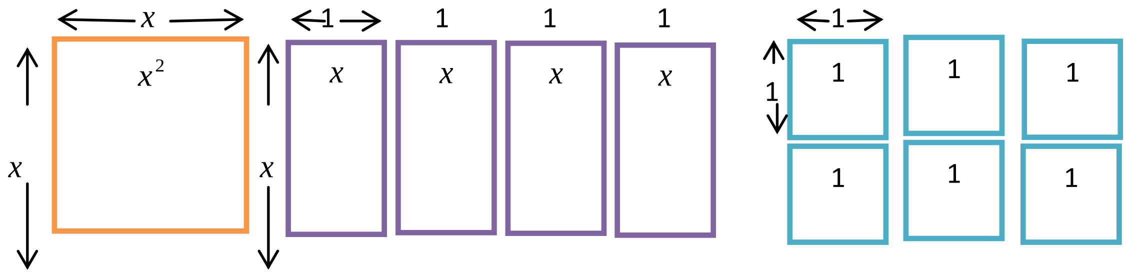
- Factorizing algebraic expressions by applying:
  - ❖ Common Factor method.
  - ❖ Grouping method
  - ❖ Difference of squares method.
  - ❖ Perfect square method.

Activity:

Let us make various rectangles out of squares and rectangles.

HOW?

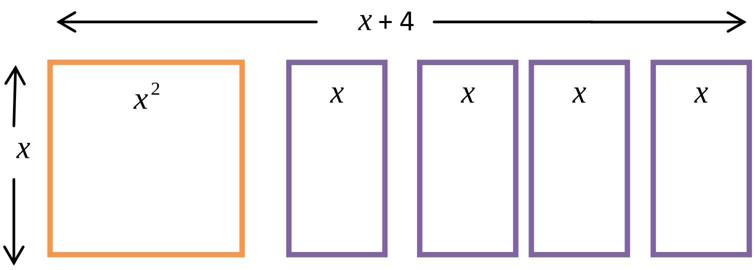
Step: 1 Cut out the following squares and rectangles.



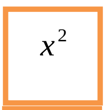
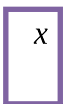
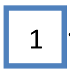
Step: 2 Make rectangles from the cut out squares and rectangles in step 1 above.

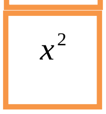
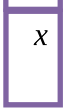
Let's say "I use one piece of x^2 and 4 pieces of x"

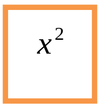
What is the area of the new rectangle?

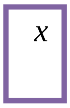


Step: 3 Now make some more rectangles using the square and rectangular pieces indicated below.

a) 1 piece of  , 2 pieces of  and 2 pieces of  .

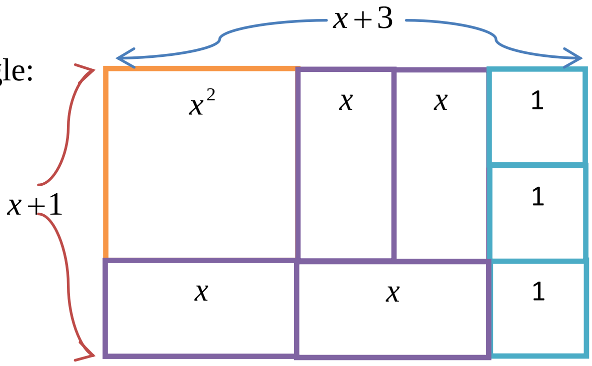
b) 1 piece of  , 3 pieces of  and 2 pieces of 

Note: If we are to make a rectangle out of 1 piece of 

, 4 pieces of 

and 3 pieces of 

Then we would get this rectangle:



Therefore:

1. The sum of the areas of all the pieces would be x^2+4x+3
2. The area of the new and bigger rectangle would be $(x+1)(x+3)$

Finally: x^2+4x+3 is the **product** of $(x+1)(x+3)$ which means that $(x+1)$ and $(x+3)$ are **factors** of x^2+4x+3.

Factorization is rewriting an expression or polynomial as a product of its factors.

Factorization

$$x^2+4x+3=(x+1)(x+3)$$

Expansion

Using Formula for Factorization:

1. Common Factor Method:

When all terms in a polynomial share a common factor, the expression can be factorized by taking the common factor outside the brackets as in the distributive law.

Example 2.1

Factorize: $2x^2+3xy$

$$\therefore 2x^2 = 2\times x\times x$$

And $$3xy = 3\times x\times y$$

The two terms have x as a common factor which is put outside the brackets to be distributed to the left over terms as shown: $2x^2+3xy = x(2x+3y)$

Example 2.2

Factorize: $3x^2 + 6xy - 9x$

$$\therefore 3x^2 = 3 \times x \times x$$
$$6xy = 2 \times 3 \times x \times y \quad \Rightarrow \quad 3x^2 + 6xy - 9x = 3x(x + 2y - 3)$$
$$-9x = 3 \times -3 \times x$$

2. Grouping Method

This is done by grouping a pair of terms. Then, factor each pair of two terms

Example 2. 3

Factorize: $p^2 + 6p + 2p + 12$

Note: A. Make 2 groups having the same common factor then factorize:

$$(p^2 + 6p) + (2p + 12)$$

Group 1: $p^2 + 6p = p(p+6)$ + Group 2: $2p + 12 = 2(p+6)$

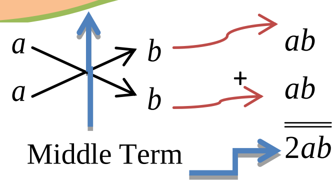
B. What is common to both groups will make one factor and the second factor would be made up of the left over factors.

$$p^2 + 6p + 2p + 12 = (p+6)(p+2)$$

Generalization:

Find factors of the first term and the second term separately, and then cross multiply. The middle term should come from adding the diagonals.

i.e. $a^2 + 2ab + b^2$



3. Quadratics

3a Difference of Squares: $a^2 - b^2 = (a+b)(a-b)$

When the sum of two numbers multiplies their difference $(a + b)(a - b)$ then the product is the difference of their squares, $a^2 - b^2$.

Proof

1. $a^2 - b^2 = (a + b)(a - b)$

Use the distributive property to expand the right - hand side.

$$= a^2 - ab + ba - b^2.$$

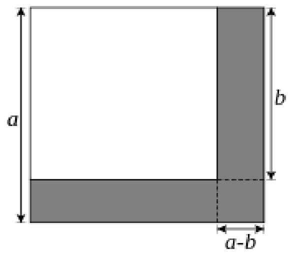
Apply the commutative Law to *ba* so that $ba = ab$ $(b \bullet a = a \bullet b)$ or $ba - ab = 0$

$$= a^2 - b^2$$

Thus *LHS* $= RHS$

2. The difference of two squares can also be shown geometrically.

In the diagram given below, the shaded part represents the difference between the bigger square (a^2) and the smaller square (b^2) which is equal to $a^2 - b^2$.



The area of the shaded part can be worked out by adding the following:

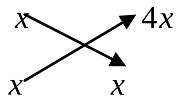
Rectangle	1	$= a(a - b)$
Rectangle	2	$= b(a - b)$
		$= a(a - b) + b(a - b)$

This can be factorized to $(a + b)(a - b)$

Thus $a^2 - b^2 = (a + b)(a - b)$

Example 2.4

Factorize: $x^2 - 16$



$-4x$

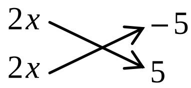
$0x$

$= (x-4)(x+4)$

Note: $0x$ **means that there is no middle term**

Example 2.5

Factorize: $4x^2 - 25$



$-10x$

$+10x$

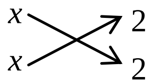
$0x$

$= (2x-5)(2x+5)$

3b Perfect Squares: $a^2 + 2ab + b^2 = (a+b)^2$ **or** $a^2 - 2ab + b^2 = (a-b)^2$

Example 2.6

Factorize: $x^2 + 4x + 4$



$+$ $2x$

$2x$

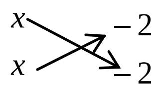
$4x$

$= (x+2)(x+2)$

$= (x+2)^2$

Example 2.7

Factorize: $x^2 - 4x + 4$



$+$ $-2x$

$-2x$

$-4x$

$= (x-2)(x-2)$

$= (x-2)^2$

Exercise 2.1

Factorize the following expressions

1. a) $2x+2$ b) $x^2 - 3x$ c) $-3xz + 6yz$

   d) $9x - 6x^2$ e) $4x - 8y + 6$ f) $4y^2 - 2y^3$

   g) $a^2 + a + 2a + 2$ h) $b^2 - 4b + 2b - 8$ i) $y^2 - 3y - 4y + 12$

2. a) $x^2 + 12x + 36$ b) $x^2 - 22x + 121$ c) $x^2 + 16x + 64$

   d) $4x^2 + 12x + 9$ e) $9x^2 - 12x + 4$ f) $\frac{x^2}{4} - 2x + 4$

3. a) $x^2 - 36$ b) $x^2 - 9^2$ c) $9x^2 - 81$

   d) $\frac{1}{4}x^2 - 9$ e) $\frac{x^2}{36} - \frac{y^2}{49}$ f) $4a^2 - \frac{y^2}{16}$

4. Use the pattern $a^2 - b^2 = (a+b)(a-b)$ to evaluate $32^2 - 31^2$

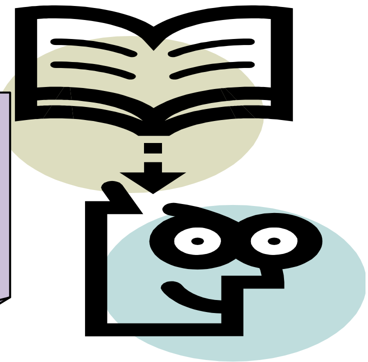
5. Marika bought a square plot of land whose length is $(4y+1)$ cm and then made a footpath on which he could walk around while tending to his vegetable garden. If the vegetable plot which is also square in shape but inside the square plot of land has length $(3y)$ cm, calculate the area of the footpath.

Square Roots

LEARNING OUTCOME

Students should be able:

- Introduce the formal treatment of square roots



WHAT CAN I MULTIPLY BY ITSELF TO GIVE THIS????

SYMBOL: $\sqrt{\ }$ - makes mathematics looks important and is called the **radical.**

The **square root** of a number is a value that can be multiplied by itself to give the original number.

Example 2.8

$\sqrt{4} = \pm\sqrt{4}$

$= \pm 2$

i.e. $2 \times 2 = 4$
$-2 \times -2 = 4$

Example 2. 9

$\sqrt{25} = \pm\sqrt{25}$

$= \pm 5$

i.e. $5 \times 5 = 25$
$-5 \times -5 = 25$

Square Root Generalizations:

1. $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

   e.g. $\sqrt{12} = \sqrt{3} \times \sqrt{4}$

2. $\sqrt{a^2} = \sqrt{a \times a}$ e.g. $\sqrt{2^2} = \sqrt{2 \times 2}$

   $= \sqrt{a} \times \sqrt{a}$ $\quad \sqrt{2} \times \sqrt{2}$

   $= a$ $\quad = 2$

3. A positive number has 2 square roots i.e absolute values are equal but signs are different. e.g square root of 4 are $\sqrt{4}$ and $-\sqrt{4}$ or $\pm\sqrt{4}$

4. $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$ e.g $\dfrac{\sqrt{25}}{\sqrt{16}} = \sqrt{\dfrac{25}{16}} = \dfrac{5}{4}$

5. $\sqrt{0} = 0$

6. $\sqrt{a+b} \neq \sqrt{a}+\sqrt{b}$ e.g $\sqrt{4+9} \neq \sqrt{4}+\sqrt{9}$

$$\sqrt{13} \neq 2+3$$

Exercise 2.2

Calculate the following:

a. $\sqrt{5}\times\sqrt{10}$

b. $\sqrt{9}\times\sqrt{16}$

c. $\sqrt{6}\times\sqrt{42}$

d. $2\sqrt{3}\times\sqrt{3}$

e. $3\sqrt{5}\times 2\sqrt{5}$

f. $\sqrt{x^2}\times\sqrt{x^2}$

g. $\sqrt{(x+1)^2}$

h. $\sqrt{(2x-1)^2}+\sqrt{x^2}$

i. $\dfrac{\sqrt{4x^2}}{\sqrt{16}}$

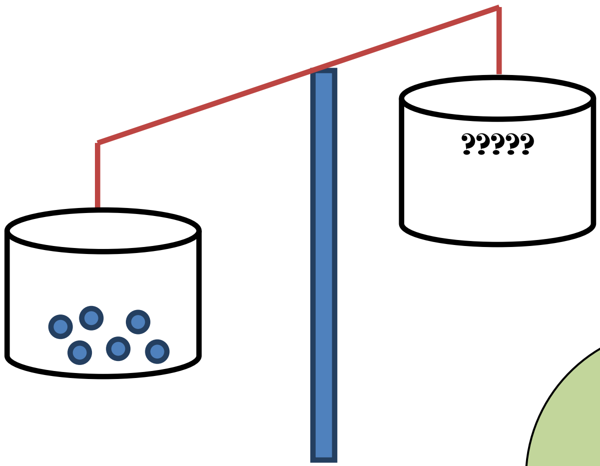
j. $\dfrac{\sqrt{9}}{\sqrt{4}}(\sqrt{(x-1)^2})$

2.2 Solving Equations & Inequations

LEARNING OUTCOMES

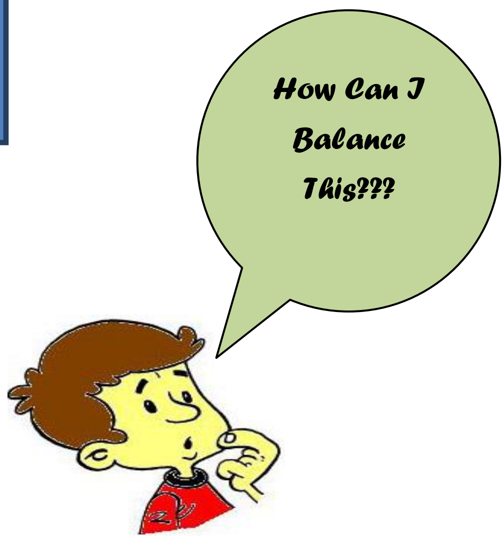
Students should be able to:

- Solve linear equations involving variables on both sides of the equation.
- Solve linear inequations involving variables on both sides of the inequality sign.
- Solve equations by applying



What is an equation?

A mathematical statement that says two things are equal.



2.2.1 Linear Equations Having Variables on Both Sides

Example: 2.10

Solve for the missing variable in the given equation

a. $2x+3=x-2$

Step: 1 Collect the like terms

$2x-x=-2-3$

When collecting the like terms; to move one term from the right of the equal sign to the left or vice-versa, the sign changes.

Step: 2 Simplify both sides

$x=-5$

b.
$$x-2=4x+4$$
$$\therefore -2-4=4x-x$$
$$\therefore -6=3x$$
$$\therefore \frac{-6}{3}=\frac{3x}{3}$$
$$\therefore -2=x$$

- Remove brackets
- Collect like terms
- Simplify like terms
- Find the unknown

c.
$$\therefore \frac{2x}{3}=x+4+2$$
$$\therefore 2x=3(x+6)$$
$$\therefore 2x-3x=18$$
$$\therefore -x=18$$
$$\therefore \frac{-x}{-1}=\frac{18}{-1}$$
$$\therefore x=-18$$

Variable should be alone on one side by removing other terms to the other side using opposite operations

Exercise 2.3

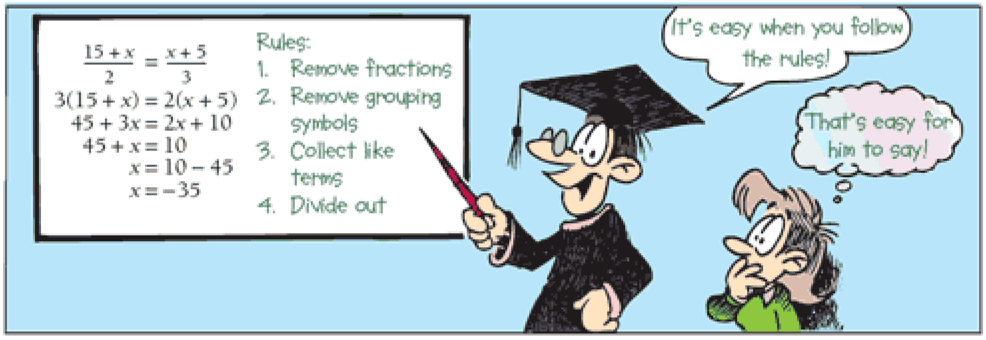
Solve for the missing variable in the following equations.

a. $6y=4y+12$	b. $5b+7=3b+15$	c. $4a-10=2a+14$
d. $15-5n=6-2n$	e. $12+5x=x-2$	f. $11w-9=7w-30$
g. $2(3x-1)=5(2x-6)$	h. $h-7=2(h-4)$	i. $2(2q-5)=q+11$
j. $3x-2=2(x+4)-1$	k. $z+3(z+2)=14+2z$	l. $2(x-5)+3(x+3)=24$

Linear equations in real life situations

Equations may look scary, but you use and solve linear equations every day of your life, whether you know it or not.

One of the realities of life is how so much of the world runs by mathematical rules. As one of the tools of mathematics, linear systems have multiple uses in the real world.



Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+cartton+linear+equations

Life is full of situations when the output of a system doubles if the input doubles, and the output cuts in half if the input does the same. That's what a linear system is, and any linear system can be described with a linear equation.

In the Kitchen

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+kitchens

If you've ever doubled a favorite recipe, you've applied a linear equation. If one cake equals 1/2 cup of butter, 2 cups of flour, 3/4 tsp. of baking powder, three eggs and 1 cup of sugar and milk, then two cakes equal 1 cup of butter, 4 cups of flour, 1 1/2 tsp. of baking powder, six eggs and 2 cups of sugar and milk. To get twice the output, you put in twice the input. You might not have known you were using a linear equation, but that's exactly what you did.

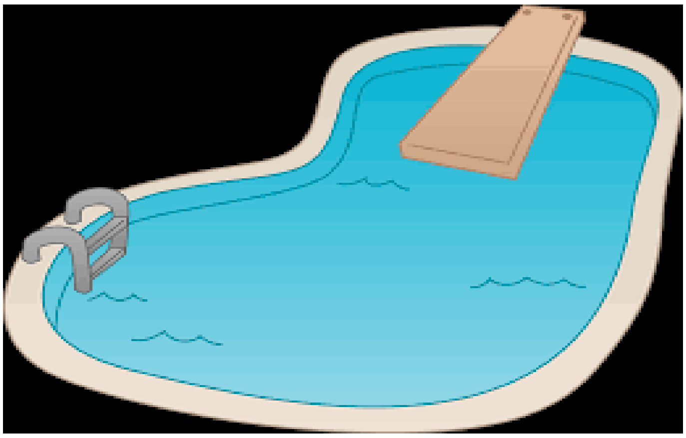
Melting Snow

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=image+of+melting+snow+

*Suppose a water district wants to know how much snowmelt runoff it can expect this year. The melt comes from a big valley, and every year the district measures the snowpack and the water supply. It gets 60 acre-feet from every 6 inches of snow pack. This year surveyors measure 6 feet and 4 inches of snow. The district put that in the linear expression (60 acre-feet/6 inches) * 76 inches. Water officials can expect 760 acre-feet of snowmelt from the water.*

Question

It's springtime and Mrs Bula wants to fill her swimming pool. She sees that it takes 25 minutes to raise the pool level by 4cm. She needs to fill the pool to a depth of 1metre; she has 44 more cm to go.



Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=pictures+of+cartoon+swimming+pools

She figures out her linear equation: 44 cm (25 minutes/4 cm is 275 minutes, so she knows she has four hours and 35 minutes more to wait.*

Question
Brian has noticed that it's springtime. The grass has been growing. It grew 2 cm in two weeks. He doesn't like the grass to be taller than 2 1/2 cm, but he doesn't like to cut it shorter than 1 3/4 cm. How often does he need to cut the lawn?

Source: http://landscaping.about.com/od/lawns/a/spring_lawns.htm

*He needs to put that calculation in his linear expression, where (14 days/2 cm) * 3/4 cm tells him he needs to cut his lawn every 5 1/4 days. He just ignores the 1/4 and figures he'll cut the lawn every five days.*

Question
A 45 feet of wood to use for making a bookcase. If the height and width are to be 10 feet and 5 feet, respectively, how many shelves can be made between the top and bottom of the frame?

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+bookcases

To solve this equation, we can use a linear relationship:

$$Nv+Mh=45$$

where v and h respectively represent the length in feet of vertical and horizontal sections of wood. N and M represent the number of vertical and horizontal pieces, respectively. Knowing that there will be only two vertical pieces, this formula can be simplified to:

$$2\cdot10+M\cdot5=45$$

Question

Consider a shirt that costs $24 when on a 40% discount. If the original price is x, find x

Answer

$x - 0.4 \cdot x = 24$

Solving for x, we find that the original price was $40.

Using similar models we can solve equations pertaining to distance, speed, and time (Distance = Speed * Time); density (Density = Mass/ Volume); and any other relationship in which all variables are first order.

Source: Boundless. "Linear Equations and Their Applications." *Boundless Algebra*. Boundless, 03 Jul. 2014. Retrieved 03 Feb. 2015 from https://www.boundless.com/algebra/textbooks/boundless-algebra-textbook/functions-equations-and-inequalities-3/linear-equations-and-functions-22/linear-equations-and-their-applications-121-5519/

Everywhere

It's not hard to see other similar situations. If you want to buy drinks for the big party and you've got $60 in your pocket, a linear equation tells you how much you can afford. Whether you need to bring in enough wood for the fire to burn overnight, calculate your paycheck, figure out how much paint you need to redo the upstairs bedrooms or buy enough gas to make it to and from your Aunt Sylvia's, linear equations provide the answers. Linear systems are, literally, everywhere

2.2.2 Linear Inequations Having Variables on Both Sides

What is an inequation?

It is an algebraic sentence which has the inequality signs ($<, >, \leq, \geq$) instead of the equal sign (=).

Very Important Note!!!

- *Inequations are solved in the same way as equations.*
- *Inequations usually have many solutions as compared to equations.*

  e.g. $x+5=7$ $\quad x+5>7$

  $\therefore x=2$ $\quad \therefore x>2$
- *When an inequation is divided by -1, the inequality sign is reversed to make the statement true.*

One solution

Infinite solution

i.e. any number greater than 2

Symbol	*Meaning*
$\leq$	*Less than or equal to*
$\geq$	*Greater than or equal to*
$<$	*Less than*
$>$	*Greater than*

Exercise 2.4

Solve the following inequations.

a. $5x+4>x+10$ b. $4-y\leq 7-2y$ c. $w+2<2w-9$

d. $15-q\geq 2q+27$ e. $3(h+7)\leq 2(h+5)$ f. $\frac{y}{3}>7-\frac{y}{2}$

Linear inequations in real life situations

Inequalities are very common in daily life. For example:

You can work a total of no more then 41 hours each week at your two jobs. House cleaning pays \$5 per hour and your sales job pays \$8 per hour. You need to earn at least \$254 each week to pay your bills. Write a system of inequalities that shows the various numbers of hours you can work at each job.

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+house+cleaning

x = housecleaning
y = sales job
Hours: $x + y \leq 41$
Money: $5x + 8y \geq 254$

Fuel x costs $2 per gallon and fuel y costs $3 per gallon. You have at most $18 to spend on fuel. Write and graph a system of linear inequalities to represent this situation.

x = fuel x
y = fuel y

Price: $2x + 3y \leq 18$
Gallons of x: $x \geq 0$
Gallons of y: $y \geq 0$

A salad contains fish and chicken. There are at most 6 pounds of fish and chicken in the salad. Write and graph a system of inequalities to represent this situation.

Source: http://photobucket.com/images/salad?page=1

x = fish
y = chicken

Total Pounds: $x + y \leq 6$
Pounds of fish: $x \geq 0$
Pounds of chicken: $y \geq 0$

Mary babysits for $4 per hour. She also works as a tutor for $7 per hour. She is only allowed to work 13 hours per week. She wants to make at least $65. Write and graph a system of inequalities to represent this situation.

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+babysitter

x = babysitting
y = tutoring

Hours: $x + y \leq 13$
Money: $4x + 7y \geq 65$

Exercise 2.5

1. Seru has $500 at his savings account at the beginning of summer. He wants to have at least $200 at the end of the summer. He withdraws $25 each week for food and clothes.

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+bank

(a) Write an equation that represents Seru's situation
(b) How many weeks can Seru withdraws money from his account? Justify your answer.

2. Marshall Taxi charges a $1.50 rate in addition to $0.85 for every kilometer. Mr Maklu has no more than $10 to spend on a ride.

   (a) Write an inequation that represents Mr Maklu's situation
   (b) How many kilometers can he travel without exceeding his limit? Show your working.

2.2.3 Solving Quadratic Equations

2.2.3.1 Using the Square Root Law

What would be the difference between the solutions in a linear equation to that of a quadratic equation?

Linear equations have one solution while quadratic equations have 2 solutions.

Example 2: 11

Find all solutions to the following equations.

a)

$$x^2 = 9$$

$$\therefore \sqrt{x^2} = \pm\sqrt{9}$$

$$\therefore x = \pm 3$$

$$\text{i.e.} \begin{pmatrix} x = 3 \\ x = -3 \end{pmatrix}$$

b)

$$x^2 - 5 = 11$$

$$\therefore x^2 = 11 + 5$$

$$\therefore x^2 = 16$$

$$\therefore \sqrt{x^2} = \pm\sqrt{16}$$

$$\therefore x = \pm 4$$

$$\text{i.e.} \begin{pmatrix} x = 4 \\ x = -4 \end{pmatrix}$$

To remove x^2 we will have to $\sqrt{\ }$ on both sides since square root ($\sqrt{\ }$) is the opposite of square.

The square root of a number will therefore have 2 values i.e. the positive value and the negative value.

2.2.3.2 Using the Null Factor Law

If multiplying any two numbers is zero, then one or both of the numbers are zero, i.e. if $ab = 0$ **,then** $a = 0$ **or** $b = 0$. This is the Null Factor Law which is often used to solve quadratic functions or other functions which could have more than 2 solutions.

Example 2. 12

Find all solutions to the following equations.

a. $(x+2)(x-3) = 0$

using the null factor l

$\therefore x + 2 = 0$ and $x - 3 = 0$

$\therefore x = 0 - 2$ and $x = 3$

$\therefore x = -2$ and $x = 3$

b. $(x+3)(x-1)(x+4) = 0$

$\therefore x + 3 = 0$, $x - 1 = 0$ and $x + 4 = 0$

$\therefore x = 0 - 3$, $x = 0 + 1$ and $x = 0 - 4$

$\therefore x = -3$, $x = 1$ and $x = -4$

2. $(x-4)(x+5)(x$

4. $(x-4)^2 = 25$

5. $2x(x-2) = 0$

6. $(2x+1)(3x+6) = 0$

7. $(x-2)^2 + 2 = 18$

8. $x^2 - 64 = 0$

9. $x^2 - 10x = -25$

2.3 Formula Manipulation

LEARNING OUTCOME

Students should be able to:

- manipulate the original formula using the inverse operations

$A = \pi r^2$ what will r equal to if $A = 25cm^2$ and $\pi = 3.14$

Definition:1

Formula: An equation which tells how variables are related to one another.

Definition:2

Subject of formula: Single variable on the left hand side of the equation with everything else going on the right hand side

Definition: 3

Changing the subject of formula: Begins with the variable to become the new subject and applying inverse operations on the other variables.

Example 2. 13

Given the formula for the area of a circle is $A = \pi r^2$, make r the subject of formula.

$$A = \pi r^2$$

$$\therefore \frac{A}{\pi} = \frac{\pi r^2}{\pi}$$

Divide by π on both sides of the equation to remove π from the right hand side and move the variable to the left hand side using the same operation.

$$\therefore \frac{A}{\pi} = r^2$$

$$\therefore \sqrt{\frac{A}{\pi}} = \sqrt{r^2}$$

$\sqrt{\ }$ on both sides to remove square (^{2})from the right hand side and move the variable to the left hand side using the same operation

$$\therefore \sqrt{\frac{A}{\pi}} = r$$

Now r is the new subject of formula

Exercise 2.6

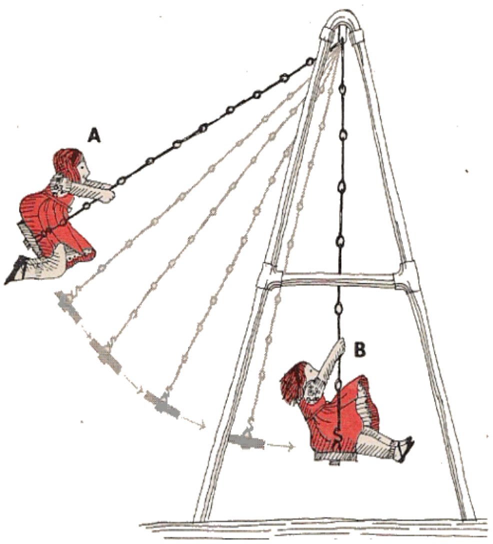
For each of the equations given, make the variables given in brackets the new subject of formula.

1. $v = u + at\ (u)$	2. $v = u + at\ (t)$	3. $y = mx + c\ (x)$
4. $2p - a = 3p + b\ (p)$	5. $P = (m + M)f\ \ (M)$	6. $V = \frac{1}{3}\pi r^2 h\ (r)$
7. $A = \frac{1}{2}b \times h\ (b)$	8. $y = 5x - \frac{z}{3}\ (x)$	9. $2s = (u + v)t\ \ (t)$

Formula manipulation is also applied in others subjects such as Physics, Chemistry, Economics

Physics

Kinetic and Potential Energy



$$E = mgh + \frac{1}{2}mv^2$$

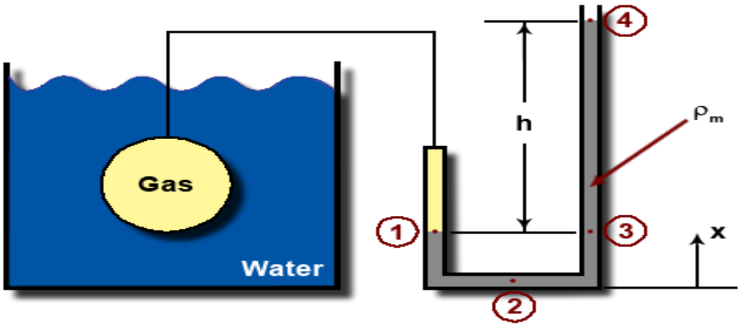
Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=kinetic+and+potential+energy

Make *m* the subject of the formula
Where m = mass of the body,

h is the height attained due to the body's displacement and
g is the acceleration due to gravity which is constant on earth
v is the velocity of the body

Chemistry

Ideal gas law



$$PV = nRT$$

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+ideal+gas

Make n the subject of the formula

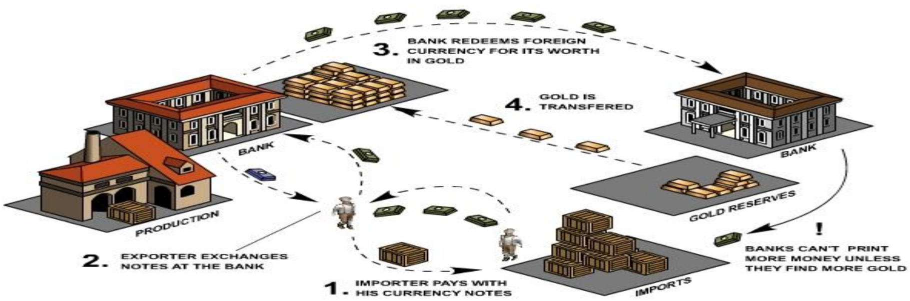
- n = number of <u>moles</u>
- R = universal gas constant = 8.3145 J/mol K
- P = Pressure
- V = Volume
- T = Temperature

Economics

Velocity Of Circulation

Make Q the subject of the formula

$$M.V = P \, . \, Q$$



Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+velocity+of+circulation+of+money

Where *M* is Money Supply, *V* is Velocity of Circulation (the average number of times money changes hand), *P* is Average Price Level (a measure of inflation), and *Q* is Quantity of Goods and Services bought or sold in the economy in a year (also known as the Gross National Product [GNP]).

GLOSSARY		
11	Absolute values	The magnitude of a quantity irrespective of sign The distance of a quantity from zero
18	Acceleration	Rate at which the speed is changing
1	Algebra	A branch of mathematics in which symbols represent numbers of a specified set and are used to represent quantities and to express general relationships that hold for all members of the given set
4	Common factor	A number or quantity that divides two or more numbers exactly
7	Commutative law	Swap numbers and still maintain the same answer
6	Distributive law	Multiplying a number by a group of numbers added together is the same as multiplying each separately
26	Economy	Consists of production distribution or trade and consumption of limited goods and services by different agents
15	Energy	Property of objects, transferable among themthrough fundamental interactions, which can be converted into different forms but not created or destroyed
9	Evaluate	To find the numerical value
2	Factorise	The resolution of an expression into factors such that when these factors are multiplied together, they give the original expression
12	Formula	Relationship between two or more variables
13	Formula manipulation	Involves rearranging variables to make an algebraic expression better suit the requirement. During this arrangement, the value of the expression does not change
20	Ideal gas	A gas whose pressure, volume and pressure are related by the ideal gas law
25	Inflation	The rate at which the general level of prices for goods and services is rising and subsequently purchasing power is decreasing
16	Kinetic energy	Energy of motion
21	Moles	A unit of measurement used in chemistry to express the amount of a chemical substance
8	Perfect square	A number that can be expressed as the product of two equal integers
5	Polynomial	An expression consisting of variables and coefficients that involves the operations of addition, subtraction and multiplication and non - negative integer exponents
17	Potential energy	Energy that an object has due to its position in a force field
22	Pressure	A measure of the force applied over a unit area
10	Radical	An expression that has a square root or cube root
3	Simplify	To rewrite an expression as simple as possible
14	Subject of a formula	The variable on its own, usually on the left hand side of a formula
23	Universal gas	A physical constant used in many thermochemical equations and relationships
19	Velocity	Rate of travel of an object along with its direction
24	Velocity of circulation	The average number of times a unit of money changes hands in an economy during a given period.

STRAND 3 → NUMBERS

HISTORY OF INDICES

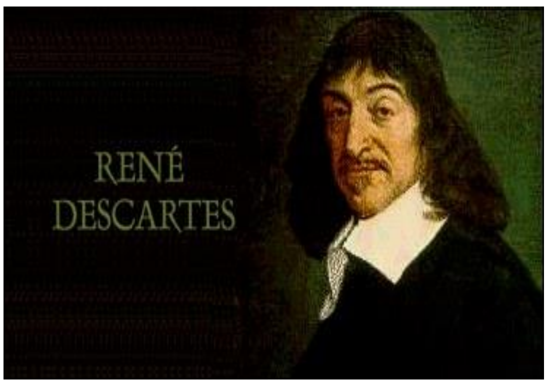
*The word **exponere** (exponent) originated from Latin, **expo**, meaning **out of**, and **ponere**, meaning **place.** While the word **exponent** came to mean different things, the first recorded modern use of exponent in mathematics was in a book called "Arithemetica Integra," written in 1544 by English author and mathematician Michael Stifel. But he worked only with a base of two, so the exponent 3 would mean the number of 2s you would need to multiply to get 8. It would look like this $2^3=8$. The way Stifel would say it is kind of backwards when compared to the way we think about it today. He would say "3 is the 'setting out' of 8." Today, we would refer the equation simply as 2 cubed. He was working exclusively with a base or factor of 2 and translating from Latin a little more literally than we do today.*

The idea of squaring or cubing goes all the way back to Babylonian times, part of Mesopotamia now Iraq. The earliest mention of Babylon was found on a tablet dating to the 23rd Century BC. And they were using the concept of exponents even then, although their numbering system used symbols to denote mathematical formulas.

What the Earliest Exponents Looked Like

The word "raised" is usually omitted, and very often "power" as well, so 3^5 is typically pronounced "three to the fifth" or "three to the five". The exponentiation b^n can be read as b raised to the n-th power, or b raised to the power of n, or b raised by the exponent of n, or most briefly as b to the n.

The modern notation for exponentiation was introduced by René Descartes in his Géométrie (Geometry) of 1637



Source: http://en.wikipedia.org/wiki/Exponentiation

Source: http://www.ehow.com/about_5134780_history-exponents.html

3.1 Expressing Numbers in Indices Form

LEARNING OUTCOMES

Students should be able to:

- To introduce indices
- To write numbers in the base index form and vice versa.

Activity:

A long time ago in the Duavata Kingdom lived a beautiful and hardworking girl. She was working for a King who was the strict ruler of Duavata Kingdom. The King was really appreciative of the work being done by the girl that he summoned her to his office to offer a reward. Below is the conversation that took place between the King and the girl.

King: "Girl, I am going to give you anything you want from my Kingdom. Please tell me what is it you really want me to give you".

Girl: "My Lord, I come from a very poor family. My requests are as follows: Today- 1 gold coin, 1st day of work tomorrow-2 gold coins, 2nd day-4 gold coins, 3rd day-8 gold coins, 4th day-16 gold coins and to continue for 30days".

King: "Is that all? You must be joking! You mean to tell me that that is enough for me to reward you for all the work that you have been doing for me. Fine, may your request be granted. Here is the 1st gold. My advisors will do the calculations and you will come and collect your rewards on a daily basis as you have requested".

The King called his advisors who then made all the calculations. After receiving the breakdown for each day until the 30th day, the King fainted.

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf 8#q=images+of+a+cartoon+king+and+his+money

Questions:

1. How many gold coins would the girl receive on the 5th day of work?
2. Why did the King faint? [Clue: Do all the calculations for 30 days?

Solutions:

1. 1st day: no work done= 1 gold coin[$2^0 = 1$], 1st day: after work = 2 gold coins[$2^1 = 2$], 2nd day: after work= 4 gold coins[$2^2 = 4$], 3rd day: after work= 8 coins[$2^3 = 8$], 4th day: after work= 16 gold coins [$2^4 = 16$], 5th day: after work= 32 coins [$2^5 = 32$]

2. The King fainted because of the amount of gold coins that he has to give after the 30days beginning from the day of their conversation i.e. 1,073,741,823 gold coins

The girl used the concept of base index to formulate her request!!!!!! A very smart girl.

Index → 4 ; Base → x

$$x^4 = \overbrace{x \times x \times x \times x}^{\text{expanded form}}$$

x^4 Is called the power of x

Exponent: another name for index

Example 3. 1

Write the following in base-index form

a. $c \times c \times c \times c$ b. $4 \times 4 \times 4 + 5 \times 5$

Solution:

a. $c \times c \times c \times c = c^4$ [4 factors so the index is 4]

b. $4 \times 4 \times 4 + 5 \times 5 = 4^3 + 5^2$

Example 3.2

Write in expanded form:

a. $2h^5$ b. $3g^5 + 4f^2$

Solution:

a. $2h^5 = 2 \times h \times h \times h \times h \times h$

b. $3g^5 + 4f^2 = 3 \times g \times g \times g \times g \times g + 4 \times f \times f$

Exercise 3.1

1. Write in base-index form

a. $q \times q \times q \times q \times q \times q$ b. $6 \times y \times y \times y$ c. $2 \times a \times a \times a + 3 \times z \times z$

d. $2 \times 2 \times 2 \times 2$ e. $-5 \times -5 \times -5$ f. $3 \times 3 \times 3 + 4 \times 4 \times 4 \times 4$

g. $p \times p \times q \times p \times q \times p$ h. $-3 \times g \times h \times g \times h \times 4 \times g \times h \times g$

2. Write in expanded form

a. x^6 b. $2d^4$ c. $3n^3 + 4a^7$

d. $3e^5 - 6e^5$ e. $9r^2 - 2r^3 + 3r^2 + 4r^3$

LEARNING OUTCOMES

Students should be able to:

- To establish rules of indices using numerals and pronumerals.
- To apply law of indices

As with all mathematical formulae, to simplify numerals and pronumerals with indices, certain laws need to be followed.

3.2 Index Rules/ Laws

When multiplying numbers or variables having the same base, we add the index.

1. Index Law: 1

$$x^2 \times x^3$$
$$= x^{2+3}$$
$$= x^5$$

Example:

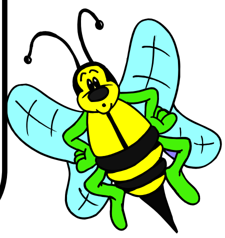
$$2^4 \times 2^3$$
$$= 2^{4+3}$$
$$= 2^7$$

2. Index Law: 2

$$x^5 \div x^3$$
$$= x^{5-3}$$
$$= x^2$$

Example:

$$2^5 \div 2^2$$
$$= 2^{5-2}$$
$$= 2^3$$



When dividing numbers or variables having the same base, we subtract the indices.

3. Index Law: 3

$$(x^2)^4$$
$$= x^{2\times4}$$
$$= x^8$$

Example:

$$(2^3)^4$$
$$= 2^{3\times4}$$
$$= 2^{12}$$

Where the index form is raised to another power, the indices are multiplied.

4. Index Law: 4

$$x^0$$
$$= 1$$

Example:

$$2^0$$
$$= 1$$

Any number or variable raised to the power of zero is equal to 1.

5. Index Law: 5

$$x^{-2}$$
$$= \frac{1}{x^2}$$

Example:

$$2^{-3}$$
$$= \frac{1}{2^3}$$
$$= \frac{1}{2\times2\times2}$$
$$= \frac{1}{8}$$

A negative index will give a fraction when simplified.

Exercise 3.2

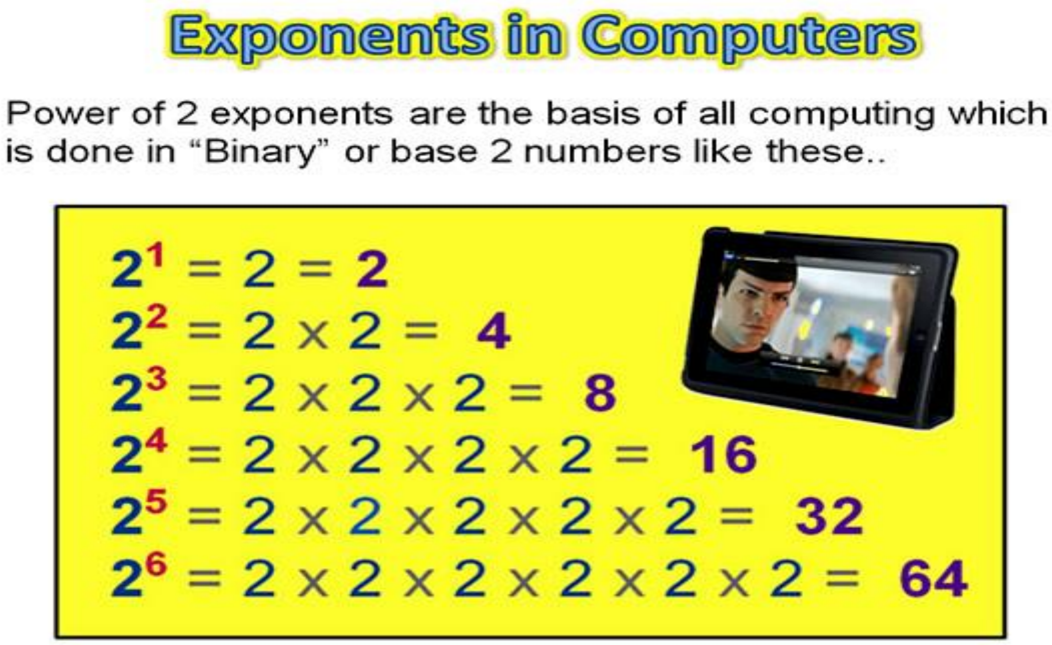
Simplify. Your answer should contain only positive exponents.

1. $a^2 \times a$	2. $w^3 \times w^4$
3. $k^6 \times k^3$	4. $p^2 \times p^3 \times p$
5. $r^6 \div r^5$	6. $t^7 \div t^7$
7. $\frac{q^5}{q^2}$	8. y^0
9. $(g^5)^2$	10. $(2^3)^2$
11. 2^{-3}	12. 3^{-2}

Indices in real life situations

Most people who use Exponents are Economists, Bankers, Financial Advisors, Insurance Risk Assessors, Biologists, Engineers, Computer Programmers, Chemists, Physicists, Geographers, Sound Engineers, Statisticians, Mathematicians, Geologists and many other professions

Exponents are fundamental, especially in Base 2 and Base 16, as well as in Physics and Electronics formulas involved in Computing.



Source: http://passyworldofmathematics.com/exponents-in-the-real-world/

There has been an Exponential increase in the speed and power of computers over recent years, and by around 2030 computing power is predicted to match that of the human brain.

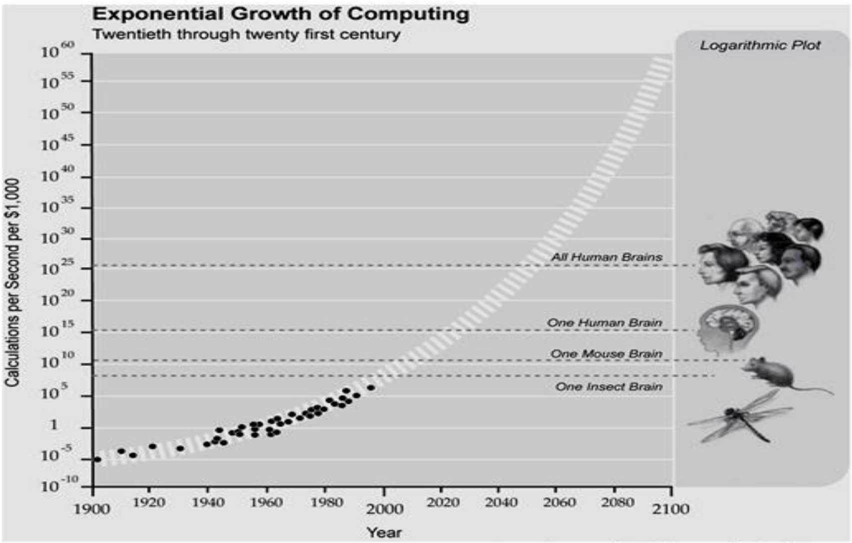
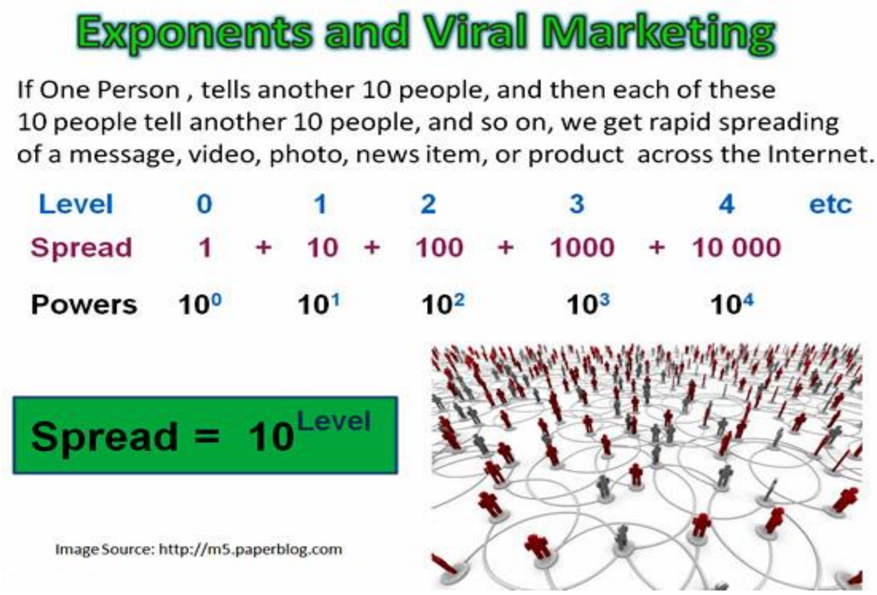
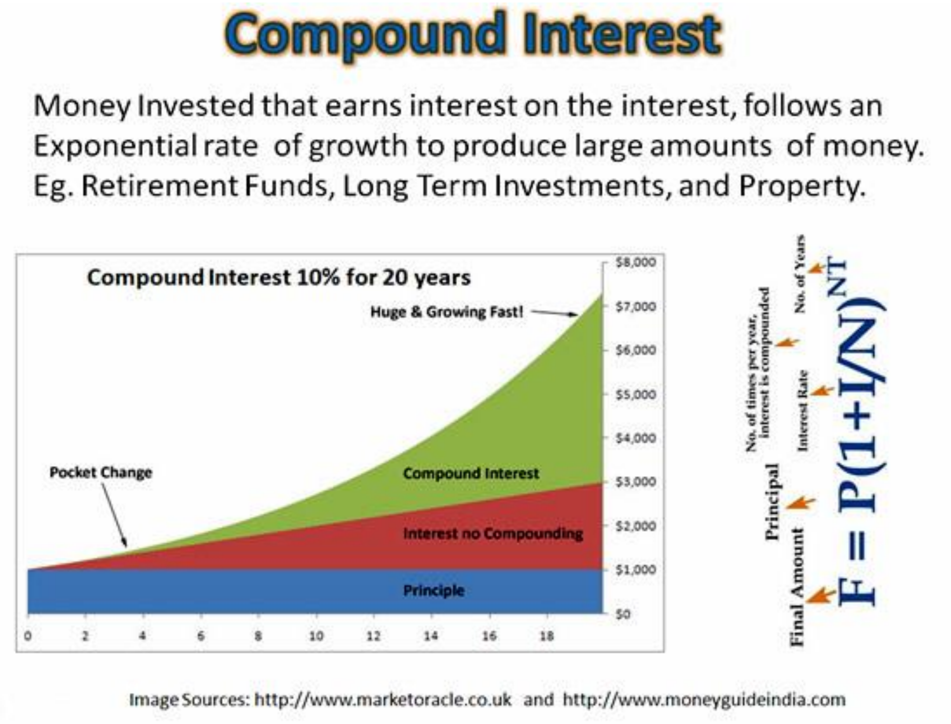


Image Source: http://www.singularity.com

Exponents are critically important in modern Internet based Sales and Marketing,

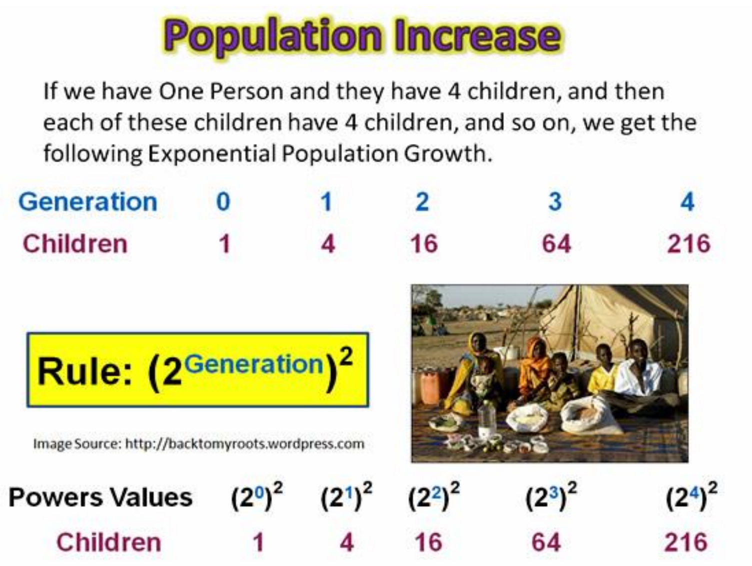


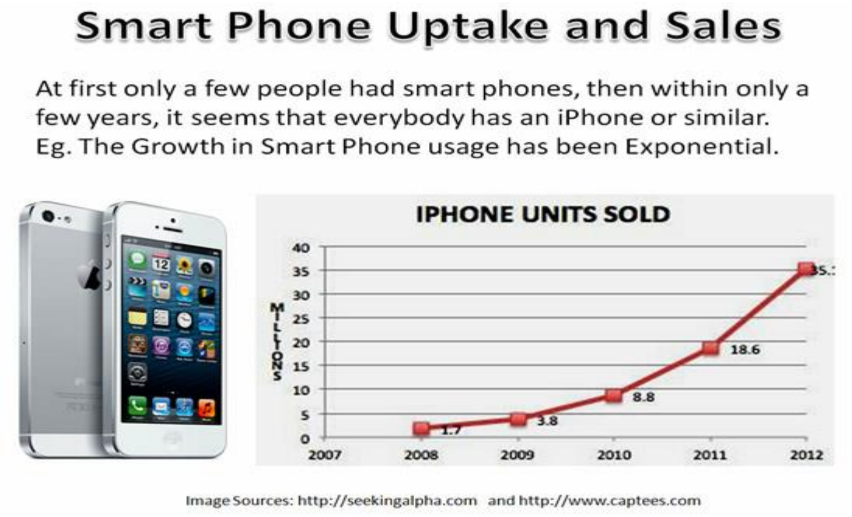
Exponents are important in Investing and Finance.



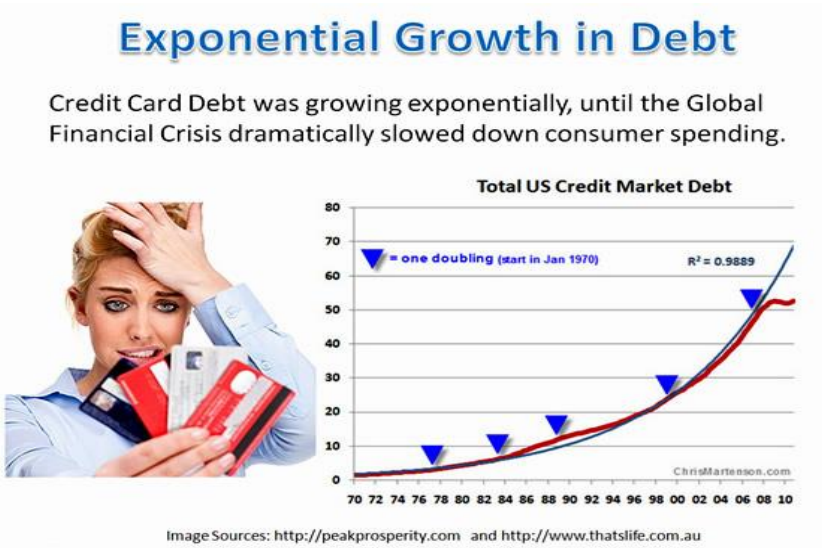
Compound Interest also works against people with a Credit Card debt they do not pay off, because the debt grows faster and faster each billing period and can quickly become out of control.

Exponents are the basis of "Demographics" (Population Growth)

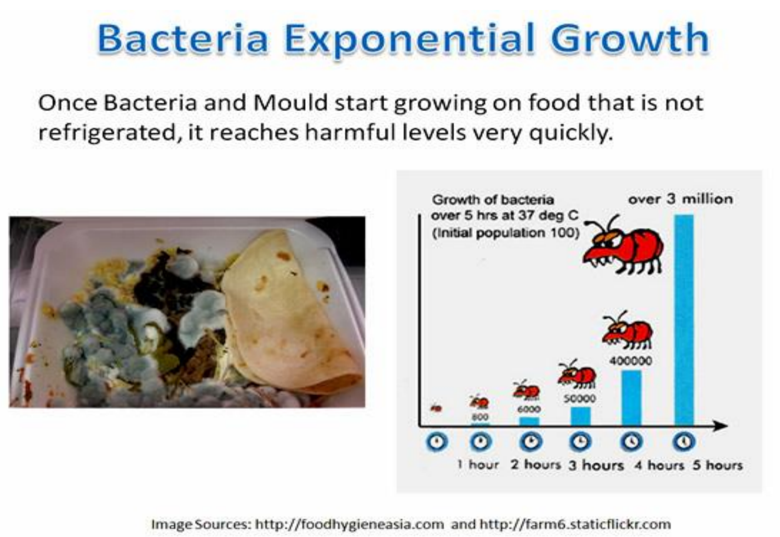




Consumer Credit Debt has increased over recent years to record high levels.



Exponents are also part of Food Technology and Microbiology.



Virus Illness, (as well as many email and computer viruses), can spread at ever increasing rates causing major widespread infected areas.

This happens the same way that Viral Marketing branches out in ever increasingly wide branches of more and more people passing something onto more and more other people.

Spread of Viruses and Disease

Viruses such as Flus, and HIV AIDS can spread at Exponential Rates causing Epidemics affecting millions of people.

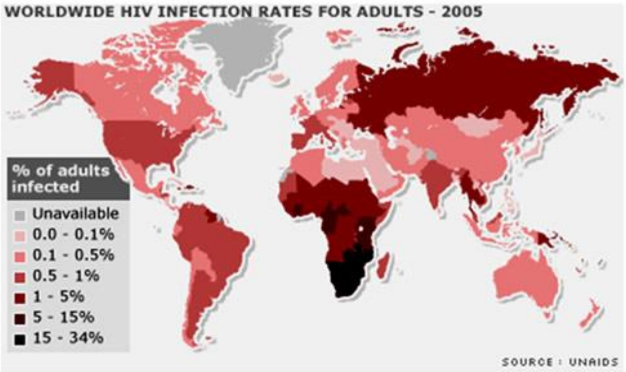


Image Source: http://news.bbc.co.uk

In explosions we get an uncontrolled massively increasing output of energy and force within a very short time period.

Picture this as a very steep exponential graph, compared to a burning match giving out energy in a fairly flat straight line graph.

Exponential Energy Release

In an uncontrolled Nuclear Explosion, Energy increases at an Exponential Rate as per Einstein's Equation: $E = mc^2$.

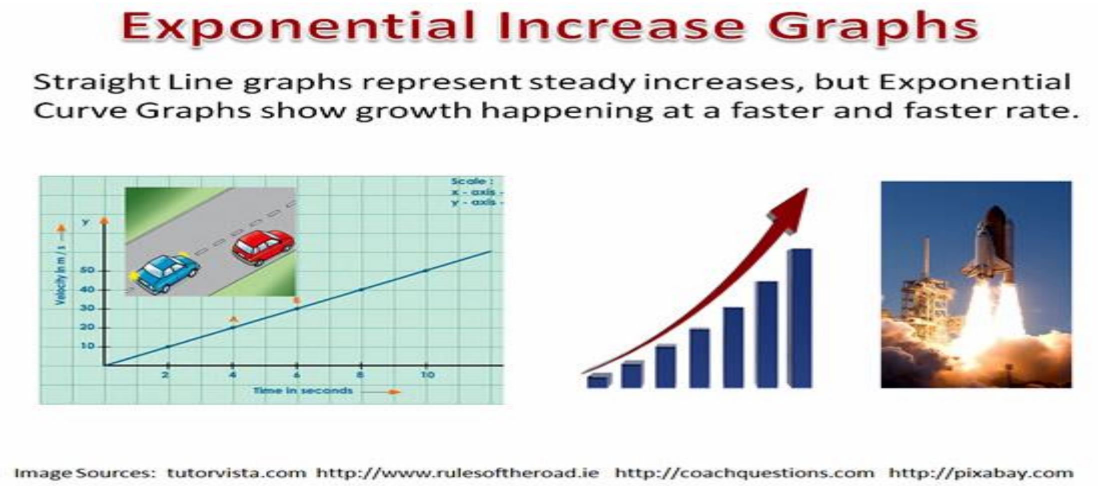
Image Source: http://www.zeusbox.com

Exponential Growth

The situations we have been considering so far involve “Exponential Growth”. The equations for graphs of these situations contain exponents, and this results in the graph starting off slow, but then increasing very rapidly.

Eg. Think of Square Numbers and how they quickly get bigger and bigger:

1 4 9 16 25 36 49 64 81 100 121 132 etc . It only takes us nine square numbers to reach 100. Exponential Growth situations when graphed look like the diagram below.

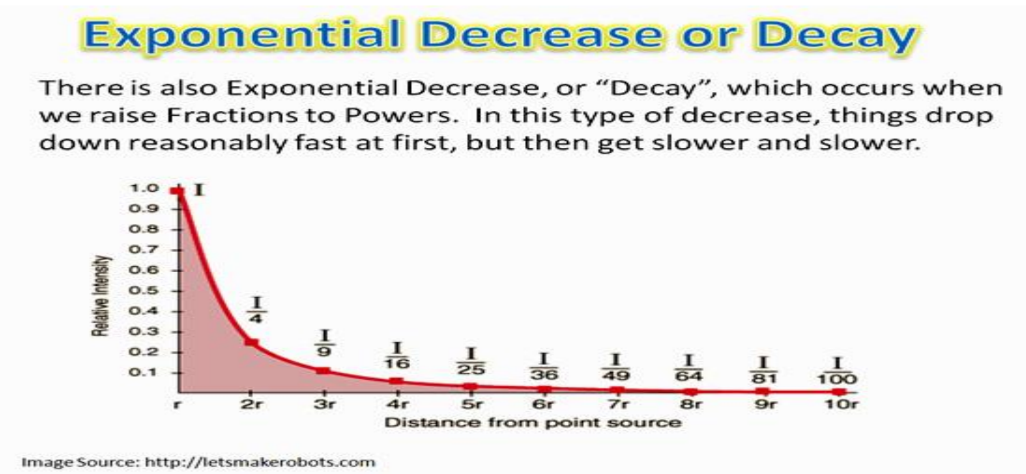


The opposite of “Exponential Growth”, is when we apply exponents to fractions which results in “Exponential Decay”.

Exponential Decay

Using negative power values results in fractions, and when these fractions have exponents applied to them we get “Decay”.

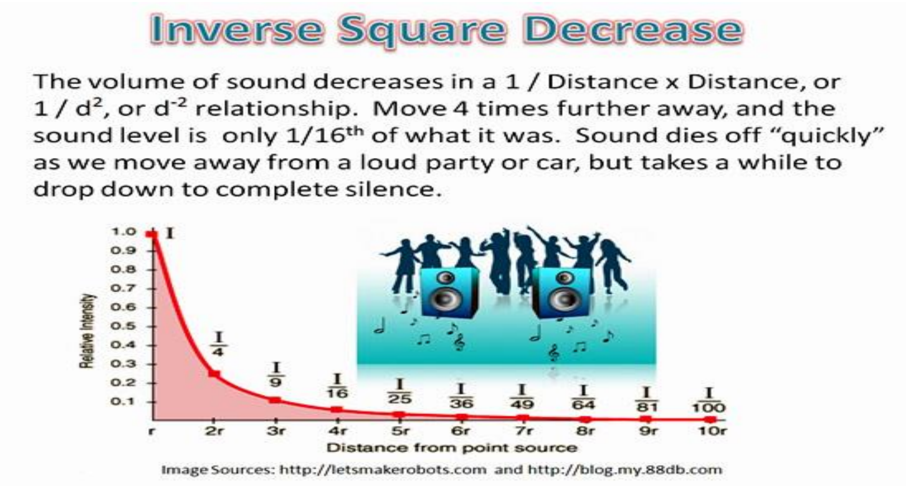
In a “Decay” process the amount involved drops off fairly quickly at the start, but then the drop off becomes slower and slower. A typical Exponential Decay graph looks like this:

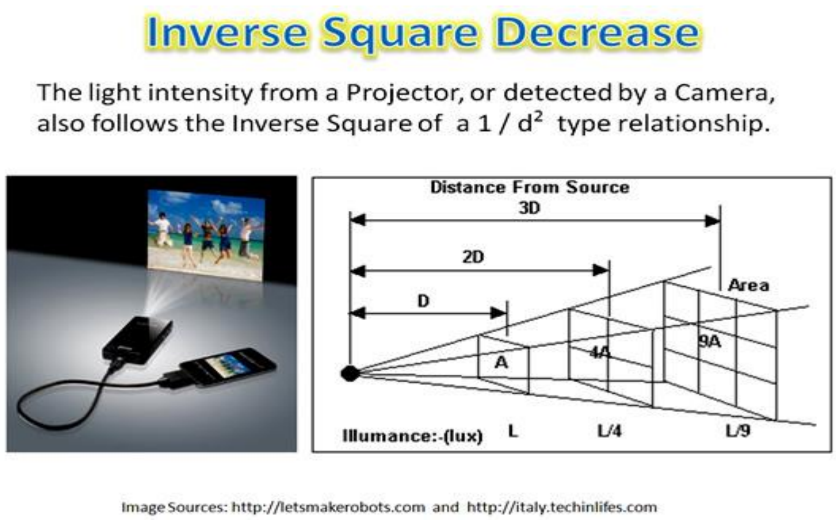


A fun way to make an Exponential decay graph is to take a pack of M&M's or Skittles and keep pouring them out of a cup, but each time removing any candies which land with the letter side showing.

Exponential Decay – Real Life Examples

Some examples of Exponential Decay in the real world are the following.





Exponential Decay and Half Life

Many harmful materials, especially radioactive waste, take a very long time to break down to safe levels in the environment. This is because these materials undergo exponential decay, and even a small amount of the material still remaining can be harmful.

Half Life Exponential Decay

Decay by half-life:

The pesticide DDT was widely used in the United States until its ban in 1972. DDT is toxic to a wide range of animals and aquatic life, and is suspected to cause cancer in humans. The *half-life* of DDT can be 15 or more years. *Half-life* is the amount of time it takes for half of the amount of a substance to decay. Scientists and environmentalists worry about such substances because these hazardous materials continue to be dangerous for many years after their disposal.

For this example, we will set the half-life of the pesticide DDT to be 15 years.

Let's mathematically examine the half-life of 100 grams of DDT.

End of Half life cycle	1 15 yrs	2 30 yrs	3 45 yrs	4 60 yrs	5 75 yrs	6 90 yrs	7 105 yrs	8 120 yrs	9 135 yrs	10 150 yrs
Grams of DDT remaining	50	25	12.5	6.25	3.125	1.5625	.78125	.390625	.1953125	.09765625
Pattern:	$\frac{100}{2^1}$	$\frac{100}{2^2}$	$\frac{100}{2^3}$	$\frac{100}{2^4}$	$\frac{100}{2^5}$	$\frac{100}{2^6}$	$\frac{100}{2^7}$	$\frac{100}{2^8}$	$\frac{100}{2^9}$	$\frac{100}{2^{10}}$

By looking at the pattern, we see that this decay can be represented as a function: $y = \frac{amount}{2^x}$

Image Source: http://www.regentsprep.org

Exponential Scales

The Richter Scale is used to measure how powerful earthquakes are.

The actual energy from each quake is a power of 10, but on the scale we simply take the index value of 1, 2, 3, 4, etc rather than the full exponent quantity.

This means that a Richter Scale 6 earthquake is actually 10 times stronger than a Richter Scale 5 quake. (Eg. 1000000 vs 100000).

Likewise, a Richter Scale 7 earthquake is actually 100 times stronger than a Richter Scale 5 quake. (Eg. 10000000 vs 100000).

Exponents and Earthquakes

The "Richter Scale" quantifies the amount of seismic energy, (as the Indexes of Powers of 10), that is released by an earthquake.

Image Source: http://lh3.googleusercontent.com

The pH Scale for measuring the Acidity of materials is also created by taking the Power Values from measured powers of 10 acid concentration values.

Exercise 3.3

1. Evaluate the following using the index laws. Leave your answers in base-index form.

a. $2^3 \times 2$

b. $3^6 \div 3^4$

c. $\left(-3^2\right)^3$

d. $6^5 \times 6^{-5}$

e. $4^{-2} \div 4^4$

d. $\left(5^{-2}\right)^1$

2. Simplify the following

a. $6x^3 \times 2x^2$

b. $18b^3 \div 3b$

c. $3a^2b \times 5ab^5$

d. $\dfrac{12x^4y^5z^8}{4x^2y^3z^6}$

e. $\dfrac{18n^3j^4}{5h^2l^6} \times \dfrac{10hl^8}{6n^2j}$

f. $\dfrac{6f^5g^3}{3r^2s} \div \dfrac{2gf^3}{12r^5s^5}$

g. $\dfrac{\left(2x^3y^2\right)^3}{12x^3y} \times \dfrac{4x^2y^3}{3xy}$

3. Write the following in base-index form using the number in brackets as the base.

a. 64 (4)

b. 125 (5)

c. 2401 (7)

d. 1/8 (2)

e. 1/81 (3)

f. 1/6 (6)

4. Evaluate the following

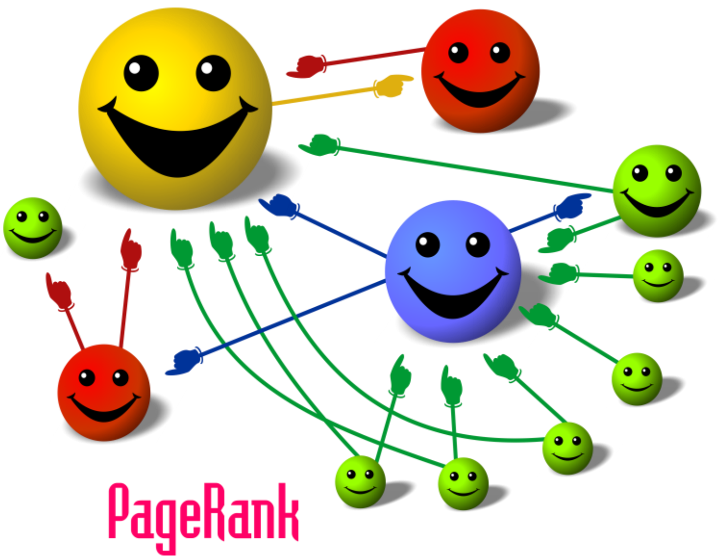
a. $4^2 \times 2^3$

b. $\left(3^6\right)^3 \div 3^5$

c. $\left(5^4\right)^2 \times 4^2 \div 5^3$

d. $2x^0 + \left(5xy^2\right)^0$

e. $-7x^0 - (100y)^0$



GLOSSARY		
6	Compound interest	When the interest rate is applied to the original principal and any accumulated interest
1	Exponent	A number placed above and to the right of another number to show that it has been raised to a power
5	Exponential Decay	It occurs when a population decreases at a consistent rate over time. For exponential decay, the total value decreases but the proportion that leaves remains constant over time
2	Exponential function	Use to model a relationship in which a constant change in the independent variable gives the same proportional change (i.e. percentage increase or decrease) in the dependent variable.
4	Exponential Growth	It is a growth that increases at a consistent rate, and it is a common occurrence in everyday life
3	Half Life	The amount of time it takes for the amount of the substance to diminish by half
9	Index (exponent, power)	The index of a number says how many times to use the number in a multiplication
7	Numeral	A symbol used to represent a number
8	Pro - numeral	A letter that is used to represent a number (or numeral) in a problem

STRAND 4 → GEOMETRY

HISTORY OF GEOMETRY

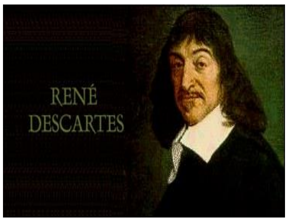
Geometry's was originated about 3,000 BC in ancient Egypt. Egyptians used an early stage of geometry in several ways, including the surveying of land, construction of pyramids, and astronomy. Around 2,900 BC, ancient Egyptians began using their knowledge to construct pyramids with four triangular faces and a square base.

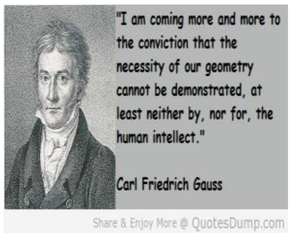
The next great advancement in geometry came from Euclid in 300 BC when he wrote a text titled 'Elements.' In this text, Euclid presented an ideal form in which propositions could be proven through a small set of statements that were accepted as true. In fact, Euclid was able to derive a great portion of planar geometry from just the first five postulates in the 'Elements.' These postulates are listed below:

- *A straight line segment can be drawn joining any two points.*
- *A straight line segment can be drawn joining any two points.*
- *Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.*
- *All right angles are congruent.*
- *If two lines are drawn which intersect a third line in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines must intersect each other on that side if extended infinitely. Euclid's fifth postulate is also known as the parallel postulate.*

The next advancement in the field of geometry occurred in the 17th century when René Descartes discovered coordinate geometry. The creation of coordinate geometry opened the doors to the development of calculus and physics.

In the 19th century, Carl Friedrich Gauss, Nikolai Lobachevsky, and János Bolyai formally discovered non-Euclidean geometry. In this kind of geometry, four of Euclid's first five postulates remained consistent, but the idea that parallel lines do not meet did not stay true. This idea is a driving force behind elliptical geometry and hyperbolic geometry.





Source: http://www.wyzant.com/resources/lessons/math/geometry/introduction/history_of_geometry

TRIGONOMETRY

Trigonometry is the study of the ratios of the sides of triangles. In Trigonometry, Trig refers to triangles and metry means to measure.

4.1 Square and Square Roots

LEARNING OUTCOME

Students should be able to:

- Calculate squares and square roots.

Square

- Is a number multiplied to itself example 3×3.
- In short it is written as 3^2(3 to the power of 2)
- Squaring a negative number always gives a positive answer.

Example 4.1

Find the following squares:

(a) $5^2 = 5 \times 5 = 25.$

(b) $(-5)^2$= -5 × -5 = (- × -) (5 × 5) = 25

(c) - 3^2 = - (3 × 3) = -9

Calculator working

Press 3

Press x^2

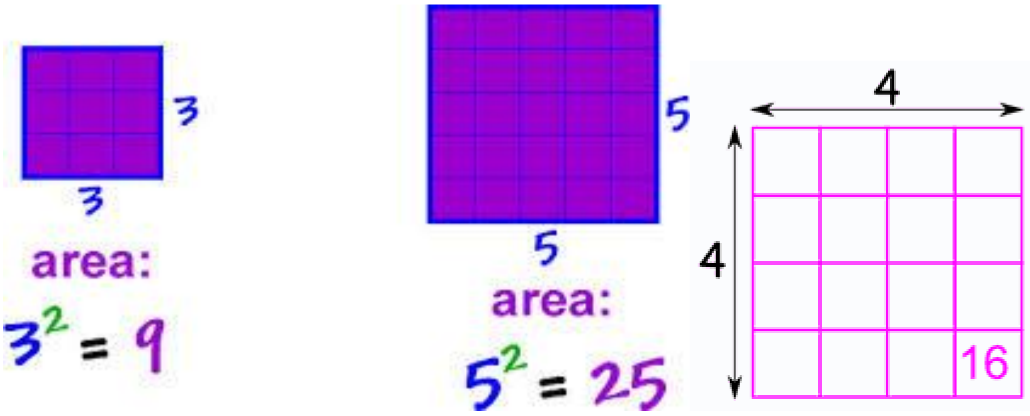
Press =

Note :

Always use brackets while squaring a negative number

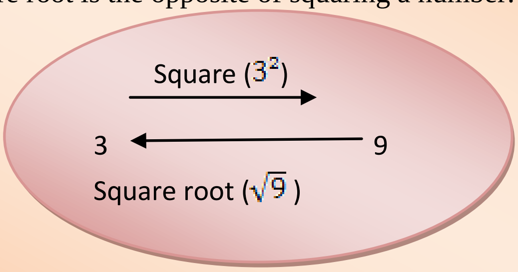
Perfect Square

Are squares of real numbers example 1, 4, 9, 16, 25, 36, 49, 64, 81……



Square roots ($\sqrt{\ }$ *or* $\sqrt[2]{\ }$)

A square root is the opposite of squaring a number.



A square root of a number is a value that can be multiplied by it to give the original answer example $\sqrt{49} = 7$ *means that* $7 \times 7 = 49$

Example 4.2

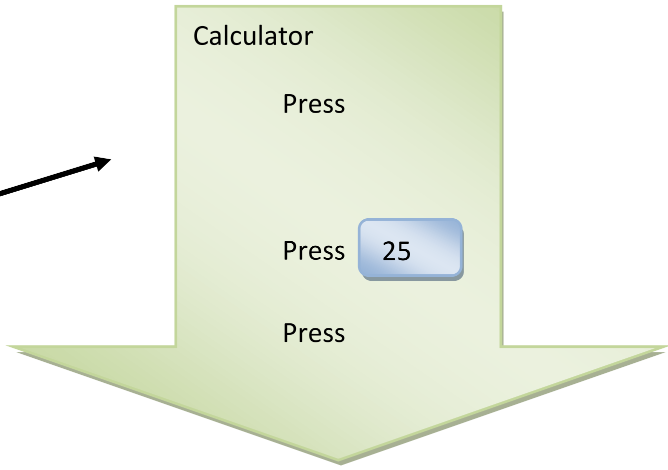
Find $\sqrt{25}$

Solution

Since 25 = 5 x 5

means $\sqrt{25}$ =

We know that 25 = 5 × 5 so $\sqrt{25}$ is 5.



Exercise 4.1

Find the following square and square roots correct to decimal places.

1. $(2)^2$ 2. $(-4)^2$ 3. -5^2 4. $\sqrt{16}$

5. $\sqrt{196}$ 6. 3.14^2 7. $\sqrt{50}$ 8. -7^2

9. $\sqrt{56.25}$ 10. $(-8)^2$ 11. $\sqrt{4}$ 12. $6^2 \times (-3)^2$

4.2 Pythagoras Theorem

Pythagoras Theorem Is a theorem that gives the relationship between the sides of a right - angled triangle.

LEARNING OUTCOMES

BRAIN TEASER

FIND THE SQUARE ROOT OF - 4

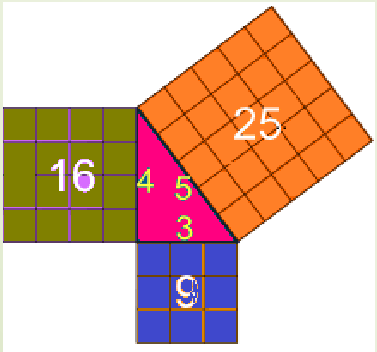
Students should be able to:

- Give the relationships between the sides of a right- angled triangle.
- Use this relationship to find the unknown sides of a right – angled triangle.
- Apply the Pythagoras theorem in real life situation,

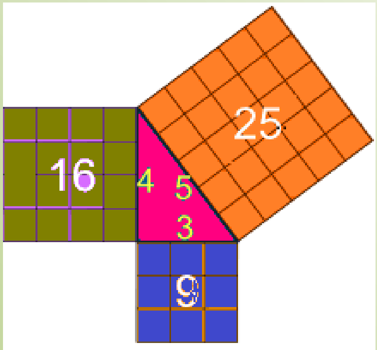
History

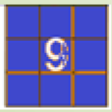
Over 2000 years ago there was an amazing discovery made by Pythagoras's about triangles.

"When the triangle has a right angle (90˚) and squares are made on each of the three sides, then the biggest square has the exact same area as the other two squares put together"



The relationship discovered by Pythagoras's is called the Pythagoras's theorem and can be written as $a^2 + b^2 = c^2$.





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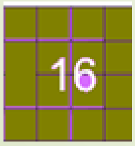

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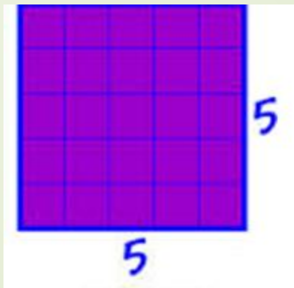
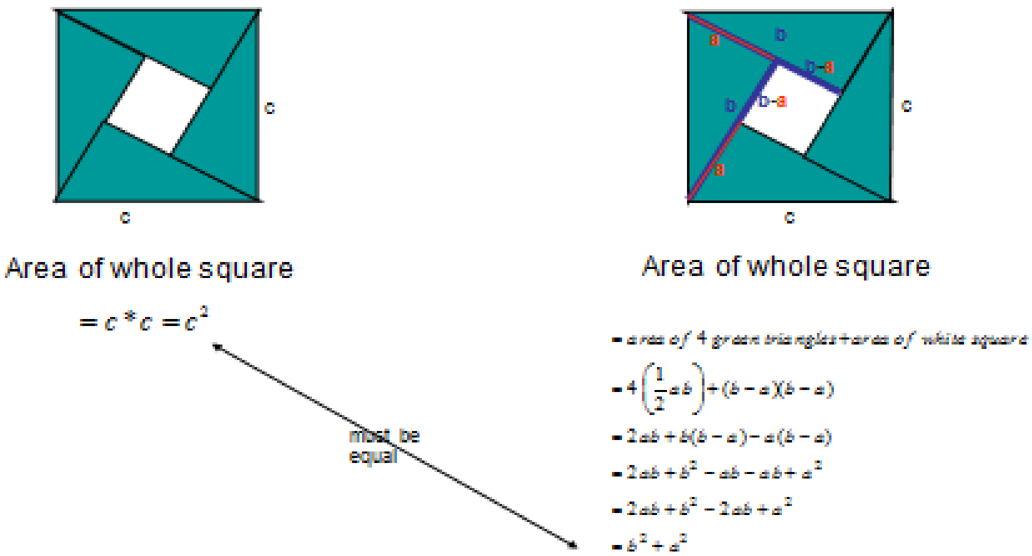
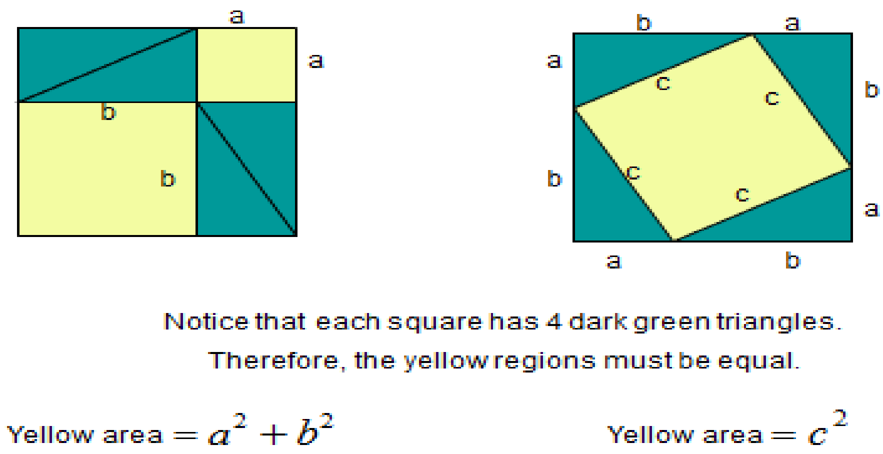


Note:

1.In this case *c* is the longest side of the triangle and is called the hypotenuse, *a* and *b* are the other two sides of the triangle. The hypotenuse is the side opposite the right angle.
2. Given any two sides of the right angled triangle the pythagoras thoerem can be used to find the length of the unknown side.
3. The theorem can also be used to determine whether a triangle is a right angled triangle or not.



Proof 2



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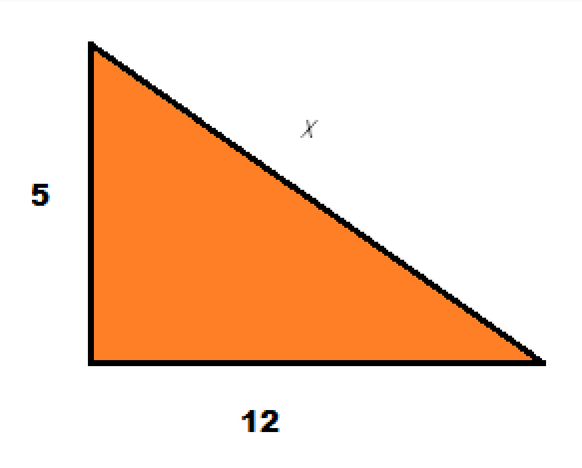
Example 4.3

(a) Find the unknown sides in the triangles given below.



Solution

$a^2 + b^2 = c^2$

$5^2 + 12^2 = x^2$

$25 + 144 = x^2$

$169 = x^2$

$x^2 = 169$

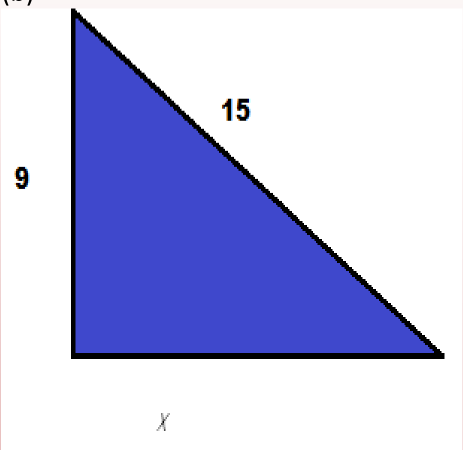
$x = \sqrt{169}$

$x = 13$

Calculate the remaining side

Solution

(b)



$a^2 + b^2 = c^2$

$9^2 + b^2 = 15^2$

$81 + b^2 = 225$ Taking 81 away from both sides gives

$81 + b^2 - 81 = 225 - 81$

$b^2 = 144$

$b = \sqrt{144}$

$b = 12$

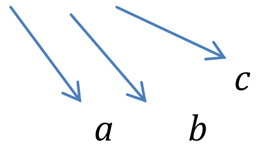
(c) A triangle has lengths 8, 15 and 16. Is it a right angled triangle?

Solution

$a^2 + b^2 = c^2$

$8^2 + 15^2 = 289$

8, 15, 16



$16^2 = 256$

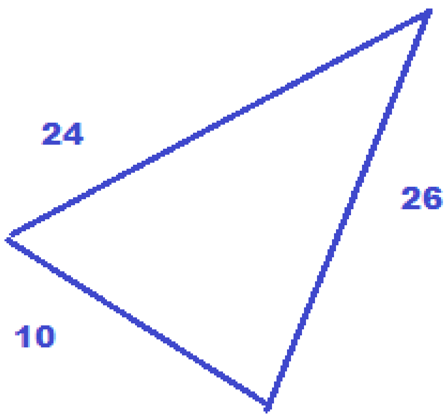
since $a^2 + b^2$ is not equal to c^2 ($a^2 + b^2 \neq c^2$)

therefore the given triangle is not a right - angled triangle.

Example 4.4

Show that the triangle given below is a right – angled triangle.

(a)



Show that $a^2 + b^2 = c^2$

Note: assign 26 to *c* since c is the longest side of the triangle.

$a = 10$ $b = 24$ $c = 26$

$10^2 + 24^2 = 676$

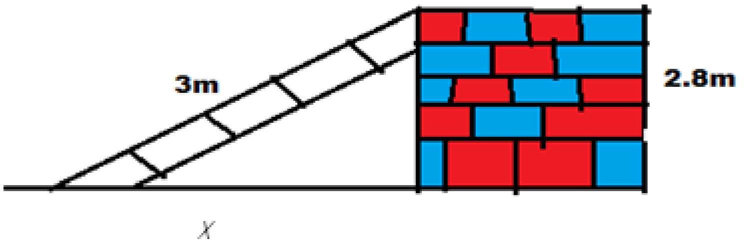
$26^2 = 676$

Since $a^2 + b^2 = c^2$ is *satisfied*

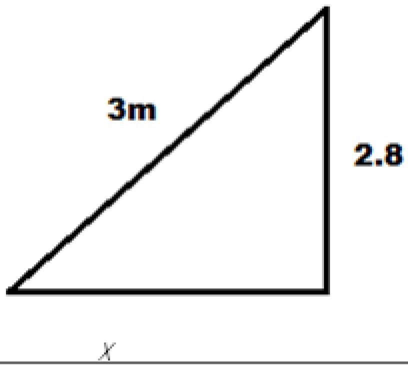
Therefore the given triangle is a right – angled triangle.

Example 4.5

A 3m ladder stands on a horizontal ground and reaches 2.8m up a vertical wall. How far is the foot of the ladder from the base of the wall?



Interpret and illustrate the question in mathematical terms.



Solution

Using the Pythagoras theorem $a^2 + b^2 = c^2$ to calculate the value of x.

Let $a = x$, $c = 3m$ and $b = 2.8m$.

$$a^2 + b^2 = c^2$$

$$x^2 + (2.8)^2 = 3^2$$

$$x^2 + 7.84 = 9$$

$$x^2 + 7.84 - 7.84 = 9 - 7.84$$

$$x^2 = 1.16$$

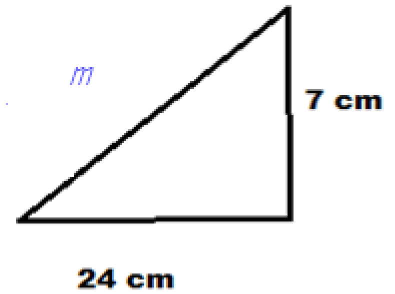
$$x = \sqrt{1.16}$$

$$\mathbf{x = 1.08m}$$

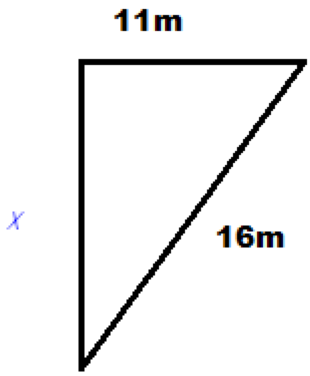
Exercise 4.2

1. Find the missing sides

(a)



(b)



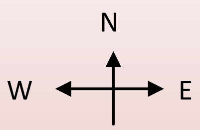
2. The side lengths of various triangles are given. Determine which ones are right angled triangles.

(a) {6, 8, 10}

(b) {3, 5, 6}

(c) { $\sqrt{3}$, $\sqrt{11}$, $\sqrt{8}$}

3. Town B is 8 miles north and 17 miles west of town A. How far are the two towns?



A rectangular field is 125 m long and the length of one diagonal of the field is 150m. What is is the width of the field.

A 8m ladder is leaned against the side of a wall. How high does the ladder reach if its base is is 3m away from the building.

4. A rectangular field is 125 m long and the length of one diagonal of the field is 150m. What is the width of the field?

5. A 8m ladder is leaned against the side of a wall. How high does the ladder reach i reach if its base is 3m away from the building?

6. Linda is mountain climbing with Allie and has just climbed a 16-metre vertical roc rock face. Allie is standing 12 metres away from the bottom of the cliff, looking up a up at Linda. How far away are Linda and Allie?

4.3 Trigonometric Functions

LEARNING OUTCOMES

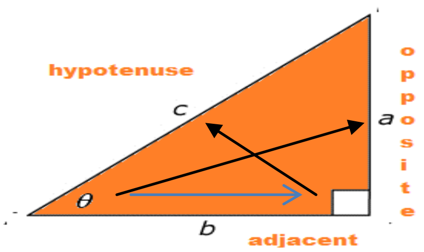
Students should be able to:

- Describing the three basic trigonometric functions.
- Calculating sine, cosine and tangent values of theta and vice versa.
- Use SOH, CAH, TOA to find unknown side and angle of right angle triangle.

4.3.1 Naming the Sides of a Right Angled Triangle

- In trigonometry the Greek letter ϴ (theta) is used as the name of an angle.
- Using ϴ the sides of the triangle can be named.

For example



The sides of the triangle can be labelled as a, b and c. Side a is opposite of ϴ, side b adjacent to ϴ and side c is the hypotenuse (the longest side) opposite the right angle. From this, 3 functions can be introduced.

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

NOTE : Sin ϴ ÷ Cos ϴ = Tan ϴ

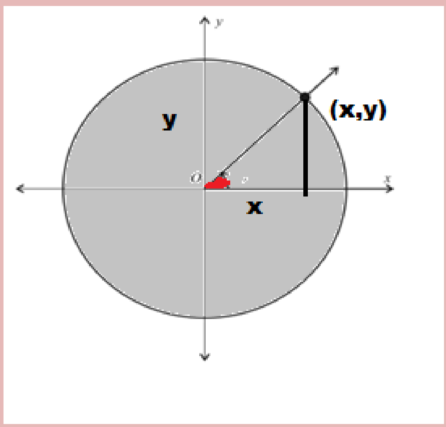
In short SOH, CAH, TOA can be used.

Example 4.6

Given below is a unit circle. A unit circle is circle of radius 1 unit. Ѳ is the angle marked in red

Questions

(i) What is the length of the hypotenuse?
(ii) Find Cos Ѳ
(iii) Find Sin Ѳθ
(iv) Find Tan Ѳ



(i) The size of the hypotenuse is I since it is the radius of a unit circle.
(ii) Cos Ѳ = $\frac{a}{h}$, so Cos Ѳ = $x/1$ = x
(iii) Sin Ѳ = $\frac{o}{h}$ = $\frac{y}{1}$ = y
(iv) Tan θ = $\frac{o}{a}$ = $\frac{y}{x}$

Sin θ and Cos θ are the x and y coordinates of the point (1, 0) as it is rotated by θ degrees about the origin

4.3.2 Using A Calculator

- A calculator can be used to find the values of the trigonometric ratios sin, cos and tan for the given angle where the angle is measured in degrees.
- In the same way if the trigonometric ratios are given then the angle can also be found using the inverse function.

Example 4.7

Example One: Find Sin 30°

Solution : Using the calculator

Press sin

Press 30

Press =

sin 30 = 0.5.

Example Two

Find sin ϴ = 0.66

Solution

Press shift

Press sin

Press 0.66 =

sin^{-1} (0.66) = 41.30 (2 dp)

Exercise 4.3

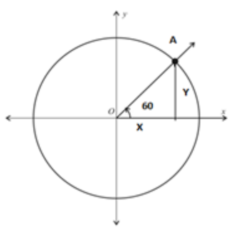
1.Evaluate the following

(a) Sin 60°
(b) Tan 34°
(c) Cos 124°

2. Find the value of ϴ

(a) Cos θ = 0.54
(b) Sin θ = 0.76
(c) Tan θ = 0.45

3. The diagram given below shows a unit circle.



(i) Find the lengths of x and y.

(ii) Write coordinates of A.

(iii) Use your calculator to find Sin 60 and Cos 60. Is it the same as the x and y coordinates of point A?

(iv) Find tan 60 and compare it with sin 60

4.3.3 Applications of SOH, CAH, TOA

LEARNING OUTCOMES

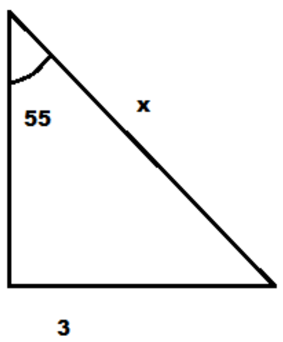
Students should be able to use SOHCAHTOA to calculate the:

- Unknown side of a right angle triangle
- Unknown angle of a right angle triangle

SOH CAH TOA CAN BE USED TO FIND THE UNKNOWN SIDE AND ANGLES OF A RIGHT ANGLED TRIANGLE

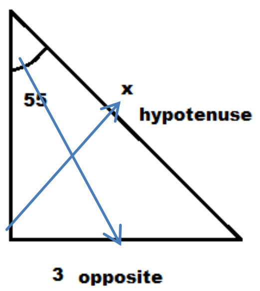
Example 4.8

Find the length of the side marked x in the right - angled triangle given below



Step one

Identify the given sides of the triangle.



Step two

Determine the trig function to be used

SOH CAH or TOA

Since O and H are given we use SOH

$$\sin \Theta = \frac{o}{h}$$

Step three

$\Theta = 55^0$ O = 3m H - x

$$\sin 55 = \frac{3}{x}$$

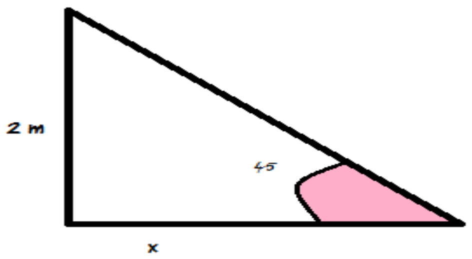
x sin 55° = 3 (multyplying x on both sides)

$x = 3/\sin 55$ (dividing by sin 55 on both sides)

<u>x = 3.66m</u> (2 dp)

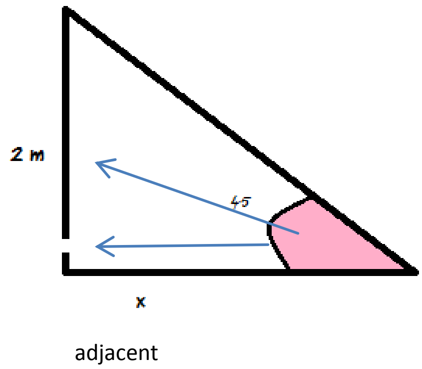
Example 4.9

Find the side marked x in the right - angled triangle given below



Step one

Identify the sides and angles given.



Step two

Identify the trig function SOH CAH or TOA

Since the opposite and adjacent are given

TOA is used.

Step three

$\tan\theta = \frac{o}{a}$ $\quad o = 2 \quad a = x \quad \theta = 45°$

$\tan 45 = \frac{2}{x}$

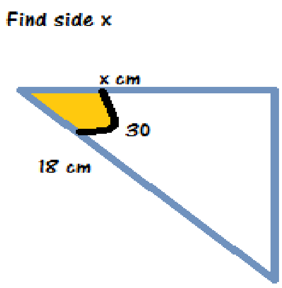
$x \tan 45 = 2$ (multiplying by x on both sides)

$x = 2/\tan 45$

$\underline{x = 2\ m}$

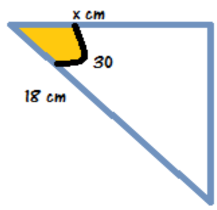
Example 4.10

A right - angled triangle is given below



Step one

Identify the sides and the angles.



Hypotenuse -18cm

Adjacent – x cm $\theta = 30^0$

Step two

Determine the trig function to use

Since a and h are given

CAH is used.

$Cos\ \theta = \frac{a}{h}$ $Cos\ 30 = \frac{x}{18}$

Step three

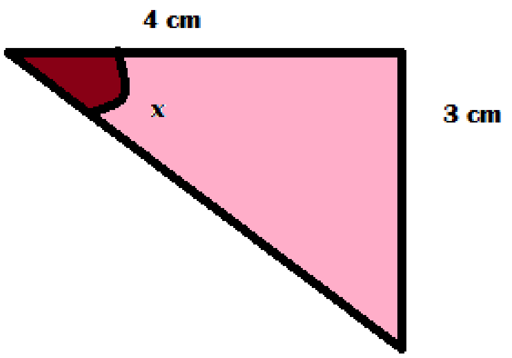
$Cos\ 30 = \frac{x}{18}$

$18 \cos 30 = x\ (multiplying\ 18\ by\ both\ sides)$

$\boldsymbol{x = 15.59\ cm}$

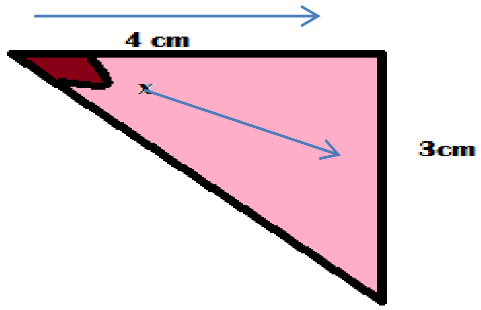
Example 4.11

For the right - angled triangle given, find angle x



Step one

Identify the sides and the angles.



$\theta = x$, opposite – 3cm, adjacent – 4cm

Step two

Determine the trig function to use

Since o and a are given

TOA is used.

$\tan \theta = \frac{o}{a}$ $\tan x = \frac{3}{4}$

Since the angle is the unknown we take $\tan^{-1}$ on both sides. $\tan^{-1} 3/4$ = **<u>36.87°</u>**

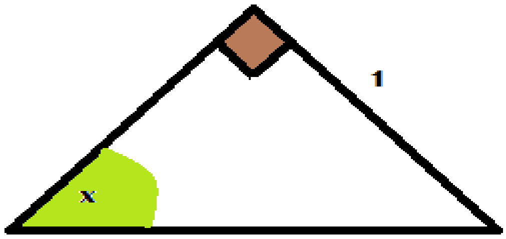
Note: while solving this types of problems ensure that the calculator is in the degree mode.

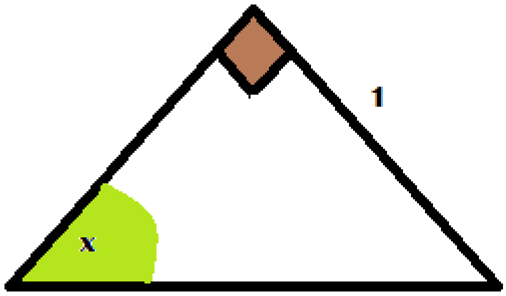
Example 4.12

The right angled triangle below is also an isosceles triangle. Find angle x

Step one

Identify the sides and the angles





Opposite – 1 θ = x hypotenuse - $\sqrt{2}$

Step two

Identify the trig function to use from

SOH CAH TOA.

Since o and h are given

SOH is used

$Sin\ \theta = \frac{o}{h}$

Step three

$\sin x = 1/\sqrt{2}$

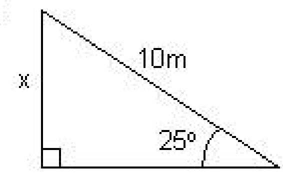
$\sin^{-1}\frac{1}{\sqrt{2}} = 45^{0}$

x = **<u>45°</u>**

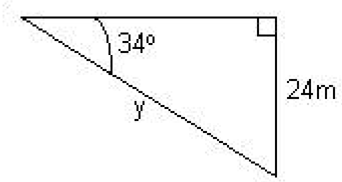
Exercise 4.4

1. Find the unknown sides and angles either using SOH CAH or TOA.

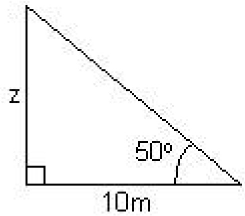
   (a)



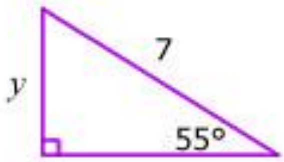
(b)



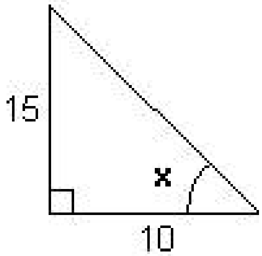
(c)



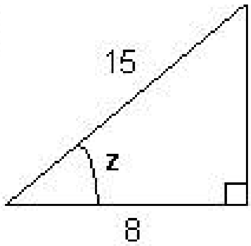
(e)



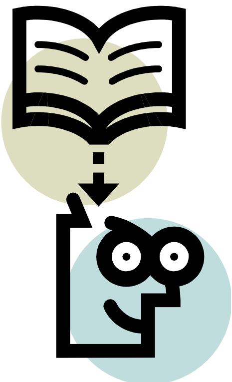
(d)

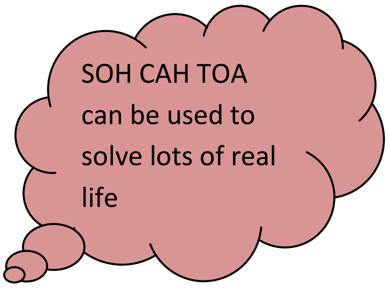


(f)



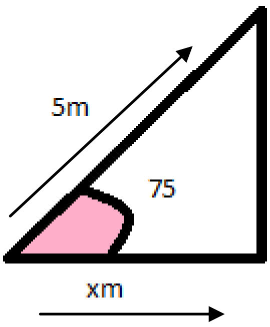
Trignometric Word Problems



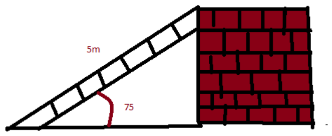


Example 4.13

A ladder leaning against a wall makes 75 ^{0} angle with the ground. If the ladder is 5m tall, how far is the base of the ladder from the wall of the house?



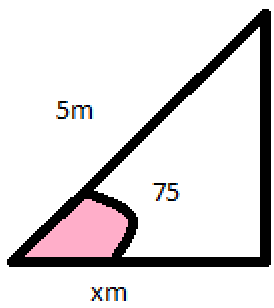
A diagramatic representation of the problem is given below.



Solution

Step one

Take out the triangle right angled triangle formed



$\theta = 75^{0}$ adjacent – x hypotenuse- 5m

Step two

Identify the sides and angles given

Step three

Identify the trig function to use

Since a and h are given

CAH is used

$\cos\Theta = a/h$

Step four

$\Theta = 75^{0}$, a = x m, h = 5m

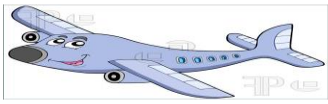
$\cos 75 = x/5$

$5\cos 75 = \mathbf{1.29\ m}$

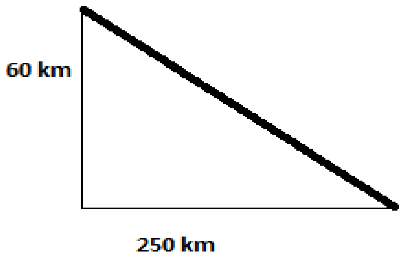
Hence the ladder is 1.29m away from the wall.

Example 4.14

An aeroplane flies 250 km west, but the wind blows it 60 km north



What is the compass bearing of the plane from the starting point?



Step one

compute angle x first

θ= x   opposite – 60 km adjacent – 250km

Use TOA since opposite and adjacent are given

Step two

$\tan x = \frac{60}{250}$

$tan^{-1}\left(\frac{60}{250}\right) = 13.5$

Step three

Total bearing = 90 +180+x

90+180+13.5 = 283.5

Exercise 4.5

1. Determine the correct formula for the sine ratio

A. $\text{Sin } x = \frac{\text{hypotenuse}}{\text{opposite}}$

B. $\text{Sin } x = \frac{\text{opposite}}{\text{adjasent}}$

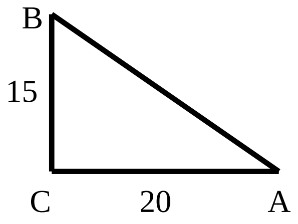
C. $\text{Sin } x = \frac{\text{opposite}}{\text{hypotenuse}}$

D. $\text{Sin } x = \frac{\text{adjacent}}{\text{hypotenuse}}$

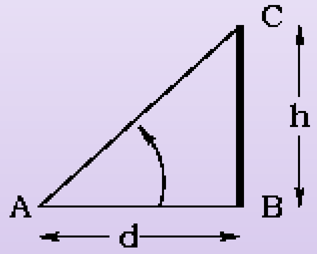
2. In $\triangle$ ABC, AB = 10 cm, $\angle B = 90^o$, and $\angle C = 60^o$. Determine the length of BC, to the nearest centimeter

A. 5 cm
B. 6 cm
C. 6 cm
D. 8 cm

3. In right triangle ABC, leg BC=15 and leg AC=20. Find angle A



4. A man is walking along a straight road. He notices the top of a tower subtending an angle $A = 60^o$ with the ground at the point where he is standing. If the height of the tower is $h = 35$ m, then what is the distance (in meters) of the man from the tower?



5. A little boy who is 1.5m tall is flying a kite. The string of the kite makes an angle of 30^o with the ground. If the height of the kite is $h = 12$ m, find the length (in meters) of the string that the boy has used.

4.3.4 Graphs of Trigonometric Functions

LEARNING OUTCOMES

Students should be able to:

- Sketch the trigonometric graphs.
- Identify the graphs of sine, cosine and tangent

Applications of the Trignometric Graphs

Music is an integral part of life of most people. Although the kind of music they prefer differs.

All music is the effect of the sound waves on the ears.

No matter what vibrating object is causing the sound wave the frequency of the wave (that is the number of waves per second) creates a sensation that is called the pitch of sound.

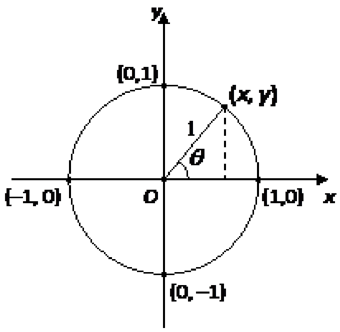
Sound is just one of the many physical entities that are transmitted by waves. Light , radio, television, X-rays, and microwaves are the others. The trigonometric Graphs that we study in this topic provide the mathematical basis for the study of waves.

The Sine Graph

The Sine Graph can be plotted by using the unit circle **.**

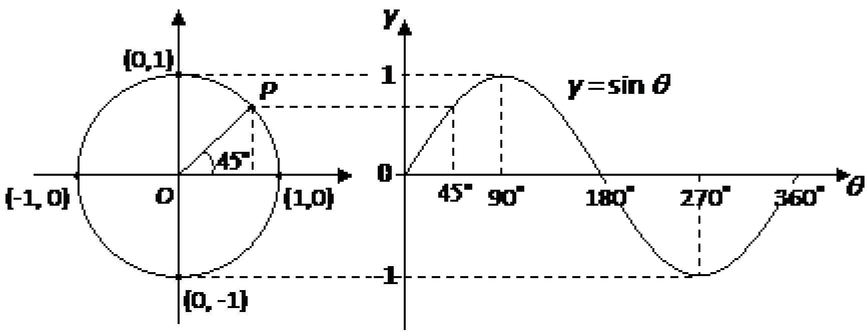
Recall:

A unit circle is a circle of radius one unit with its centre at the origin.



- ***y* = sin θ is known as the sine function.**
- Using the unit circle, we can plot the values of *y* against the corresponding values of $\boldsymbol{\theta}$.

The **graph of *y* = sin θ, for 0˚ ≤ θ≤ 360˚** obtained is as shown:



Properties of the Sine function:

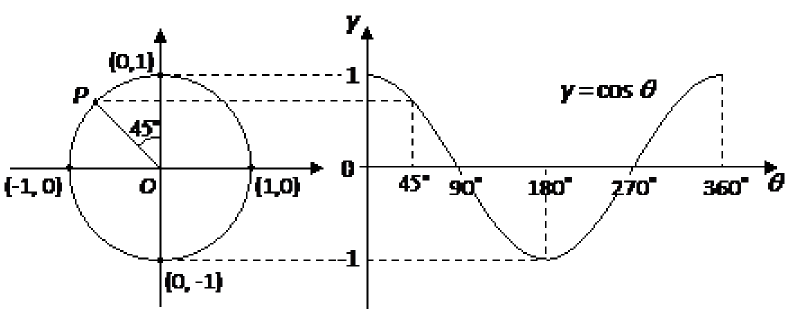
- The sine function forms a wave that starts from the origin
- sin θ= 0 when θ = 0˚, 180˚, 360˚.

Maximum value of sin θ is 1 when θ= 90˚. Minimum value of sin θ is –1 when θ = 270˚. So, the range of values of sin θ is –1 ≤ sin θ≤ 1.

The Cosine Graph

- ***y* = Cos θ is known as the cosine function.**
- Using the unit circle, the values of *y* against the corresponding values of **θ** *can be plotted.*

The **graph of *y* = cos θ, for 0˚ ≤ θ≤ 360˚** obtained is as shown:

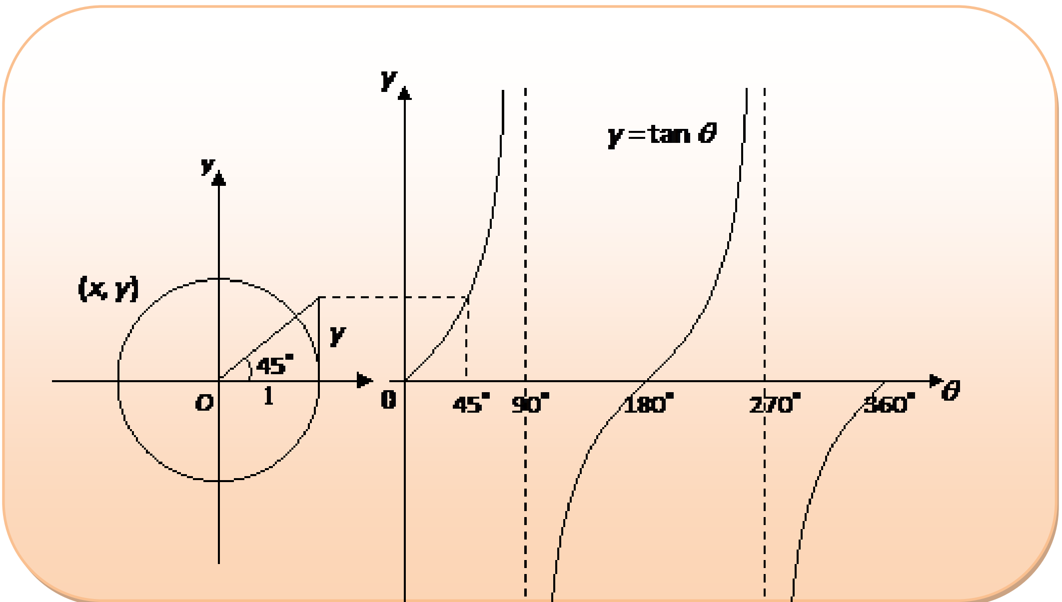


Properties of the Cosine function:

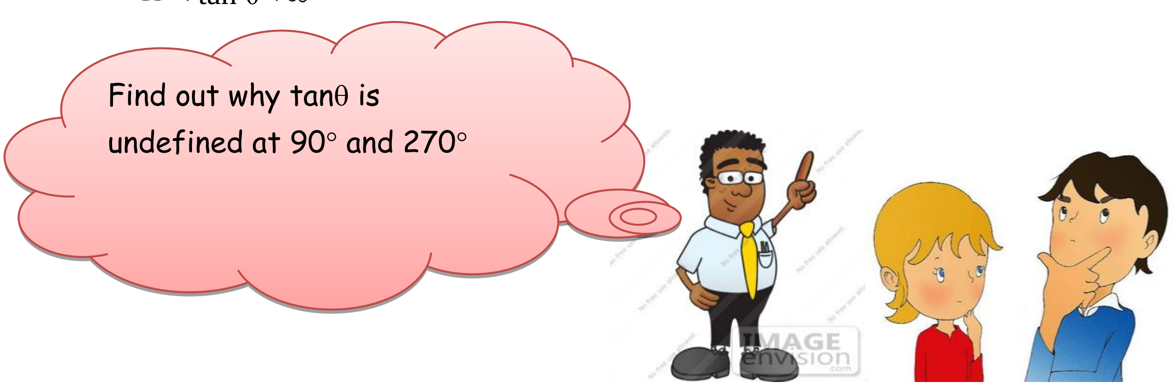
- The cosine function forms a wave that starts from the point (0,1)
- cos **θ** = 0 when **θ** = 90˚, 270˚.
- Maximum value of cos **θ** is 1 when **θ** = 0˚, 360˚. Minimum value of cos **θ** is –1 when **θ** = 180˚. So, the range of values of cos **θ** is $-1 \leq \cos\theta \leq 1$.

The Tangent Graph

- **y= tan θ is known as the tangent function.**
- Using the unit circle, values of *y* against the corresponding values of **θ** can be plotted. The **graph of *y* = tan θ, for 0˚ ≤ θ ≤ 360˚** obtained is as shown:

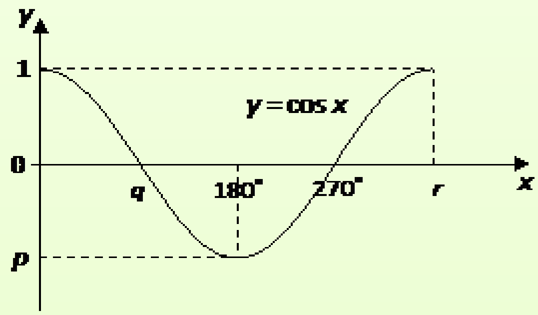


- The curve is not continuous. It breaks at **θ** = 90˚ and 270˚, where the function is undefined.
- tan **θ**= 0 when **θ**= 0˚, 180˚, 360˚. tan **θ** = 1 when **θ**= 45˚ and 225˚.
- tan **θ** = –1 when **θ**= 135˚ and 315˚.
- tan **θ** does not have any maximum or minimum values. The range of values of tan **θ** is $-\infty < \tan\theta < \infty$



Exercise 4.6

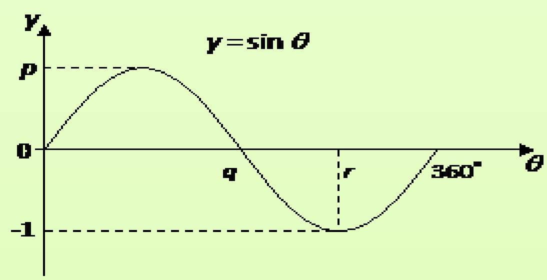
1. The diagram shows a graph of $y = \cos x$ for $0^\circ \leq x \leq 360^\circ$, determine the values of p, q and r.



2. Sketch the graph of $y = \tan x$ for $0^\circ \leq x \leq 360^\circ$ by setting up a table of values.

x	0	45	90	135	180	225	270	315	360
$\tan x$									

3. The diagram shows a graph of $y = \sin x$ for $0^\circ \leq x \leq 360^\circ$, determine the values of p, q and r.



4. Sketch the graph of $y = \cos x$ for $0^\circ \leq x \leq 360^\circ$ by setting up a table of values.

x	0	45	90	135	180	225	270	315	360
$\cos x$									

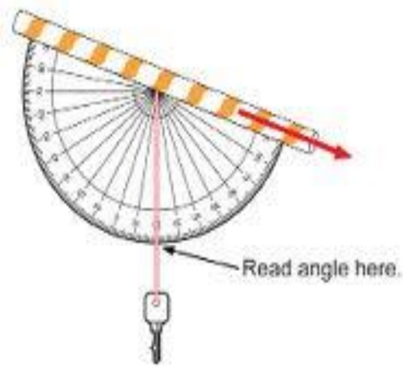
5. Sketch the graph of $y = \sin x$ for $0^\circ \leq x \leq 360^\circ$ by setting up a table of values.

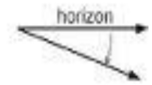
x	0	45	90	135	180	225	270	315	360
$\sin x$									

LEARNING OUTCOMES

Students should be able to:

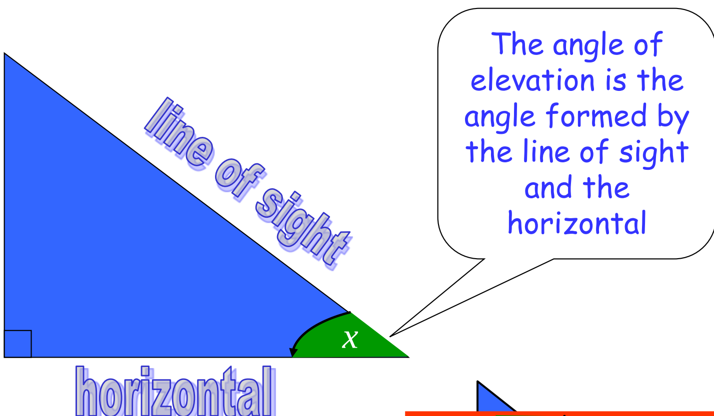
- Use a clinometer to measure angles in practical situations.
- Use angle measured above to calculate the height or length of object being studied



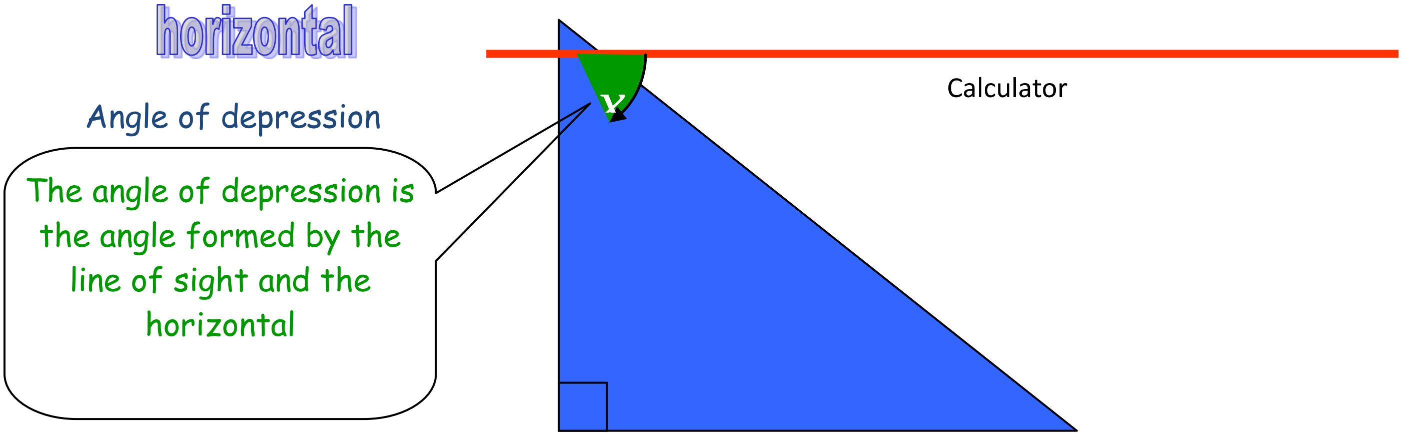


- A clinometer is an instrument for measuring angles of slope elevation or depression.

Angle of elevation



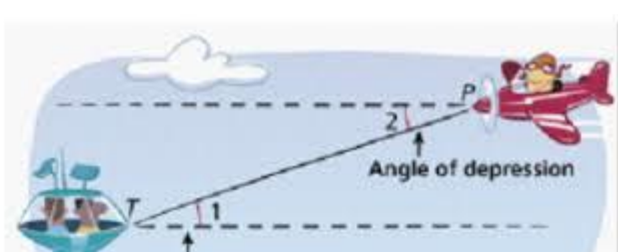
Angle of depression



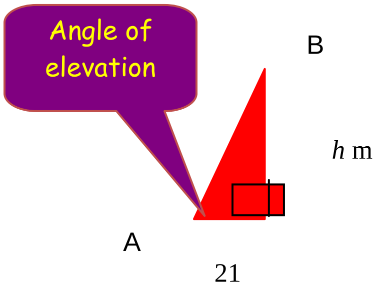
Calculator

Example 4.15

The angle of elevation of building A to building B is 25^0. The distance between the buildings is 21 metres. Calculate how much taller Building B is than building A.



Step 1: Draw a right angled triangle with the given information.



Step 2: Take care with placement of the angle of elevation

Step 3: Decide which trig ratio to use.

$$\tan 25° = \frac{h}{21}$$

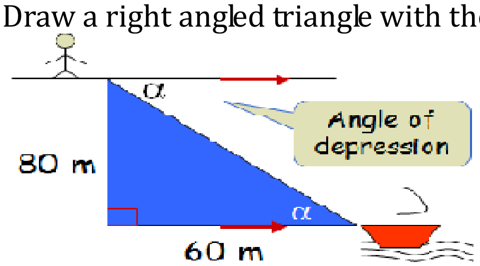
Step 4: Solve the trig equation.

$$h = 21 \times \tan 25°$$
$$h = 9.8\,m \ (1\,dec.\,pl)$$

Example 4.16

A boat is 60 metres out to sea. Madge is standing on a cliff 80 metres high. What is the angle of depression from the top of the cliff to the boat?

step One Draw a right angled triangle with the given information.



Step two Identify the angle of depression

Step 3: Decide which trig ratio to use. $\tan \alpha = \frac{80}{60}$

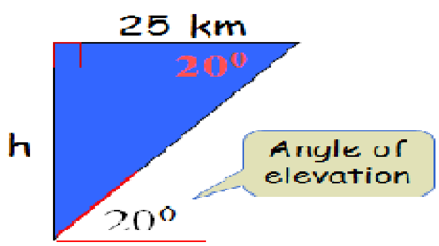
Step 4: Use calculator to find the value of the unknown. $\alpha = 53.8'$

Exercise 4.7

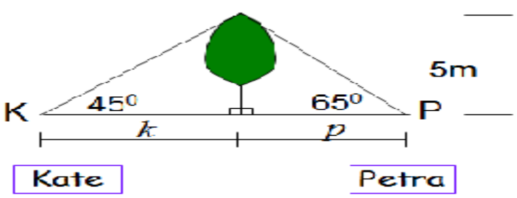
1. Neil sees a rocket at an angle of elevation of 11 degrees. If Neil is located 5 miles from the rocket's launchpad, how high is the rocket? Round your answer to the nearest hundredth.

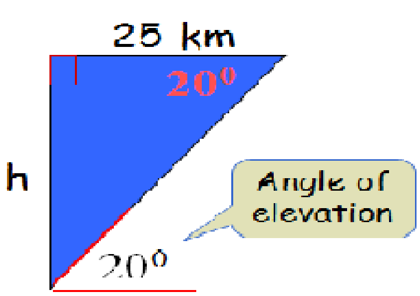
2. A boat is 500 meters from the base of a cliff. Jackie, who is sitting in the boat, notices that the angle of elevation to the top of the cliff is 32°. How high is the cliff? (Give your answer to the nearest metre).

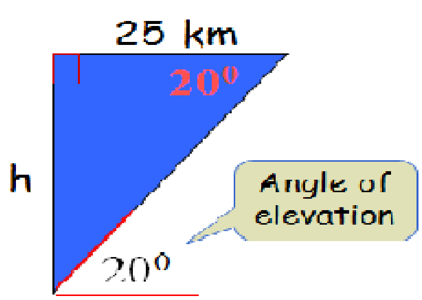
3. Marty is standing on level ground when he sees a plane directly overhead. The angle of elevation of the plane after it has travelled 25 km is 20^0. Calculate the altitude of the plane at this time

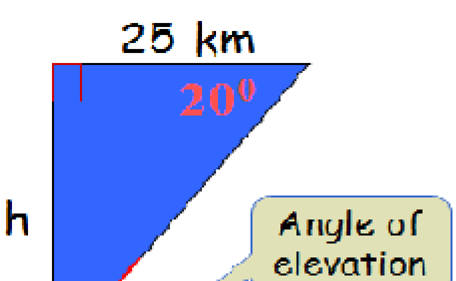


4. Kate and Petra are on opposite sides of a tree. The angle of elevation to the top of the tree from Kate is 45^o and from Petra is 65^o. If the tree is 5 m tall, who is closer to the tree, Kate or Petra





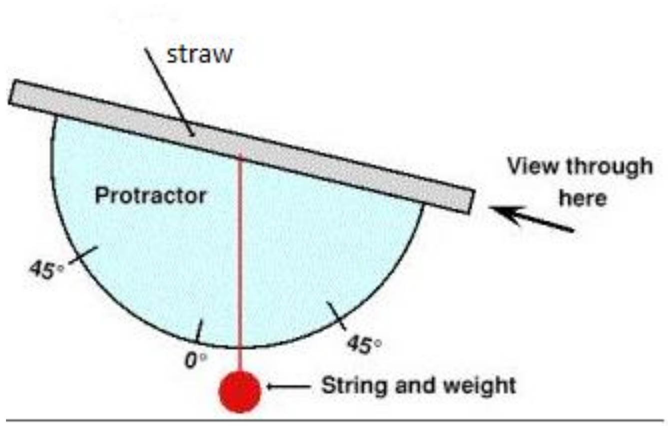




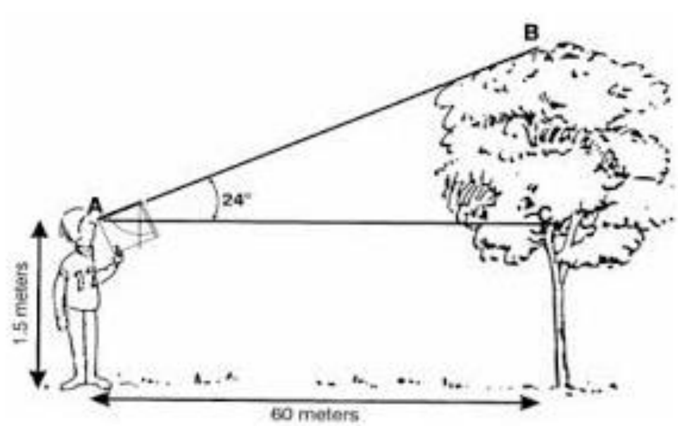
Let's make your own clinometer. To make your clinometer you need the following:

1. A drinking straw 2. A protractor 3. Some adhesive 4. A small mass 5. string

Use the adhesive to attach the straw to the protractor.



2. Let's measure some inaccessible heights in your school.
   (a) You must choose your partner.
   (b) Obtain a measuring tape and measure each other's height up to the eye level only.
   (c) Use the measuring tape to find an appropriate distance back from the object you are finding the height of.
   (d) Hold the clinometer level along the horizontal line and adjust the angle of the straw to sight the top of the object.
   Example



Exercise 4.8

Results

Object name	Height of person's eyes From ground level	Angle of elevation	Distance from base	Height of object (show all working)

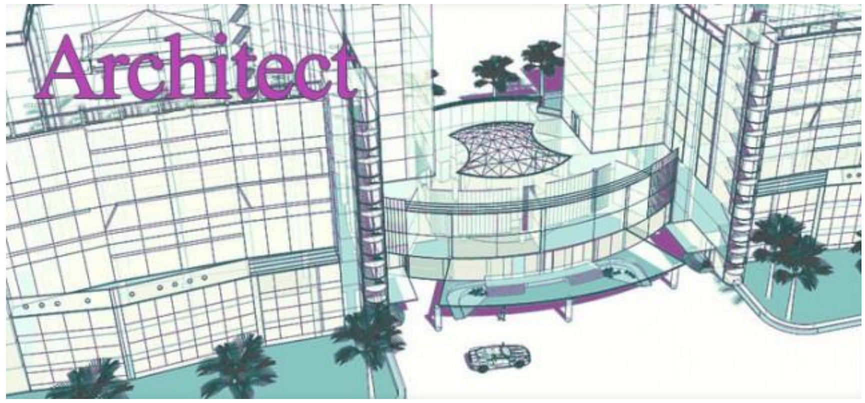
1. If you were to measure the height of a street light would you use a clinometer .Explain why or why not?
2. Jonetani is standing 15m from the base of a building and using a clinometer he measures the angle of elevation to be 37°. If his eyes are 1.65m above the ground, find the height of the building.

4.

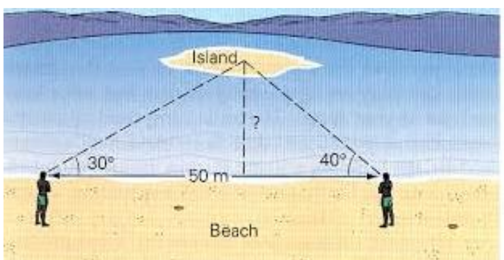
LEARNING OUTCOMES

Students should be able to:

- Identify some career opportunities related to trigonometry

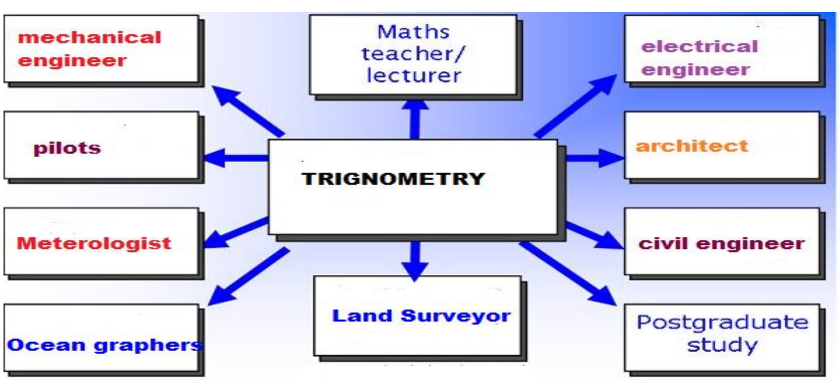


Trigonometry, out of all other topics is Mathematics is the most practical. It is almost used everywhere in our daily lives.

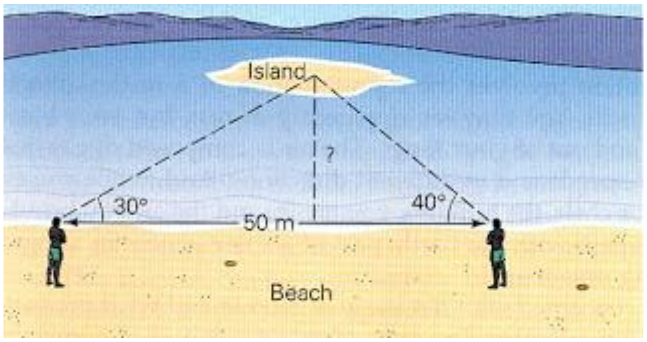


Applications of trigonometry include its study in physics, engineering and chemistry. Though trigonometry tables were created over two thousand years ago for computations in astronomy.

There are numerous jobs that require trigonometry of which some are shown below.



Trigonometry in real life situations



- Trigonometry is commonly used in finding the height of towers and mountains.

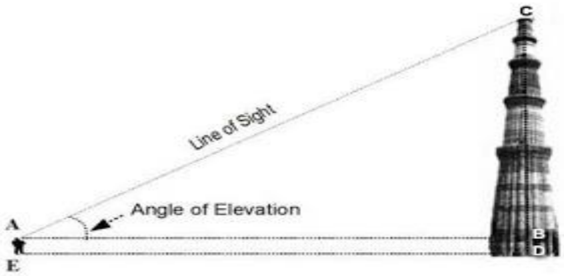
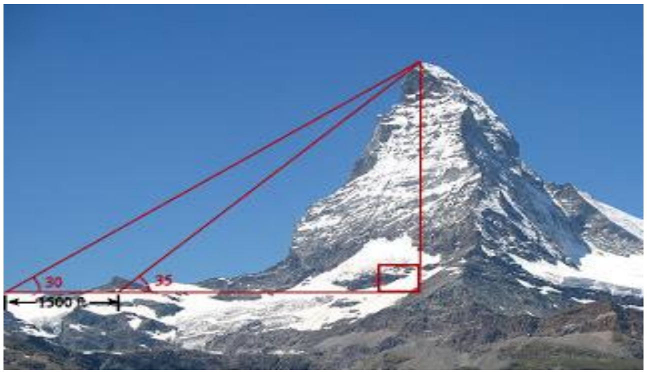
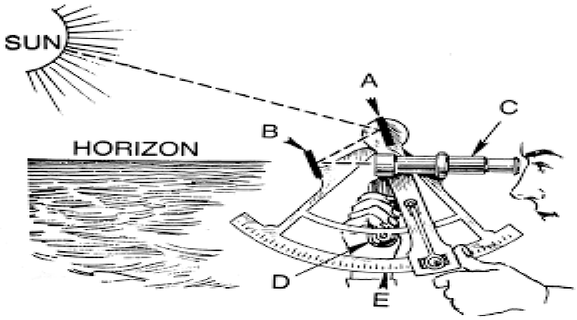


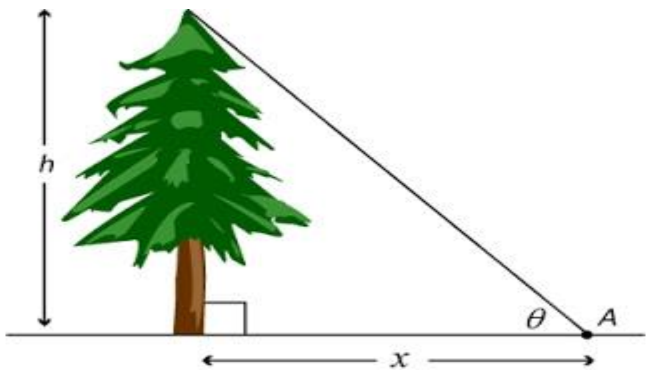
Fig 6



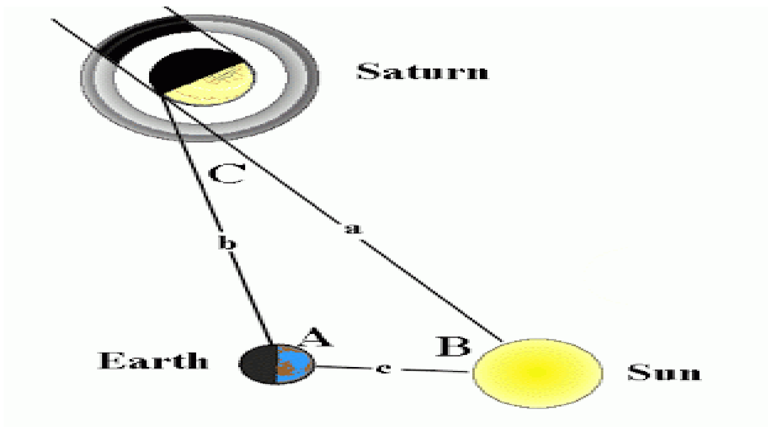
- It is used in navigation to find the distance of the shore from a point in the sea.



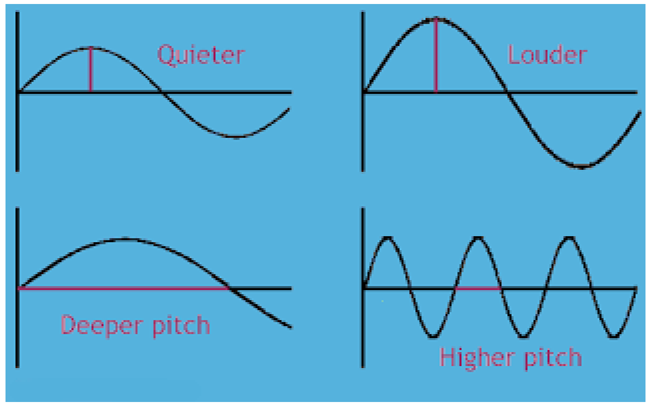
- It is used in oceanography in calculating the height of tides in oceans



- It is used in finding the distance between celestial bodies



- The sine and cosine functions are fundamental to the theory of periodic functions such as those that describe sound and light waves.



- Architects use trigonometry to calculate structural load, roof slopes, ground surfaces and many other aspects, including sun shading and light angles

Source: http://malini-math.blogspot.com/2011/08/applications-of-trigonometry-in-real.html

4.4 Constructions

4.4.1 Construct Angles

LEARNING OUTCOMES

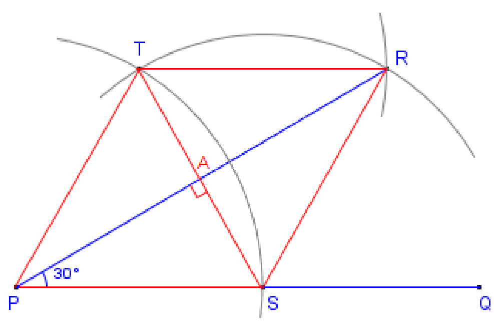
Students will be able to:

- Construct various special angles
- Construct angle bisector, midpoint and mediator of line segment
- Construct centers of triangles using ruler and compass

4.4.1.1 Constructing a 30^o angle

Steps

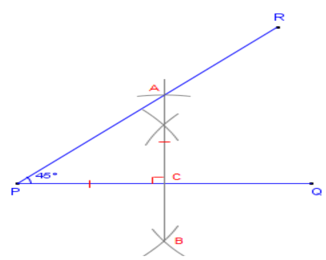
1. Draw a line PQ that will be one leg of the angle
2. With the compass point on P, set it on any width between P and Q
3. Draw an arc crossing PQ at S
4. Move the compass point to S and draw another arc crossing the first one at T
5. Move to T and make an arc crossing the previous one , labelling the intersection point R
6. Draw a straight line from P through R
7. Angle RPQ = 30^O



4.4.1.2 Constructing a 45^o angle

Steps

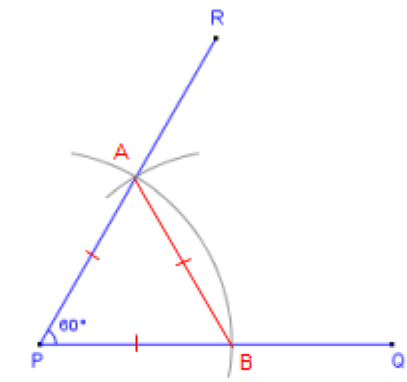
1. Draw a line PQ that will be one leg of the angle
2. With the compass point on P, set it on any width of more than half the length of PQ
3. From points P and Q, draw arcs above and below line PQ
4. Draw a straight line joinning the arc intersections. The intersection of the two straight lines is the midpoint of line PQ.
5. From the midpoint of PQ, set the compass width to point P
6. Draw an arc across the perpendicular line and label it Point C
7. Draw line PC
8. Angle PCQ = 45^O



4.4.1.3 Constructing a 60^o angle

Steps

1. Draw a line PQ that will be one leg of the angle
2. With the compass point on P, set its width to about half of PQ
3. Draw an arc from above point P and crossing PQ
4. Place the compass point to where the arc crosses PQ, draw an arc above PQ and ensure that the two arcs cross each other. Label the point of intersection R
5. Draw straight line PR
6. Angle RPQ = 60^O

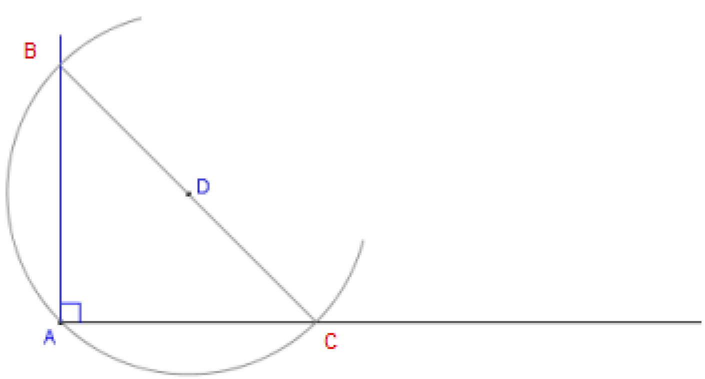


4.4.1.4 Constructing a 90^o angle

Steps

1. Draw a horizontal straight line and label one end A
2. Mark point D somewhere above and between the end points of the line drawn
3. Place the compass point on D and sets its width to point A
4. Draw an arc across the line to above point A
5. Draw a diameter through D starting from where the arc crosses the line
6. Draw a stright line from A to the other end of the diameter

7 The angle between the two straight lines is 90^O

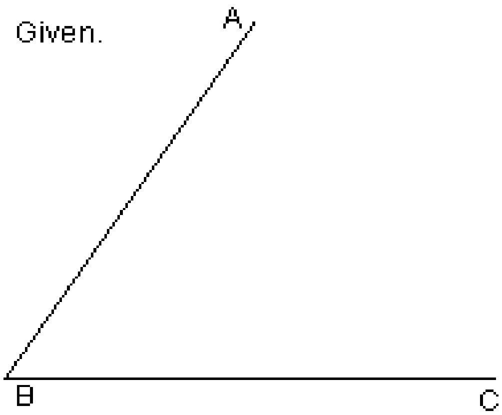


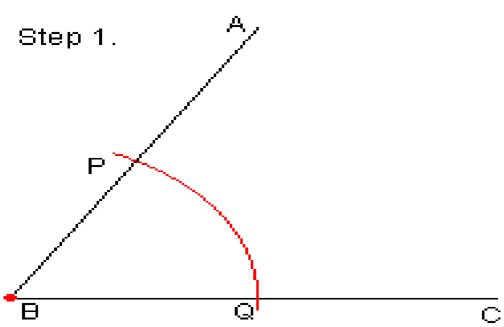
4.4.1.5 Angle Bisector

Steps

Step 1

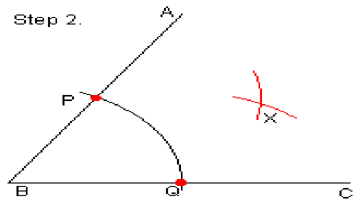
With the compass point at the centre of the vertex of the angle, draw an arc with radius of any length. The arc must be intersecting both sides of the angle. Label the intersection points P and Q





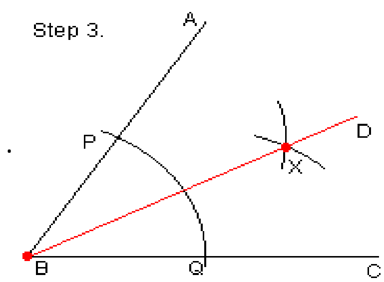
Step 2

Place the compass point at point P , draw an arc of any length within lines AB nad BC. Place the compass at point Q and do the same. The radius of the second arc must be the same as the first and the two arcs must be long enough to intersect at a point X. .



Step 3

Draw a straight line from the center of the vertex through the point of intersection X.



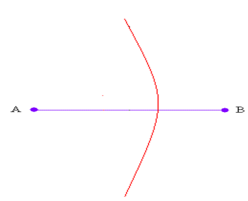
4.4.1.6 Constructing the midpoint of a line segment

The midpoint of a line segmentis its middle point which is equidistant from the end points.

Constructing the midpoint of a segment is considered to be quite simple and easier than any other straightedge construction.

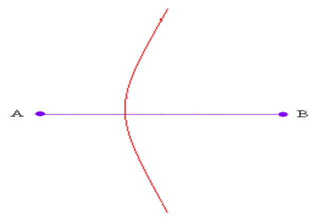
Step 1

Draw line segment AB. Place the compass point at point A and with the width of more than half the length of AB draw an arc from a point above AB right down to a point below AB

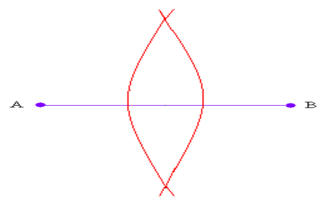


Step 2

Now place the compass point at B and draw an arc

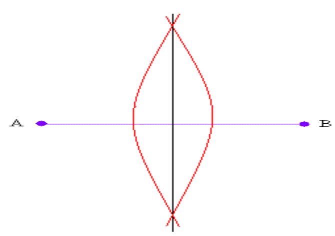


The two arcs should intersect above and below AB

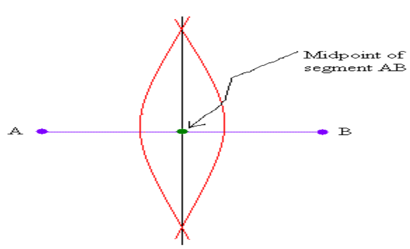


Step 3

Draw a straight line joining the two point intersection

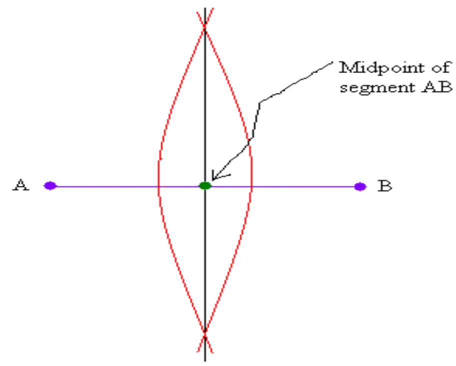


The point of intersection of the vertical line and the line segment AB shown with a green point is the midpoint



4.4.1.7 Constructing the Mediator of a line segment

A **mediator** is the same as a **perpendicular bisector**, The bisector of segement AB is the line perpendicular to this segment which passes through its midpoint

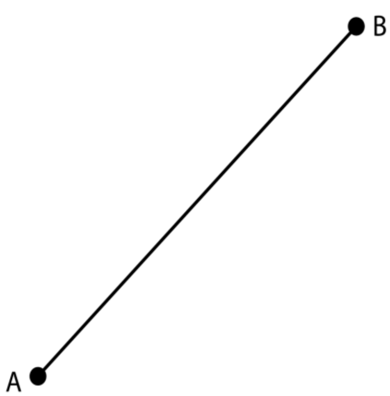


Exercise 4.8

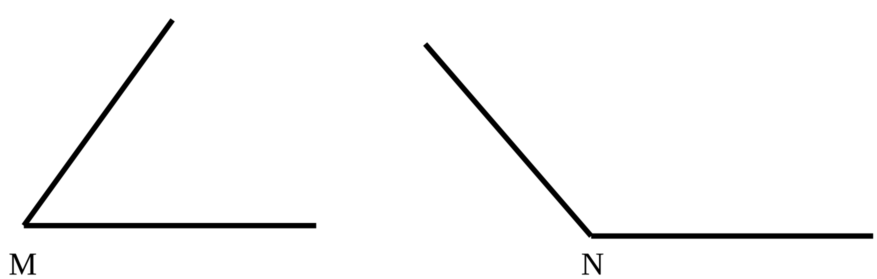
1. Bisect a 30^o angle

2. Bisect a 45^o angle

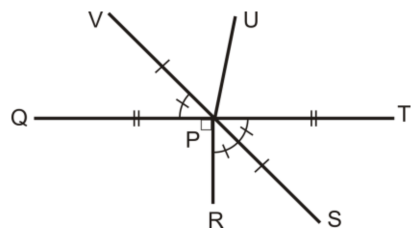
3. Bisect the line segment AB given below



4. Construct the bisector of ∠M and ∠N



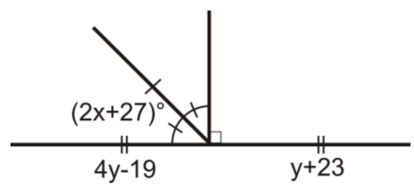
5. Use the following picture to answer the questions.



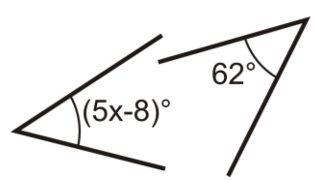
(a) What is the angle bisector of ∠TPS?
(b) P is the midpoint of what two segments?
(c) Find ∠QPR
(d) Find ∠TPS

6. Use algebra to determine the value of variable(s) in each problem.

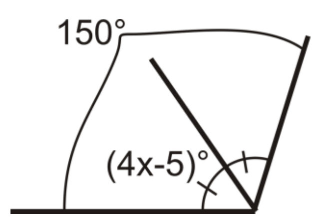
(a)



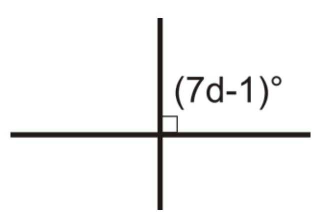
(b)



(c)



(d)

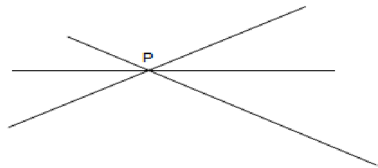


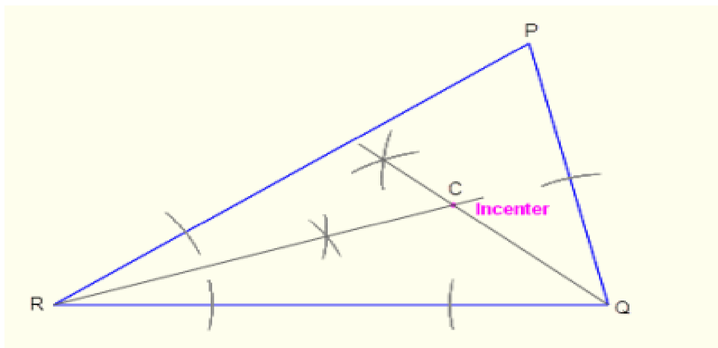
Constructing centers of triangles

4.4.2 The Incenter

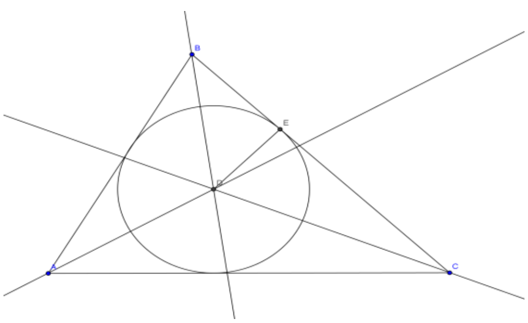
Definition: The point of concurrency of the **angle bisectors** is called the **incenter** of the triangle.

Points of currency: The point where three or more lines intersect and usually refers to various centers of a triangle.





The circle that has its center at the incenter and is tangent to each of the sides of the triangle is called the *inscribed circle*, or simply the *incircle* of the triangle.



The steps for constructing the incenter and incircle are provided below

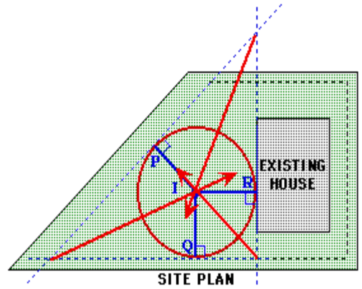
1. Draw a triangle
2. Construct the angle bisectors of each of the interior angles. These bisectors will intersect in a single point called the incenter.
3. Mark the point of intersection of these lines.

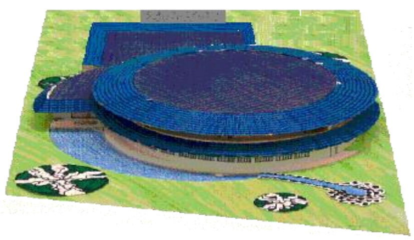
4. Draw a perpendicular line from the incenter to one of the sides of the triangle. This line becomes a radius.
5. Construct a circle that has center of the incenter and the intersection point on the circle itself. This is the incircle.

Real life examples

Question : You are an architect and are given a task to design a round office tower. The client wants the tower to be circular. The location on which it will be built is at the corner of two streets and there is another buildng located on the property Where would you place the center of the lanai (A veranda or roofed patio), to make it as big as possible?

Answer: The center of the round house needs to be equidistant from the street and the two properties lines: if we must therefore be equidistant from the 3 sides of the triangle formed by the 3 lines. If we consider 2 lines at a time, the set of points equidistant from two lines is the bisector of the angle between 2 lines. So the set of points equidistant from all 3 lines is the point where all 3 angle bisectors meet. This point is called the Incenter of the triangle. Because it is equidistant from the 3 sides of the triangle, if we use that distance as radius, we can construct a circle tangent to all 3 sides of the triangle. This circle is the outline of our lovely new round house





Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=incenter+of+a+triangle+in+real+life

- The **incenter** could be used to build a clock. You wouldn't want the hands on the clock to be off centered so you would find the middle of the circle.

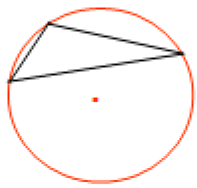
- Three pilots are flying over a triangular city. They plan to attack. The approach the city at the corners, or verticies. They bisect each angle. They all are traveling, and have a 3 plane collision in the center of the city. Too bad they hadn't calculated the incenter.

Source**:** http://www.illustrationsof.com/royalty-free-airplane-clipart-illustration-32931.jpg

4.4.3 The Circumcenter

Circumcenter is the center of a triangle's circumcircle. It is where the "perpendicular bisectors" (the lines perpendicular to the side of each triangle which passes through its midpoint) meet.

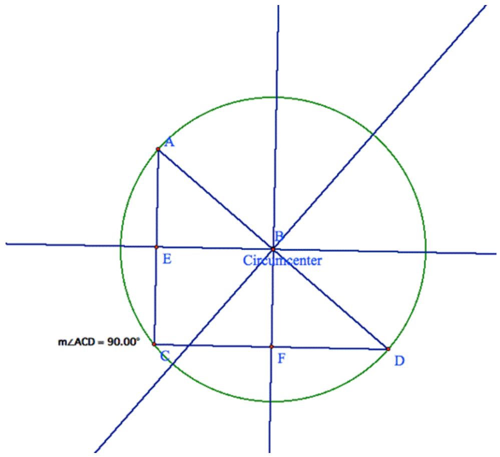
The circumcircle (circumscribed circle) of a polygon is a circle which passes through all the vertices of the polygon. The center of this circle is called the circumcenter.



In the case of a triangle, there is always a circumcircle possible, no matter what shape the triangle is.

The circumcenter is constructed by identifying the midpoints of the segments AC, CD, and DA. Then a perpendicular line is drawn through the midpoints perpendicular to the side segment.

The example shown below is of a right – angled triangle.



Note that the triangle is inside the circle and the circumcenter is in the center of the circle.

Constructing the Circumcenter
Use these steps to construct the circumcenter of the given triangle (ΔACD).

Steps

1 Construct the perpendicular bisector of one side (CD).

2. Construct the perpendicular bisector two other sides: AC and DA.
3. Where the 3 perpendicular bisectors intersect is the circumcenter. Mark this as point B.
4. Place the compass point on B. Open the compass so that the pencil is on any vertex. of the triangle. Draw a circle with this compass width with the center at B.

Real Life Examples

There are many reasons people may need to know the altitude of buildings. Pilots need to know the altitude of a building if they are going to be landing on a building, such as the pilots that fly the flight for life helicopter.

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+helicopters

Without knowing the altitude of the building the pilot wouldn't know how fast or slow to begin going and when to position the jet to get it on the building. Altitdue plays a very important role in the career of Piloting.

There are many uses of the circumcenter of a triangle in real life, but most of these revolve around one main thing. That is when you are locating a main point which is the same distance to three different locations or spots making it most relevant to people. First of all circumcenter is where three perpendicular bisectors meet at the point of concurrency meaning that spot would be equal to all three vertices.

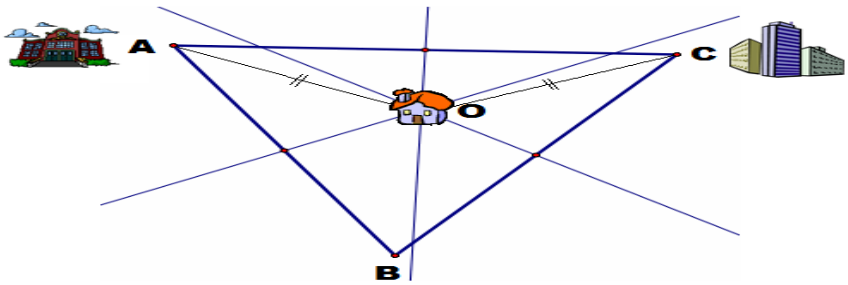
- A company is designing a mall. They want the food court to be in the very center, and equidistant from the three main stores (or verticies). They would put it at the circumcenter.

Source: http://www.picturesof.net/_images_300/People_At_A_Shopping_Mall_Royalty_Free_Clipart_Picture_081106-160290-408048.jpg

- Finding the **circumcenter** could also be used when building a house. If you wanted to put a window in the middle of a wall then you could find the circumcenter to do that.

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=circumcenter+of+a+triangle+in+real+life

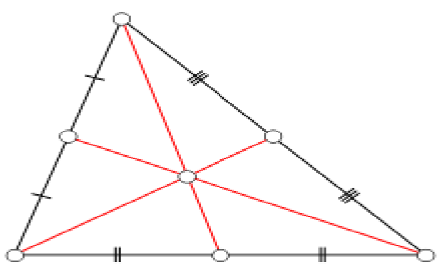
- a family wanted to buy a home, but they wanted it to be close both to both the children's school and the parents' workplace. By looking at a map, they could find a point that is equidistant from both the workplace and the school by finding the ***circumcenter*** of the triangular region.



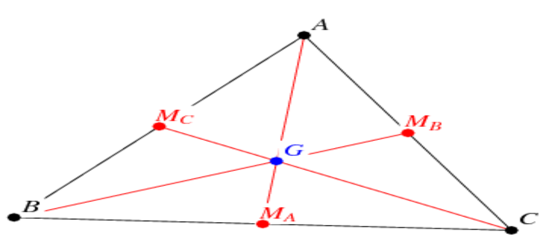
4.4.4 The Centroid

A **centroid of a triangle** is the point where the three **medians** of the triangle intersect.

A **median** of a triangle is a line segment joining the vertex to the mid - point on the opposite side of the triangle.



The **centroid** is also referred to as the center of gravity of the triangle



Centroid facts

- The centroid is always inside the triangle
- Each median divides the triangle into two smaller triangles of equal area.
- The centroid is exactly two-thirds the way along each median.
  Put another way, the centroid divides each median into two segments whose lengths are in the ratio 2:1, with the longest one nearest the vertex. These lengths are shown on the one of the medians in the figure at the top of the page so you can verify this property for yourself.

Constructing the Centroid

The steps for constructing centroid are given below:

1. Draw Triangle *ABC*

2. Construct the mid - point of each side of the triangle and label them M_A, M_B, M_C accordingly

3. Draw a line joining the mid – point of *BC* to the vertex *A* on the opposite side (median). Repeat this for the other two sides and vertices

4. Label the point of intersection of these three medians *G* which is called the **Centroid**

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=centroid+in+real+life&start=10

The **centroid** of a triangle could be used in real life by needing to find the center of a certain area. For example someone is putting a swimming pool in the center of a community they will need to find right where the middle is.

A Centroid; A magician is preforming a show, and he needs an object to remain on a triangular stand as he pulls away a cloth from beneath. The centroid is the center of gravity. He would be successful if the object was placed on the centroid.

Other real life situations

Baseball pitch

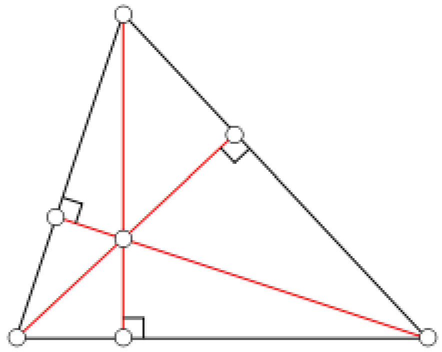
Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+baseball+fields

Roundabout

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+roundabouts

4.4.5 The Orthocenter

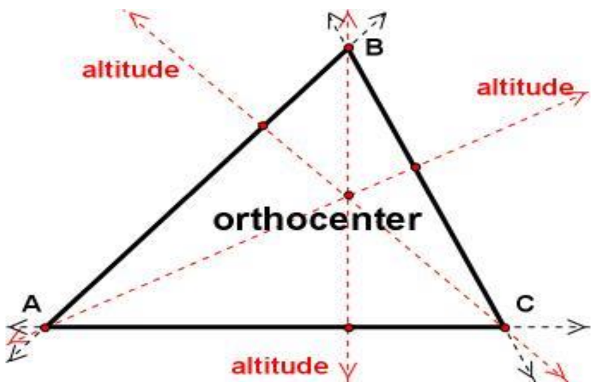
The **orthocenter** of a triangle is the intersection of the three **altitudes** of a triangle. The **altitude** of a triangle is a **perpendicular** segment from the vertex of the triangle to the opposite side.



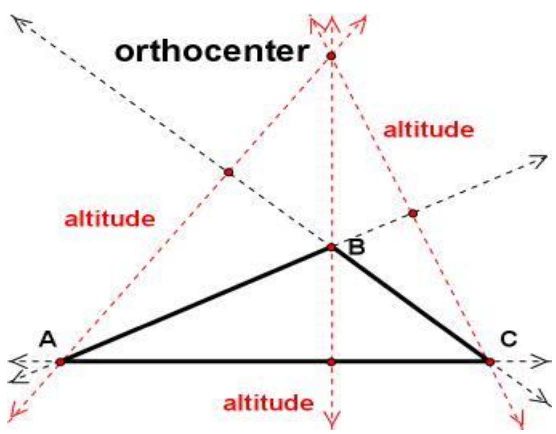
There are therefore three altitudes possible, one from each vertex.

The orthocenter is not always inside the triangle. If the triangle is obtuse, it will be outside. To make this happen the altitude lines have to be extended so they intersect.

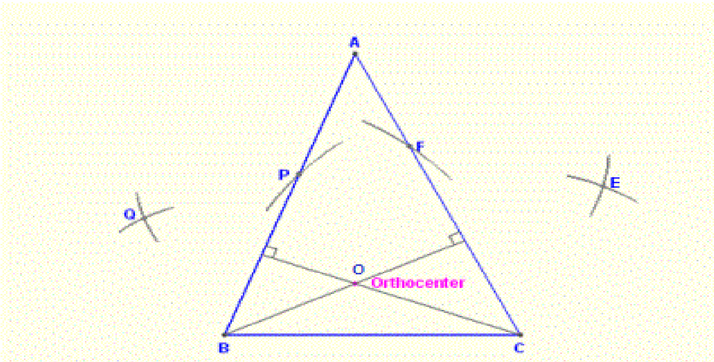
Orthocenter of an Acute triangle



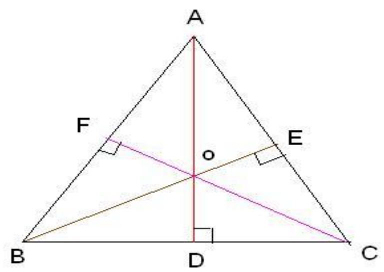
Orthocenter of an Obtuse triangle



Constructing orthocenter



If the three coordinates A, B and C are given, the following procedure is to be followed to find the orthocenter:



1. Draw triangle ABC
2. Select vertex A and segment BC
3. Construct an altitude from vertex A to segment BC
4. Repeat this for the other two vertices and segments
5. The point where these altitudes intersect is the orthocenter

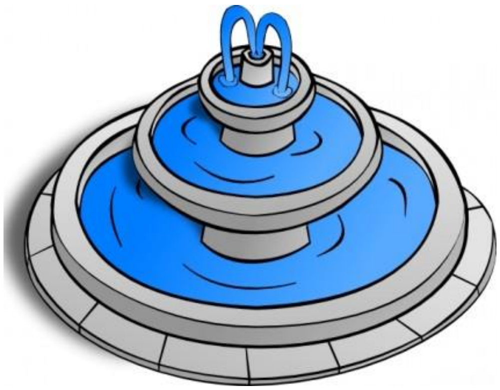
Real Life Examples

Orthocenter:
Imagine that you still live at a vertex of Denny Triangle. You want to find the shortest distance you must walk to get to the street that is the opposite side of the triangle. Since a straight line is the shortest distance, finding the street that is perpendicular to the opposite Would give you the shortest distance. Finding the orthocenter would give the perpendicular line, or altitude, from any vertex.

- Another example of **orthocenter** is the **Eiffel tower**. They might of use the orthocenter to find where all the altitudes met while building it.

- In a triangular format lay a mall, park, and hotel. The city wants to build a memorial fountain visible from every site. They find the altitudes, then the orthocenter for everyone to enjoy.

Source: http://images.all-free-download.com/images/graphiclarge/fountain_clip_art_17015.jpg



Point of concurrency

The Centroid is the point of concurrency where the 3 medians of a triangle meet. This point is also the triangle's center of gravity.

2) The Circumcenter is the point of concurrency where the perpendicular bisectors of all three sides of the triangle meet. This point is the center of the triangle's circumscribed circle.

3) The Incenter is the point of concurrency where the angle bisectors of all three angles of the triangle meet. Like the circumcenter, the incenter is the center of the inscribed circle of a triangle.

4) The Orthocenter is the point of concurrency where the 3 altitudes of a triangle meet. Unlike

the other three points of concurrency, the orthocenter is only there to show that altitudes are concurrent. Thus, bringing me back to the initial statement.

Exercise 4.9

1. Write True or False

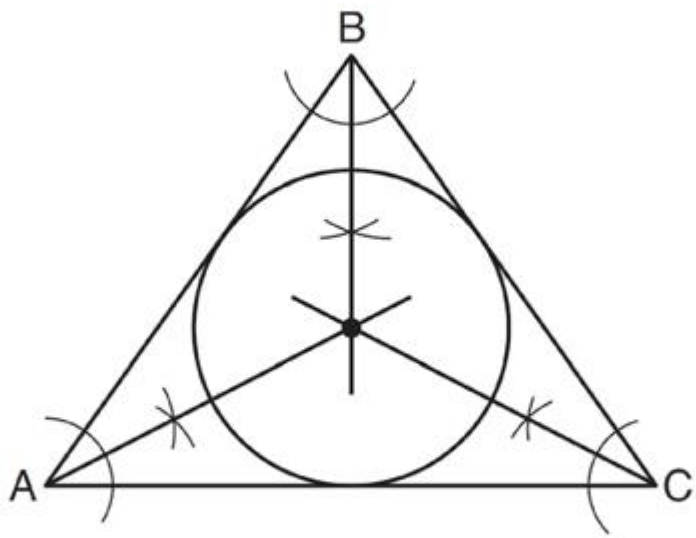
   (a) The angle bisectors of a scalene triangle intersect outside the triangle ___________

   (b) The altitude from the vertex angle of an isosceles triangle is always the median__

   (c) To find the point that is equidistant from the sides, find the circumcenter ______

   (d) The median starts at a vertex and goes to the opposite midpoint ______________

2. Which principle is used in the construction shown below?



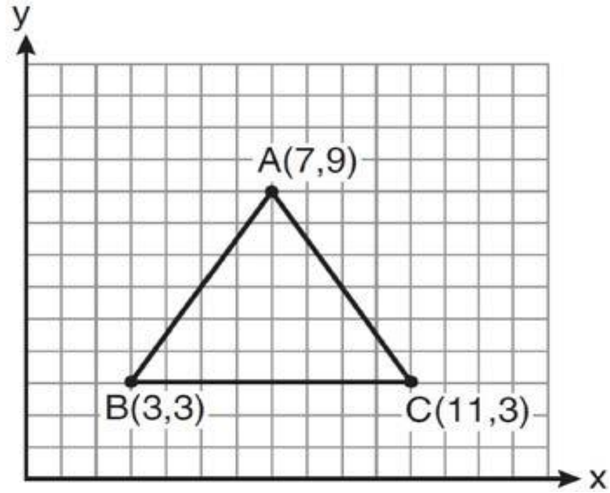
   A. The intersection of the angle bisectors of a triangle is the center of the inscribed circle

   B. The intersection of the angle bisectors of a triangle is the center of the circumscribed circle.

   C. The intersection of the perpendicular bisectors of the sides of a triangle is the center of the inscribed circle.

   D. The intersection of the perpendicular bisectors of the sides of a triangle is the center of the circumscribed circle.

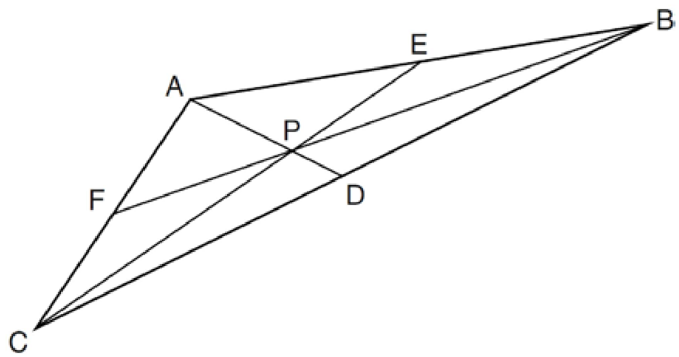
3. The vertices of the triangle drawn are $A(7, 9)$, $B(3, 3)$, and $C(11, 3)$.



What are the coordinates of the centroid of △ABC?

A. (5, 6)
B. (7, 3)
C (7, 5)
D (9, 6)

4 In the diagram below of ABC, AE ≅ BE, AF ≅ CF, and CD ≅ BD.



The point marked P is known as the

A Incenter
B Circumcenter
C Orthocenter
D. Centroid

5 The coordinates of the endpoints of *AB* are *A*(0, 0) and *B*(0, 6). The equation of the perpendicular bisector of *AB* is

A. $x = 0$
B. $x = 3$
C. $y = 0$
D. $y = 3$

4.5 Intersecting Chord Theorem

Learning outcomes

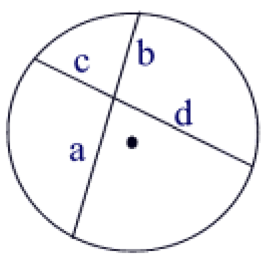
Students will be able to:

- Identify different types and properties of intersecting chords.
- Prove properties of intersecting chords
- Apply intersecting chords to real life

4.5.1 Two Chords Intersect in a Circle

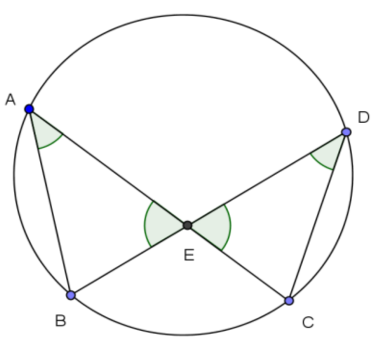
Theorem 1

If two chords intersect inside a circle then the product of the lengths of the segments of one chord equals the product of the lengths of the segments of the other chord



$a \bullet b = c \bullet d$

Theorem Proof:



Given chords AB and CD, prove that $\overline{AE} \bullet \overline{EC} = \overline{BE} \bullet \overline{ED}$

∠BAC = ∠BDC as inscribed angles subtended by the same chord BC

∠ABD = ∠ACD   as inscribed angles subtended by the same chord AD

∠AEB = ∠DEC   as a pair of vertical angles

By Triangles with Two Equal Angles are Similar we have $\triangle AEB \sim \triangle DEC$.

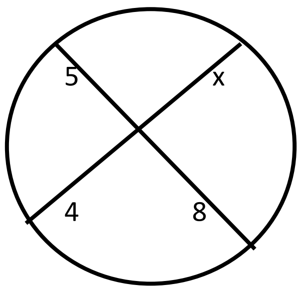
Thus:

$$\frac{AE}{EB} = \frac{DE}{EC}$$

$$\Longrightarrow\ AE \cdot EC = DE \cdot EB$$

Example 4.17 Find *x* in each of the following

(a)

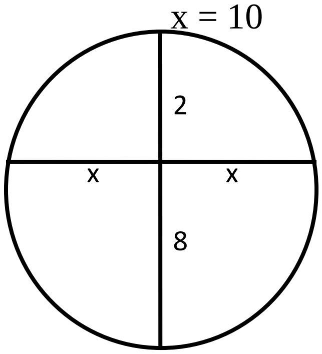


By Theorem 1, $4 \bullet x = 5 \bullet 8$

$4x = 40$

$x = 10$

(b)



By Theorem 1, $x \bullet x = 2 \bullet 10$

$x^2 = 20$

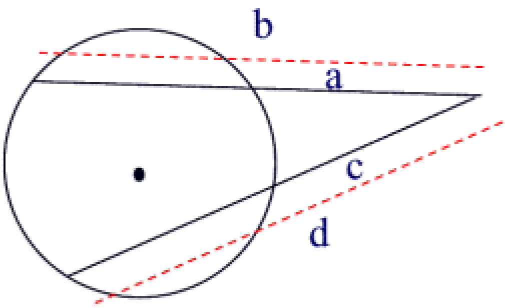
$x = \sqrt{20}$

$x = 4.47$

4.5.2 Two Secant Segments

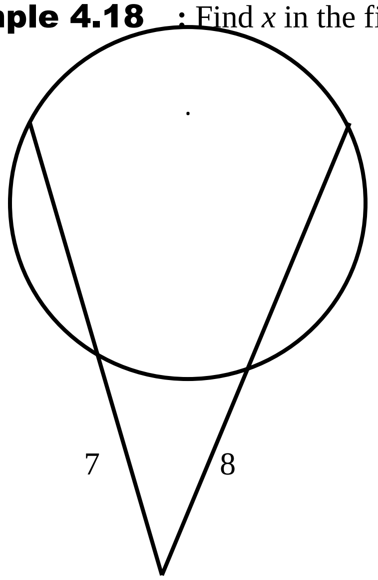
Theorem 2

If two secant segments are drawn to a circle from the same external point, the product of the length of one secant segment and its external part is equal to the product of the length of the other secant segment and its external part.



a • b = c • d

Example 4.18 : Find *x* in the figure below



By Theorem 2, **(x + 7) • 7 = (6 + 8) • 8**

7x + 49 = 14 • 8

7x = 112

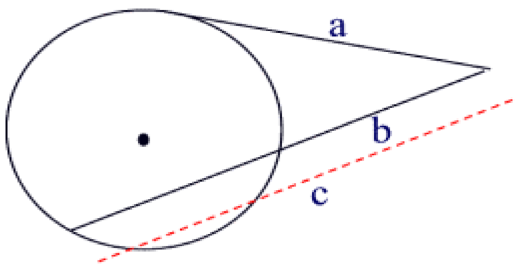
x = 112/ 7

x = 16

4.5.3 A Tangent and a Secant Segment

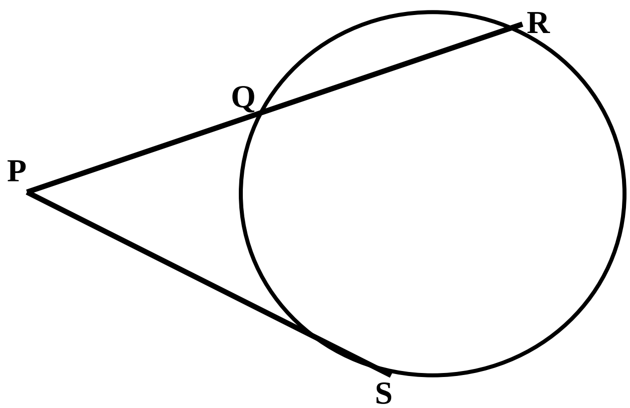
Theorem 3

If a secant segment and tangent segment are drawn to a circle from the same external point, the product of the length of the secant segment and its external part equals the square of the length of the tangent segment.



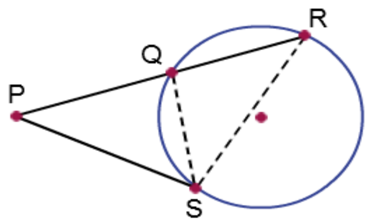
$b \cdot c = a^2$

Given secant PR and tangent PS, prove that $PS^2 = PR \times PQ$



Theorem Proof:

In order to prove this theorem, we are going to join QS and RS.



In ΔPSQ and ΔPRS

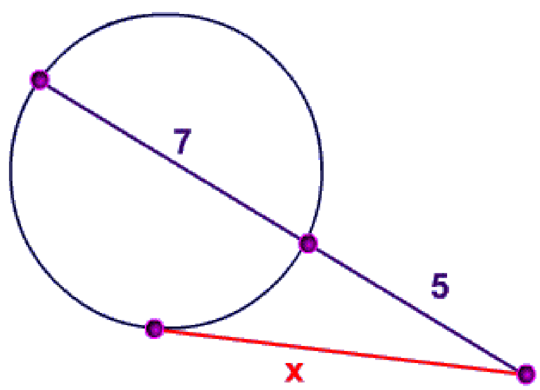
$\angle PSQ = \angle PRS$ ______ (By tangent-chord theorem, according to which an angle formed by a tangent and a chord is equal to the angle formed by that chord at another point at the circle.)

angle QPS = angle RPS ______ (Common angle)

Thus, $\triangle PSQ \sim \triangle PRS$

$$\frac{PS}{PR} = \frac{PQ}{PS}$$

$PS^2 = PR \times PQ$ **Hence proved.**



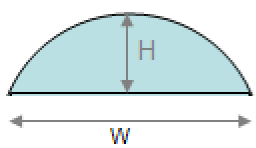
Example

Use the theorem for the intersection of a tangent and a secant of a circle to solve the problems below.
In the diagram on the left, the red line is a tangent, how long is it?

$x^2 = (7+5) \cdot 5$
$x^2 = (12) \cdot 5$
$x^2 = 60$
$x = \sqrt{60}$

$x = 7.75$

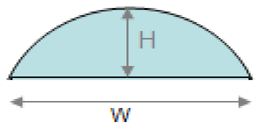
A Practical use



When making doors or windows with curved tops we need to find the radius of the arch so we can lay them out with compasses. See Radius of an Arc for a way to do this using the Intersecting Chords Theorem.

Circular arcs turn up frequently in the real world, such as the top of the window shown on the right. When constructing them, we frequently know the width and height of the arc and need to know the radius. This allows us to lay out the arc using a large compass to calculate the rad

Given an arc or segment with known width and height:



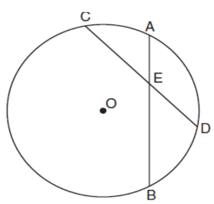
The formula for the radius is:

$$Radius = \frac{H}{2} + \frac{W^2}{8H}$$

where:
W is the length of the chord defining the base of the arc
H is the height measured at the midpoint of the arc's base.

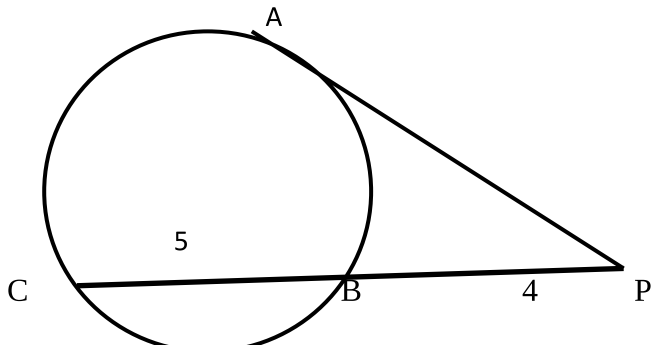
Exercise 4.10

1. In the circle given below, $\overline{AB}$ and $\overline{CD}$ intersect at E. If $\overline{CE}$ = 10 cm, $\overline{DE}$ = 6 cm and $\overline{AE}$ = 4 cm, what is the length of $\overline{EB}$?



A. 15cm
B. 12cm
C. 6.7cm
D 2.4cm

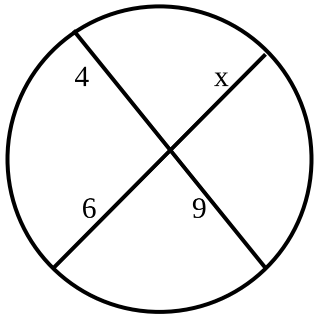
2. In the diagram below, tangent $\overline{PA}$ and secant $\overline{PBC}$ are drawn from external point P. What is the length of $\overline{PA}$?



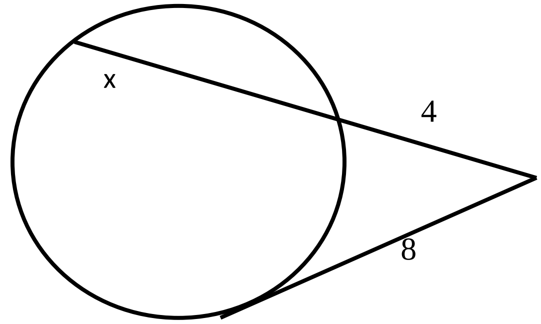
A. 20
B. 9
C. 8
D. 6

3. For each of the following, find the value of x

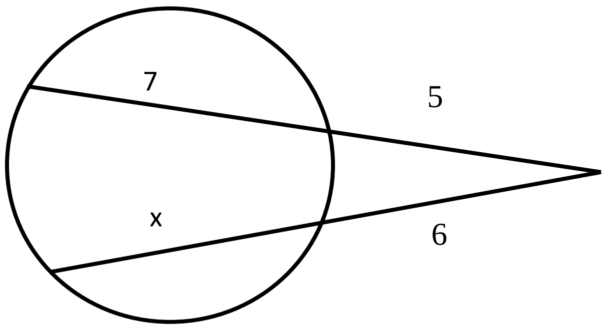
a



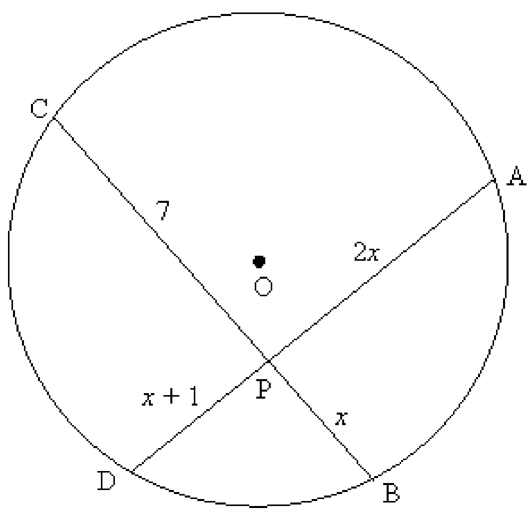
b.



c.



4. In the circle given below, chords CB and AD intersect at point P. The segments formed by these intersecting chords are CP = 7, BP = x, AP = 2x, and DP = x + 1. What is the measure of chord CB?



GLOSSARY		
1	Acute triangle	All angles are less than right angles
2	Adjacent side	Adjacent sides are those the are next to each other
3	Altitude	A line segment through a vertex and perpendicular to (i.e. forming a right angle with) a line containing the base (the opposite side of the triangle)
4	Angle bisector	A line segment that bisects one of the vertex angles of a triangle.
5	Angle of depression	Angle below the horizontal
6	Angle of elevation	Angle above the horizontal
7	Arc	A portion of the circumference of a circle
8	Bisect	To divide into two equal parts
9	Centroid	It is a point of concurrency of the triangle. It is the point where all 3 medians intersect and is often described as the triangle's center of gravity
10	Chord	It is a geometric line segment whose endpoints both lie on the circle
11	Circumcenter	The point where the three perpendicular bisectors of the sides of a triangle meet. Also, the center of the circumcircle. One of a triangle's points of concurrency.
12	Circumscribed circle	It is a circle which passes through all the vertices of the polygon. The center of this circle is called the circumcenter and its radius is called the circumradius
13	Clinometer	An instrument used in surveying for measuring an angle of inclination
14	Point of Concurrency	The point of intersection of the lines, rays, or segments
15	Construction	The act of drawing geometric shapes using only a compass and straightedge
16	Equidistant	When a point is the same distance from each figure.
17	Geometry	The branch of mathematics that treats the properties, measurement, and relations of points, lines, angles, surfaces, and solids.
18	Hypotenuse	The side of a right triangle opposite the right angle.
19	Incenter	The point of concurrency of the three angle bisectors of a triangle
21	Inscribed circle (incircle)	When a circle drawn in a triangle touches the sides
23	Median	A segment from a vertex to the midpoint of the opposite side
24	Mediator	The plane through the midpoint of a line segment and perpendicular to that segment
25	Midpoint	A point on a line segment that divides it into two equal parts
27	Orthocenter	It is where altitudes meet.
28	Perpendicular bisector	A segment, ray, line, or plane that is perpendicular to a segment at its midpoint
30	Pythagoras Theorem	The square of the length of the hypotenuse of a right triangle equals the sum of the squares of the lengths of the other two sides
31	Secant segment	A secant is a line that intersects a circle at two points.
32	Segment	The region bounded by a chord and the arc subtended by the chord
34	Straightedge construction	It is the construction of lengths, angles, and other geometric figures using only a ruler and compass.
35	Subtend	To be opposite to and delimit (an angle or side)
36	Tangent	A straight line to a plane or curve at a given point that "just touches" the curve at that point
38	Trigonometry	The branch of mathematics dealing with the relations of the sides and angles of triangles and with the relevant functions of any angles
39	Vertex	A vertex is a point where two or more straight lines meet. It is a Corner.

HISTORY OF MONEY

Money, one of the earliest and most significant inventions of civilization, is important to the development of trade. Before money was used, there was only barter, a relationship between two people each of whom has something which the other wants

The earliest currency used in commercial transactions occurred in Egypt and Mesopotamia by the third millennium BC. It consisted of gold bars which needed to be weighed to establish their value each time they were exchanged. Later these were supplemented by gold rings for smaller sums. In about 2500 BC an extensive trade, at ***Ebla*** *in modern Syria, was based on currency of this kind in silver and gold.*

The earliest known coins in the western world came from the city of Ephesus in ***Ionia*** *(in western Turkey) in about 650 BC. The metal used was electrum, a natural alloy of gold and silver found locally. The coins were bean shaped and were struck on one side with a distinguishing mark, such as the image of a lion. The underlying purpose was to ensure a stable value in this variable metal of exchange, previously traded by weight alone.*

A century later, a king of neighbouring Lydia became the first ruler to mint coins in pure gold and silver. They show the facing heads of a lion and a bull. Greek cities closed to Lydia including the Persian empire were quick to adopt the useful new technique of metal currency. By the end of the 6th century coinage was common used throughout the region

Source: http://www.historyworld.net/wrldhis/PlainTextHistories.asp?historyid=ab14

5.1 Money

LEARNING OUTCOME

Students should be able to:

- explore the application of percentages in social aspects of living.

5.1.1 Hire Purchase Agreements

LEARNING OUTCOME

Students should be able to:

- Calculate the total amount paid when goods are bought on hire purchase, credit terms and lay by .
- Discuss the advantages and the disadvantages of buying on goods on these terms.

Stores like Courts, Morris Hedstrom and Tappos are popular places in Fiji which sell televisions, furniture and many household items. The reason for their popularity is that they sell their goods in two ways either cash or credit.

Cash is the usual way for paying. When goods are bought on cash the full amount is paid and the good is taken home.

Hire purchase is buying on credit. Where the store trusts its customers and expects them to pay later. Usually the store charges a certain amount of the price as deposit which the customers may choose to pay. After which the customer pays monthly or weekly instalments. The store charges a certain amount of interest when goods are bought on hire purchase.

With lay -by the item is put away for the customer by the store, regular payments need to be made to the store. when the entire cost of the item is paid for then the customer takes the good home. Lay- by does not cost any more than the ticketed price of the item.

Using credit cards is another way of paying for items and taking the goods home. The transaction is finished from the retailer's point of view. The entire balance of the card can be paid without being charged interest if the customer is able to pay within the timeframe otherwise interest is charged on the purchase.

Example 5.1

Cash price $ 2700

Or 24 monthly payments of $150.00

The above deal is offered at a store in Suva. The customer wishes to take the set on hire purchase.

Calculate:

(I) Total amount the customer pays while buying on hire purchase.

Solution

24 x $150.00 = $3600.00

(ii) The amount the customer saves if the set is bought on cash.

Solution

$3600.00 - $2700.00 = $900.00

(iii) The percentage interest that the store charges.

Solution

($900.00 ÷ $2700.00) x 100 = 33.33%

Example 5.2

Alpana really likes a sari that she saw in Bhanlal & Sons. The sari costs $ 250 but Alpana just has $50 with her. The shopkeeper suggests that she lay – by the good. Calculate the number of payments that Alpana needs to make if she deposits $50 and makes weekly payments of $25.00.

Solution

$ 250.00 - $ 50.00 = $200.00. (The amount owed)

$200.00 ÷ $25.00 = 8 weekly payments.

Exercise 5.1

1. Define the following terms:
   (a) Hire purchase (b) deposit (c) lay – by

2. The cash price of a bedroom suite is $4200. The bedroom suite can be purchased on the following terms: 20% deposit and weekly payments of $43.94 for two years. Calculate the cost of buying the bedroom suite if you bought it on terms.

3. Robert wants to buy a used car with a cash price of $12600. The dealer offers terms of 10% deposit and monthly repayments of $812.70 for two years.

   a. Calculate the amount of the deposit
   b. Calculate the total amount to be paid in monthly repayments.
   c. What is total amount Robert pays for the car?
   d. How much more than the cash price of the car does Robert pay.

4. Miriama buys a settee on hire purchase. The cash price is $800. She pays 30% deposit and the store charges 9% interest.

   a. If she takes two years to pay for the settee, what are her monthly payments?
   b. What is the total amount that she pays for the settee.

5. Discuss the advantages and disadvantages of buying on
   (i) Hire purchase
   (ii) Lay – by

5.1.2 Tax Assessment

LEARNING OUTCOME

Students should be able to:

- Explain the need for taxes.
- Define Pay As You Earn and Normal tax
- Calculate the amount of PAYE or normal tax

WHY TAXES???

Taxes are a form of income for the government of Fiji. This income is used for social security, health care, national defence and for many other important reasons.

Pay As You Earn

- Income tax is a tax on the money that you earn and is called **Pay As You Earn.**
- In Fiji there is a new system where the correct amount of PAYE is deducted from the salaries and wages.
- The main objective of the new system is to ensure that tax payers pay the correct tax and there is no refund or liability.
- The last tax return was lodged in 2013 as in 2012 the new system was not put into place and there were refunds and liabilities.
- From 2014 the tax return forms were not lodged.
- In Fiji any individual earning up to $15600 does not have to pay tax.
- There are separate rates of normal tax for resident and non-resident individuals.
- The tables below show the tax rates that are charged on the chargeable income.
- **Chargeable income =Total income – Total allowances & deductions**
- **Normal tax** is a basic rate of taxation (as on income) applied to taxpayers.
- The calculation of normal tax will be based on actual income and the tax rates applicable to the relevant period

Resident tax rates (2013)

Chargeable income	Tax payable
0-16000	nil
16,001-22,000	7% of excess over $16000
22,001-50,000	420 +18% of excess over $22,000
50,001-270,000	5460+20% of excess over $50,000
270,001-300,000	49460+20% of excess over $270,000
300,001-350,000	55460+20% of excess over $300,000
350,001-400,000	65460+20% of excess over $350,000

Source : http://www.frca.org.fj/residents-tax-rates

Non Resident tax rates (2013)

Chargeable income	Tax payable
0-16000	20% of excess of $0
16,001-22,000	3200 + 20% of excess over $16000
22,001-50,000	4400 + 20% of excess over $22,000
50,001-270,000	10,000 + 20% of excess over $50,000
270,001-300,000	54,000 + 20% of excess over $270,000
300,001-350,000	60,000 + 20% of excess over $300,000
350,001-400,000	70,000 + 20% of excess over $350,000

Source : http://www.frca.org.fj/nonresidents-tax-rates

Example 5.3

Mr.Mikaele Waqa is resident of Fiji and is a school teacher and his chargeable income is $24,493. Using the tables above calculate his normal tax.

SOLUTION

Step 1

Since Mr. Mikaele is a resident we will use the table for resident tax rates and we choose the row in which Mr. Mikaele's income falls.

Chargeable income	Tax payable
0-16000	nil
16,001-22,000	7% of excess over $16000
22,001-50,000	420 +18% of excess over $22,000
50,001-270,000	5460+20% of excess over $50,000
270,001-300,000	49460+20% of excess over $270,000
300,001-350,000	55460+20% of excess over $300,000
350,001-400,000	65460+20% of excess over $350,000

Step 2

Find the excess over $22,000.

$24,493 - $22,000 = $2493

Step 3

Find 18% of excess over $22000.

$\frac{18}{100}$ x 2493 = $448.74

Step 4

420 + 18% of excess over $22000.

420 + 448.74 = **<u>$868.74</u>** Normal tax.

Example 5.4

Mr. David Rodney is a non- Fiji resident and works in Fiji as a lecturer at one of the local University. His chargeable income is $85,890. Calculate his normal tax

SOLUTION

Step 1

Since Mr. David Rodney is a non- resident we will use the table for non- resident tax rates and we choose the row in which his income falls.

Chargeable income	Tax payable
0-16000	20% of excess of $0
16,001-22,000	3200 + 20% of excess over $16000
22,001-50,000	4400 + 20% of excess over $22,000
50,001-270,000	10,000 + 20% of excess over $50,000
270,001-300,000	54,000 + 20% of excess over $270,000
300,001-350,000	60,000 + 20% of excess over $300,000
350,001-400,000	70,000 + 20% of excess over $350,000

Step 2

Find the excess over $50,000.

$85,890 - $50,000 = $35,890

Step 3

Find 20% of excess over $50,000.

$\frac{20}{100}$ x 35890 = $7178

Step 4

10000 + 18% of excess over $50000. 10000 + 7178 = **<u>$17178.00</u>** Normal tax

Exercise 5.2

Use the the tables for Resident and Non Resident tax rates (2013) above to answer the following questions

1. Mr. Pradeep Kumar's chargeable income is $51,435.00. Calculate his normal tax

2. Ms. Katie Brown is a New Zealand resident who is hired by one of the local hair salons in Fiji. Her chargeable income is $26,567.00 dollars. Find her normal tax

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+cartoon+hair+salons

3. Mr. Kesar's chargeable income is $53,569.00 and is a Fijian resident. Calculate his normal tax

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+cartoon+guy+busy+working

5.1.3 Percentage Increases and Decreases

LEARNING OUTCOMES

Students should be able to:

- Increase or decrease a given quantity by a percentage.
- Work out the percentage increase or decrease

Example 5.5

Increase $10 by 5%

Step one

$$\frac{5}{100} \text{ x } \frac{10}{1} = \frac{5}{10} = 0.5$$

Step two

$10 + $0.5 = **<u>$10.50</u>**

Example 5.6

Decrease 15kg by 4%

Step one

$$\frac{4}{100} \text{ x } \frac{15}{1} = \frac{60}{100} = 0.6$$

Step two

15 - 0.6 = **<u>14.4 kg</u>**

Example 5.7

$20 was increased to $25. Workout the percentage increase.

Step one

$25 - $20 = $5

Step two

$$\frac{5}{20} \; x \; \frac{100}{1} = 25\%$$

Original amount is always the denominator

Example 5.8

Katie earns $150 per week. She is to be given a 5% pay rise. How much will she earn per week after the pay rise

Step One

Workout 5% of $150

Step two

$150 + $7.5 = $157.50

Exercise 5.3

1 a) increase $1000 by 20%

b) increase $400 by 30%

c) increase 40kg by 3%

d) increase 250m by 7%

2 a) decrease $60 by 30%

b) decrease 8m by 5%

c) decrease 80kg by 10%

d) decrease 90m by 2%

3. (a) 96m is increased to 120m. find the percentage increase.

(b) 56kg is reduced to 50kg. find the percentage decrease.

4. A company increases the cost of all its products by 5%. Calculate the new price of each item listed

(a) a tent that previously cost $450

(b) a sleeping bag that previously cost $250

5. Joe was paid $30 per week for delivering papers. He was given a 3% pay rise. How much will he now earn each week?

6. Samy's grandmother wants to enlarge Samy's graduation photo. The photo now has dimensions 25cm x 20cm. Find the new dimensions if the photo is to be enlarged by 25%.

7. The price of all television in a shop are to be increased by 8%. Calculate the new price of a television that originally cost $899.

8. In a sale the cost of a computer is reduced by 30%. The normal price of the computer was $1499. Calculate the sale price of the computer.

9. A mountain bike was priced at $180. Its price was increased by 8%. Later, this increased price was reduced by 20% in a sale. Calculate the sale price of the bike.

10. This is how Caryl works out 15% of 120 in her head.

10% of 120 is 12,

5% of 120 is 6,

so 15% of 120 is 18.

how Caryl can work out $17\frac{1}{2}$ % of 240 in her head?

6.1.2

Applications of Percentages in Real life

LEARNING OUTCOME

Students should be able to:

- Discuss applications of percentages in real life situations

Percentages are an important part of our everyday lives.
Percentage is a very handy way of writing fractions.
Percentages can be compared more easily than fractions.

- Shops advertise discounts on products. These discounts are percentages.
  "Up to 50% off marked prices"
- Companies describe their success or failure as an increase or decrease in profit levels.
  "C-Company profit down by 15% for the last financial year"
- A salesperson may be given a commission as payment for selling goods. The commission can be a percentage of the sales made
- Financial institutions quote interest charged to the client on loans, or interest paid for money invested, as a percentage.
  "Housing Loans-4.95% p.a. for the first 12 months"
  Interest paid may be as Simple Interest or Compound Interest.

"I'm sure you recall the penalty for early withdrawal."

A bank is a business that borrows money in form of deposits and it lends money in the form of loans. They direct money from people who are saving to people who want to build houses, buy cars and start business. The bank pays interest to all money that is deposited for a fixed term and charges interest on all money that is borrowed. Interest can either be simple or compound.

5.1.4 Simple Interest

LEARNING OUTCOMES

Students should be able to calculate:

- Simple interest
- The total amount at the end of a fixed term.

Saving money is a lot better than borrowing money. When money is saved and deposited, interest is paid out.

Example 5.9

Mr. Hiralal deposits $10000 in a bank for 3 years. The bank pays him 5% interest per annum. Calculate the total amount that Mr. Hiralal has in his account after 3 years.

Solution

Calculate the interest in dollars.

$\frac{5}{100} x \$ 10000 = \$500$ Interest for one year.

Interest for 3 years.

$500 x 3 = $1500 (interest remains the same for the 3 years)

Total amount after 3 years = principal amount + interest for 3 years.

$10000 + $1500 = **$11500**

Example 5.10

Sunil wishes to buy a car worth $12000. He applies for a loan from a local bank which is going to charge him 6% interest per annum. If Sunil takes 3 years to finish his loan what is the total amount that he will have to pay?

Solution

Calculate the interest in dollars.

$\frac{6}{100} \times \$12000 = \$720$ Interest for one year.

Interest for 3 years.

$720 x 3 = $2160 (interest remains the same for the 3 years)

Total amount after 3 years = principal amount + interest for 3 years.

$12000 + $2160**= $14160**

Example 5.11

Villiame puts $2000 in a two year term deposit that earns simple interest. When the account matures he has $2200. What interest rate was charged.

Solution

(i) Find the interest

$2200 - $2000 = $200

(II) find interest per year

$200 ÷ 2 = $100

(ii) Find the rate

$\$2000 \times x = \$100$

$2000x = 100$

x = 100/2000

x = 0.05

(iv) convert to a percentage

0.05×100 = **5%**

Exercise 5.4

1. If $200 is deposited in a savings account that earns 7.5% simple interest, how much money will be earned after:

   (a) 1 year (b) 3 years (c) 15 years

2. Inoke deposits $1500 in a savings account that earns 5% simple interest. How much money would he have if he withdrew after:

   (a) 1 year (b) 6 months (c) 9 months

3. Vilimaina invests $2000 in a term deposit account for 3 years. After 3 years she has a total of $2388. What interest rate was paid on the account?

4. If $1000 is put on a term deposit that earns 8% simple interest, how many years will it take the balance to reach $2000?

5.1.5 Compound interest

LEARNING OUTCOMES

Students should be able to:

- Calculate the compound interest.
- Find the total amount.
- Differentiate between simple and compound interest

Compound interest is paid by most banks. This type of interest is not only paid on the principal , but also on the interest that has been already earned.

Population growth and depreciation also work according to the rules of compound interest.

Example 5.12

06/2010 Mr. Prasad deposits $10000 in a local bank which pays out 3.5% compound interest. Find the total amount after 3 years.

Date	Deposit and interest	balance
06/ 2010	$10000	$10000
06/ 2011	$\frac{3.5}{100} \times \$10000 = \$350$ $10000 + $350 = $10350	$10350
06/2012	$\frac{3.5}{100} \times \$10350 = \$362.25$ $10350 + $362.25 = $10712.25	$10712.25
06/2013	$\frac{3.5}{100} \times \$10712.25 = \$374.93$ $10712.25+ $374.93= $11087.18	$11087.18

Compound interest Vs Simple interest

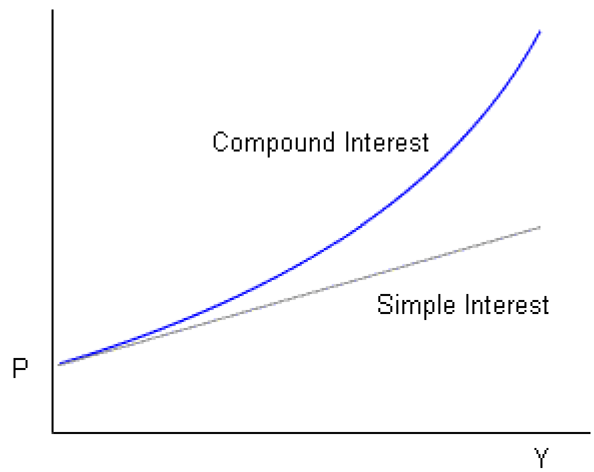
Suppose In the above example if Mr. Prasad was offered 3.5% simple interest on his money for 3 years than his total amount would be - $\frac{3.5}{100} \times 10000 = \$350$

Interest for 3 years would be $350 $\times 3 = \$1050$

Total amount that Mr. Prasad gets is $10000 + $1050.00 = $11050.00

It can be seen that compound interest earns more for Mr. Prasad than simple interest.

Simple interest is said to have a linear growth where as compound interest has an exponential growth.



Exercise 5.5

1. Tonasi deposited $2000 into a savings account at 5% compound interest.

Principal Plus interest (5%)	$2000
Principal (2nd year) Plus interest (5%)	
Principal (3rd year) Plus interest (5%)	
Balance after 3 years.	

   a. Copy and complete the table to find the balance after 3 years.
   b. How much interest is paid altogether?
   c. How much would have been paid if the account earned only simple interest.
   d. Which is better compound or simple interest? Explain why.
2. Find the value of $8500 invested for 5 years at 10% compound interest.

3. Alka invests $5500 at 7% compound interest per annum. After how many years does she have $6737.74 in her account.

4. Danny has $12500 to invest. An investment over a 2 year term will pay interest of 7% per annum.
   a. Calculate the compounded value of Danny's investment if the period is
      (i) One year (ii) 6 months (iii) 3 months
   b. Explain the advantage (s) in having the interest compounded on a more frequent basis.

5.1.6 Careers

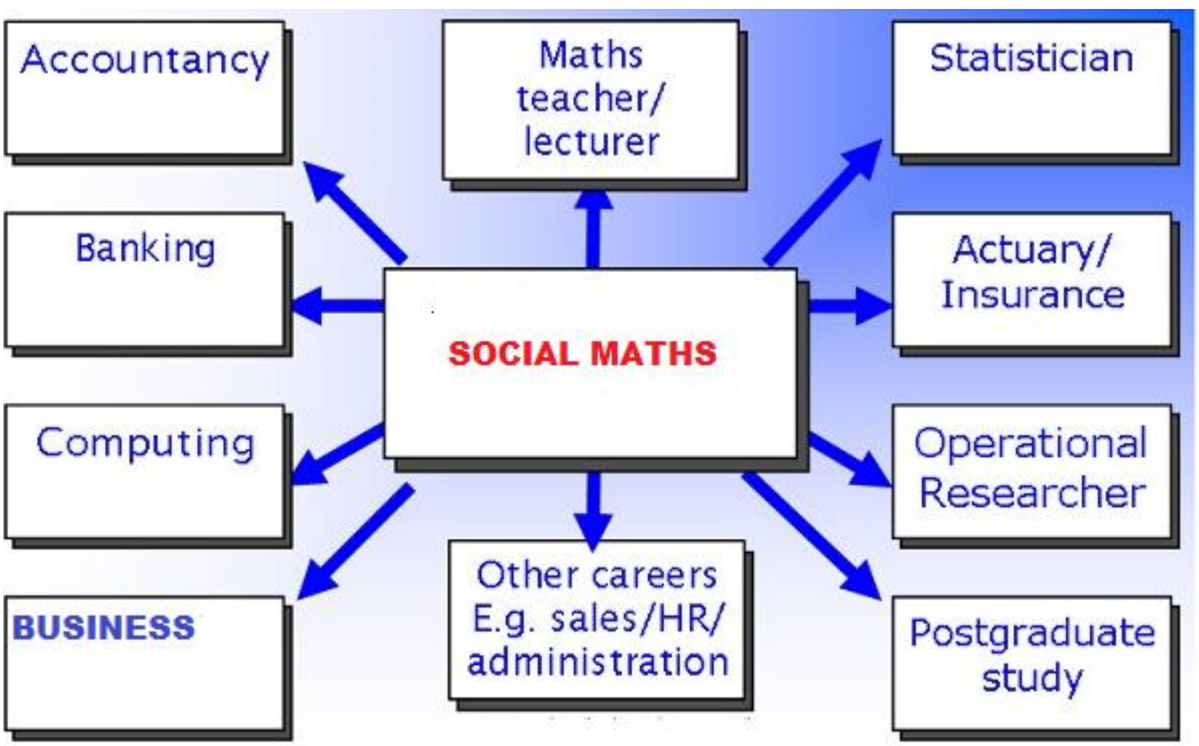
LEARNING OUTCOME

Students should be able to:

- identify some careers related to social mathematics.

Of course money has lots of relevance in all our lives. Many calculations with money involve mathematics like finding the discount, calculating the markups, finding the interest etc.

Social Maths can help in many ways and some career opportunities related to social maths are:

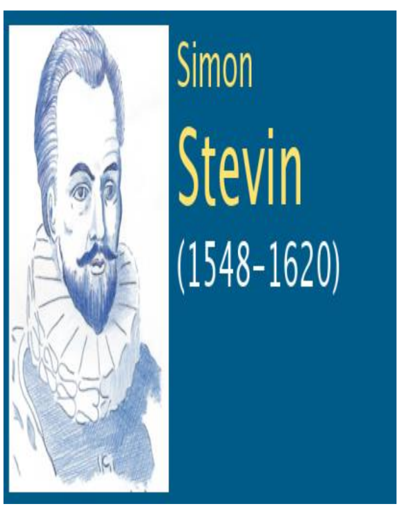


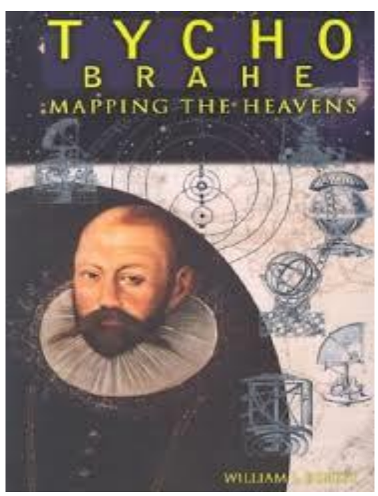
GLOSSARY		
1	Cash price	The actual amount of money that is exchanged when commodities are bought and sold
2	Chargeable income	It is your 'assessable income' after deducting 'personal reliefs (deductions which help you to save tax)
3	Compound interest	When the interest rate is applied to the original principal and any accumulated interest
4	Deposit	To put or leave something especially money in a particular place
5	Depreciation	Reduction in the value of a tangible asset over specified period of time
6	Discount	Reducing the price or value of an object or item
7	Hire purchase	A system by which a buyer pays for a thing in regular installments while enjoying the use of it.
8	Money	Coins or bills used as a way to pay for goods and services.
	Normal tax	A basic rate of taxation (as on income) applied to taxpayers.
9	PAYE	Tax payment method in which an employer is required by law to deduct income tax (and national insurance, if applicable) from an employee's taxable wages or salary
10	Percentage	A percent is a ratio whose second term is 100. Percent means parts per hundred
11	Principal value	A value selected at a point in the domain of a multiple-valued function, chosen so that the function has a single value at the point
12	Simple interest	The original amount invested, separate from earnings
13	Tax	A compulsory contribution to state revenue, levied by the government on workers' income and business profits or added to the cost of some goods, services, and transactions
14	Tax payable	A type of account in the current liabilities section of a company's balance sheet. This account is comprised of taxes that must be paid to the government within one year.
15	Tax return	A form on which a taxpayer makes an annual statement of income and personal circumstances, used by the tax authorities to assess liability for tax

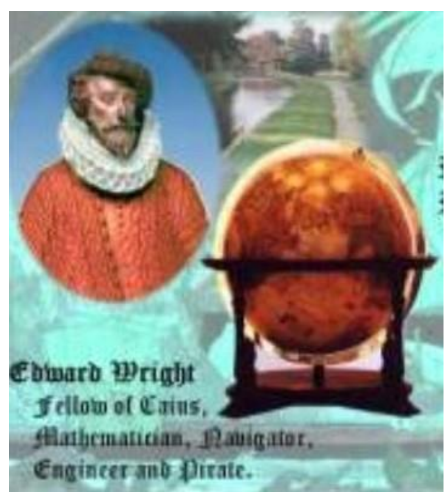
HISTORY OF MEAN, MEDIAN

*The arithmetic **mean**, although a concept known to the Greeks, was not generalised to more than two values until the 16th century. The invention of the decimal system by Simon Stevin in 1585 seems likely to have facilitated these calculations. This method was first adopted in astronomy by Tycho Brahe who was attempting to reduce the errors in his estimates of the locations of various celestial bodies.*

*The idea of the **median** originated in Edward Wright's book on navigation (Certaine Errors in Navigation) in 1599 in a section concerning the determination of location with a compass. Wright felt that this value was the most likely to be the correct value in a series of observations.*







Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+mean+and+median

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images

6.1 Data Representation

LEARNING OUTCOMES

Students should be able to:

- Organise numerical data into a frequency table
- Recognise different types of graphs and how they are sketched
- Identify suitable graphs for various data

We will study various graphical forms of data representation which are often used by statisticians. Some forms of representation are used more commonly than others.

We will now have a look at five different forms of graphs used by statisticians. These are:

- frequency tables
- bar graphs
- line graphs
- pie charts
- Pictograms

A worked example will be given for each one followed by an example for you to try on your own.

a) Frequency tables:

- Data that is collected is generally tabulated in frequency tables. These tables might contain data that is either grouped or ungrouped.
- The type of representation that is used depends on:
  - the nature of the data, i.e., discrete or continuous data
  - the format in which the data is given ungrouped or grouped

Definition

A Frequency Table is a table that lists items and uses tally marks to record and show the number of times they occur.

Example 6.1

- The following frequency table shows the number of Grade 2 students who chose cat, dog or hamster as their favorite pet.

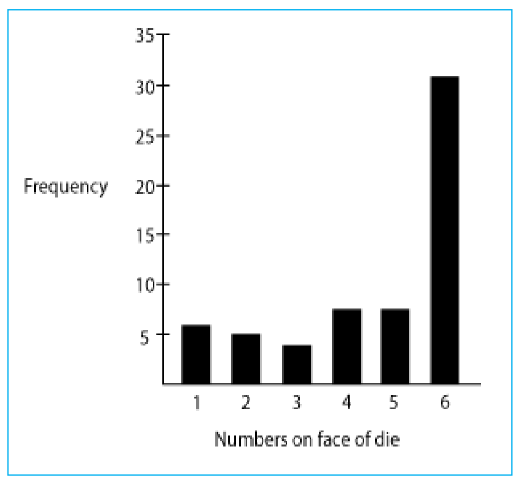
Favorite Pets		
Pet	**Tally Marks**	**Number**
	卌 卌	10
	||||	4
	卌 |	6

b) Bar graphs

A **bar graph** is a graph made of vertical columns or rectangles with equal widths. The height/length represents the frequency of the category.
Here is an example of a bar graph to represent the data from the following table.

Example 6.2

Occurrence of the numbers thrown with a die	
Number	**Frequency**
1	6
2	5
3	4
4	7
5	7
6	32



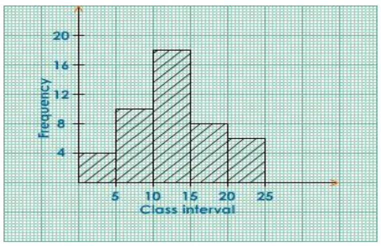
When the bars are joined to each other, the graph is called a **histogram**.

Example 6.3

Draw a histogram for the following data:

Class Interval	Frequency
0 - 5	4
5 - 10	10
10 - 15	18
15 - 20	8
20 - 25	6

Solution:



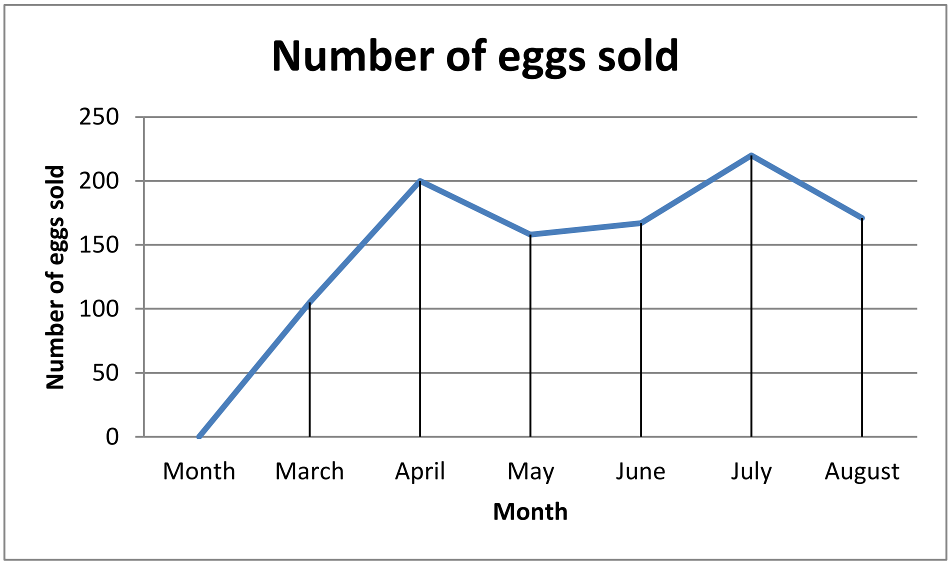
Source: http://math.tutorvista.com/statistics/histogram.html

c) **Line graphs**

A **line graph** is a graph made up of straight line segments joined together between points which are marked according to frequencies.

Month	Number of Eggs sold
March	105
April	200
May	158
June	167
July	220
August	171

Eggs sold at Corner Shop March to August



d)Pie charts

A **pie chart** is a circular representation, a bit like a pie which has been cut up into various sizes of slices. The different segments (the slices) represent the various **relative frequencies**.

The **relative frequency** of a class is the fraction of the total number of data of items belonging to the class. For example, a data set with a total number of *n* items, the relative frequency of each class is given by the following formula.

$$\textbf{Relative Frequency} = \frac{\text{number of successful trials}}{\text{total number of trials}}$$

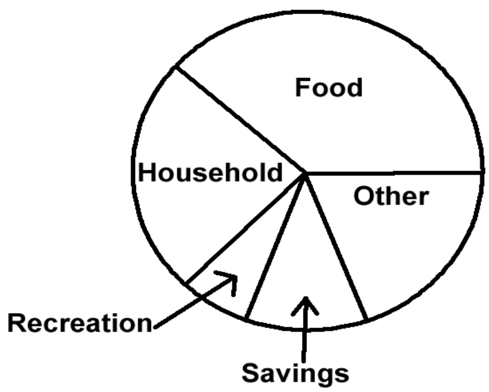
The segments are usually labeled in percentages, and the total of all the percentages should be 100%.

Example 6.4

The pie chart represents amounts spent by a family on various items in the month of September in a certain year. Take note of how the angle sizes of the segments are calculated.

Item	Amount Spent ($)	Amount Spent (%)	Angle Size of Segment
Food	2000	40	$\frac{40}{100} \times 360 = 144$
Household	1250	25	$\frac{25}{100} \times 360 = 90$
Recreation	250	5	$\frac{5}{100} \times 360 = 18$
Savings	500	10	$\frac{10}{100} \times 360 = 36$
Other	1000	20	$\frac{20}{100} \times 360 = 72$

Amounts spent from family budget on various items



e) Pictogram

A **pictogram** looks a like a bar graph. The difference is that the bars are made up of little pictures which represent certain numbers of things as is indicated in the key, which must accompany the pictogram. The picture usually relates in some way to the data being represented.

Below is a frequency table for the number of cars passing through various police check points on 22nd September in a certain year.

Check points	Number of Cars Passed through
A	250
B	520
C	135
D	330
E	405

Below is a pictogram representing the number of cars passing through the Check points A – E:

Checkpoint	No of Cars						
A							
B							
C							
D							
E							

KEY	
	Represents 100 cars
	Represents 10 cars
	Represents 5 cars

Exercise 6.1

1. i. Using the data in the table below, draw a bar graph to illustrate the information. Use graph or grid paper.

Favorite cakes in Class 4B	
Type	**Frequency**
chocolate	15
vanilla	11
carrot	6
coconut	7
apple pie	6
coffee	3

ii. What is an advantage of using a bar graph here than a pie chart?

2. Consider the following information relating to the number of buses going and arriving at various bus stations in Fiji.

City/Town	Number of buses per day
Suva	120
Lautoka	67
Nadi	45
Nausori	90
Ba	55

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+cartoon+buses

Which would be a better way to represent this in a line graph or a pictogram? Why? (**Show your answer**)

6.2 Measures Of Central Tendency

LEARNING OUTCOMES

Students should be able to:

- Define measures of central tendency
- Calculate measures of central tendency
- Work out measures of central tendency from frequency table
- Identify median from a histogram
- Relate measures of central tendency to real life situations

One of the most simple ways of describing and analyzing data is to use measures of central tendency.

To a statistician, the arithmetic mean is only one of several measures of central tendency to get an idea about the center of a distribution. The idea of the center of a distribution and what it reflects depends on a number of some factors. That is why statisticians believe that there should be other measures of central tendency.

i. Mean or average

A single number, obtained through a mathematical calculation which is very often used to describe a set of data.

To calculate the mean, **we add all of the data items together and then divide that sum by the number of terms which we added.**

Example 6.5

Find the mean of 56, 34, 25, 38, 49, 80 and 73.

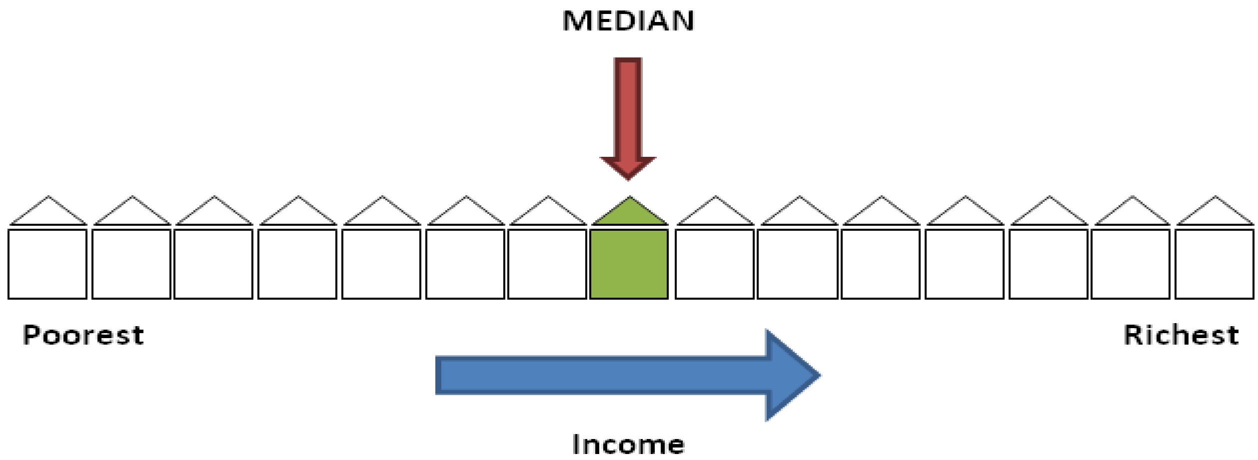
To do this you should add 56, 34, 25, 38, 49, 80 and 73 and then divide the sum of these numbers by 7.

So $56 + 34 + 25 + 38 + 49 + 80 + 73 = \frac{355}{7} = 50.71$

The mean is 50.71.

ii. Median

The median of a set of data is the value which divides the set into two equal numbered parts when the data has been ranked according to size. To rank data, we put it into ascending (or descending) numerical order. If the number of scores is odd, then the central score will be the median. If the number of scores is even, then the median will be the average of the two central scores.



Source:
https://www.google.com/search?q=images+of+median&biw=1760&bih=843&tbm=isch&tbo=u&source=univ&sa=X&ei=GIfFVNGRNIz-8QXNhoKwCw&ved=0CDMQ7Ak

Example 6.6

Calculate the median of the following data:

2, 4, 9, 10, 13, 15, 19

The data has been ranked (it is in ascending numeric order), there are seven data items (scores) and so 10, which is the central most score (the one in the middle), is the median.

Example 6.7

Calculate the median of the following data:

12, 16, 80, 0, 0, 0, 0, 0, 0, 0, 0, 0

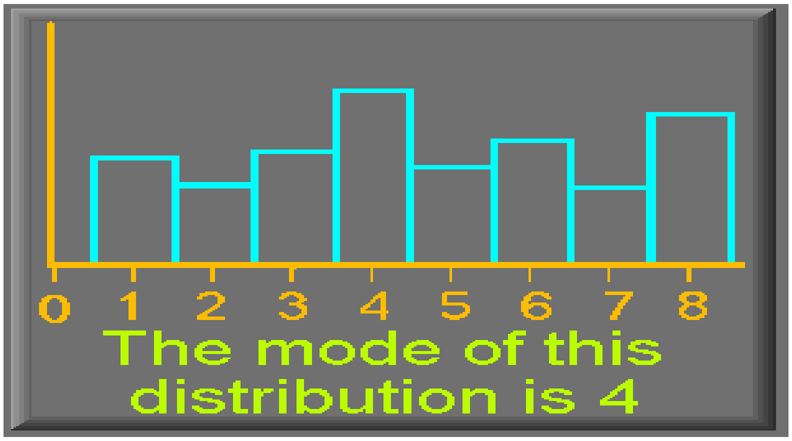
Rank the data: 0, 0, 0, 0, 0, 0, 0, 0, 0, 12, 16, 80.

There is an even number of data items. The two central data items are both zero. The average of these two numbers is zero and so the median is zero.

iii. Mode

The mode is the score that appears the most often. It is the ***most common score***.

Why do you think educators might find the mode a useful measure of central tendency?



Source: https://www.google.com/search?q=images+of+mode+of+scores&biw=1760&bih=843&tbm=isch&tbo=u&source=univ&sa=X&ei=t4jFVPzNM6LdmAXQ9YGwBw&ved=0CDMQ7Ak

Example 6.8

What is the mode of 1, 2, 2, 3, 3, 3, 3, 4 and 15?

The 3 appears more often than any other score and so it is the mode.

What is the mode of 2, 4, 9, 9, 10, 10, 13, 13, 13, 15, 19 and 19?

The 13 appears more often than any other score and so it is the mode.

Data may have two modes, in which case it is called ***bi-modal***.

Exercise 6.2

1. A marathon race was completed by 5 participants in the times given below.

   3.2 hr, 7.1 hr, 5.4 hr, 5.8 hr, 4.7 hr

   What is the mean race time for this marathon?

2. i. Find the mean of the following numbers: 32, 24, 14, 18, 11, 10, 31. Show your working.

   ii. Is the order of numbers important while calculating the mean? Why?

3. If the rainfall in Koro Island in a certain year was 12mm in January, 16 mm in February, 80 mm in March and in the remaining months of the year no rainfall was recorded, calculate the mean rainfall for the Koro Island for that year. Show your working.

4. Calculate the median of: (Show all working)

a) 32, 24, 14, 18, 11, 10 and 31.
b) 50, 65, 35, 60, 40, 90, 50, 48, 63, 27, 68 and 53.
c) Compare your working in a) and b) above. What is different about the two workings? Why is this so?

5. What is the mode of:

1,1,2,2,3,3,3,3,4,4,4,5 and 6?

6. Below is a table showing the Maths marks of Year 9 students in a test

Marks	Frequency
0	2
2	2
3	8
4	**A**
5	2

a. If the mean test mark is 3, find the value of **A.**

b. What is the:

i. Median mark?

ii. Modal mark?

6.2.1 Class Intervals And Histograms

When data does not fall into evenly spaced categories, we can make our own categories. These categories are called class intervals. Class intervals should be equal in width and non-overlapping. A histogram is a frequency graph of class intervals.

To make a frequency distribution table, first divide the numbers over which the data ranges into intervals of equal length. Then count how many data points fall into each interval.

A *histogram* is "a representation of a frequency distribution by means of rectangles whose widths represent class intervals and whose areas are proportional to the corresponding frequencies." Online Webster's Dictionary

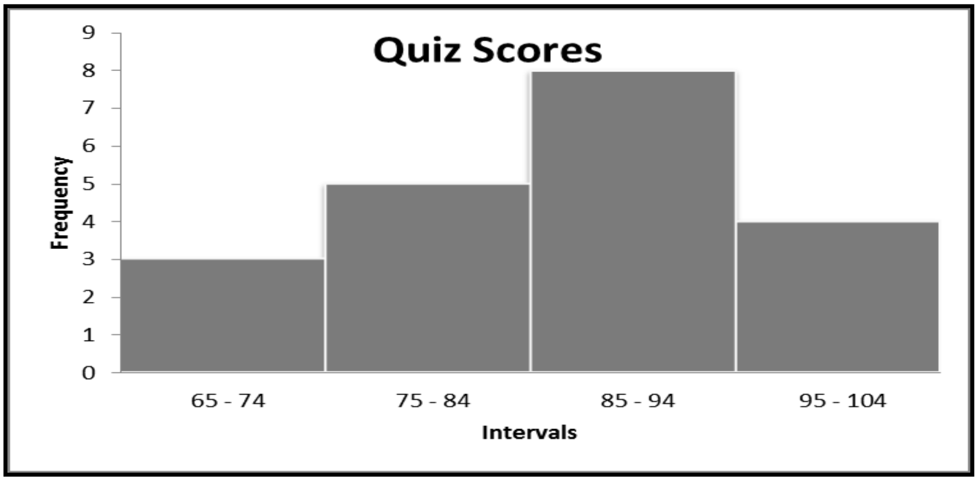
Finding The Mean Range, Mode And Median From Class Intervals.

Example 6.9

Create a frequency table and histogram of the data below on quiz grades.
67, 99, 88, 87, 95, 92, 100, 89, 73, 84, 85, 84, 93, 91, 72, 75, 98, 93, 83, 79

Solution

Intervals	Frequency
65 - 74	3
75 - 84	5
85 - 94	8
95 - 104	4
Total	20



Calculate the range and mean, median and mode

Answers

i. **Range**= Highest score – Lowest score

100 – 65 = 35

ii. **Mean**= $\frac{\text{Sum of Centre of Interval x Frequency}}{\text{Sum of frequency}}$

Intervals	Centre of Interval	Frequency	Total [Centre x Freq]
65 - 74	69.5	3	208.5
75 - 84	79.5	5	397.5
85 - 94	89.5	8	716
95 - 104	99.5	4	398
Total		**20**	**1720**

Mean= $\frac{1720}{20} = 86$

iii. **Median**= $\frac{\text{Sum of Cumulative Frequency + 1}}{2} = \frac{21}{2} = 10.5$

Intervals	Frequency	Cumulative Frequency
65 - 74	3	3
75 - 84	5	8
85 - 94	8	16 }10th and 11th scores
95 - 104	4	20

The median is between the 10th and the 11th scores which is in the 85-94 score range so the median interval is 85-94.

iv. **Mode**=The most common score would be shown by the highest bar so the modal interval is 85-94

Exercise 6.3

1. Below are the marks obtained by learners in Year 10, Term 4.

51 64 56 53 72 45 42 46 57 41

51 63 50 45 64 53 48 47 34 48

39 52 36 55 58 46 50 39 48 44

57 47 54 41 36 54 46 44 57 52

49 77 70 49 41 60 54 69 53 65

i. Draw a frequency table for the data.

ii. Show the data on a histogram.

iii. Find the:

- Mean
- Range
- Median
- Mode

2. Apenisa grows two different types of cucumber plants (Type A and Type B) in his greenhouse. One week he keeps a record of the number of cucumbers he picks from each type of plant.

Day	**Mon**	**Tue**	**Wed**	**Thu**	**Fri**	**Sat**	**Sun**
Type A	5	5	4	1	0	1	5
Type B	3	4	3	3	7	9	6

i. Calculate the mean, median and mode for the Type A cucumber plants

ii. Calculate the mean, median and mode of the Type B cucumber plants

iii. Which measure would you argue that there is no difference between type A and Type B plants?

iv. Which measure would you argue that Type A is the best plant?

v. Which measure would you argue that Type B is the best plant?

6.3 Measures of Dispersion

LEARNING OUTCOMES

Students should be able to:

- Identify and describe different measures of dispersion
- Calculate the measures of dispersion from an ungrouped data
- Calculate the measures of dispersion from frequency tables
- Interpret the meaning of numerical values representing measures of dispersion
- Relate the measures of dispersion to real life situations

For a more informative numerical summary, we need some measure of dispersion, sometimes called the *spread*, of observations. This gives us the information of how spread out the values of a data set is.

i. The Range

The ***range*** is defined as the difference between the largest score in the set of data and the smallest score in the set of data.

Formula: Range = Highest Score – Lowest Score

Source:
https://www.google.com/search?q=images+of+range+of+scores&biw=1760&bih=843&tbm=isch&tbo=u&source=univ&sa=X&ei=eIvFVNmnJ4GsmAWvwIHQDQ&ved=0CDEQ7Ak

Example 6.10

What is the range of the following data:
4 8 1 6 6 2 9 3 6 9

The largest score is 9; the smallest score is 1; the range is 9 - 1 = 8

ii. Quartiles and the interquartile range

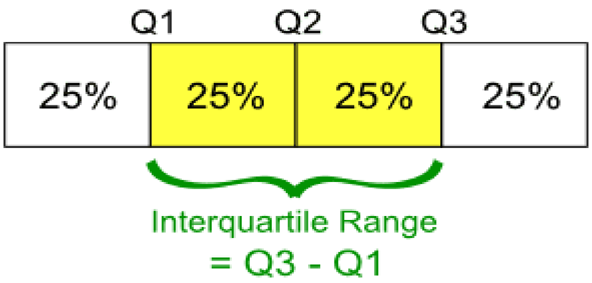
The **median** divides the data into two equal sets, the first (usually to the left of the median) and the second.

- The lower quartile is the value of the middle of the first set, where 25% of the values are smaller than Q_1 and 75% are larger. This first quartile takes the notation Q_1.
- The upper quartile is the value of the middle of the second set, where 75% of the values are smaller than Q_3 and 25% are larger. This third quartile takes the notation Q_3.
- It should be noted that the median takes the notation Q_2, the second quartile

The **interquartile range** is the difference between the lower and upper quartiles. The quartiles split the data into quarters. In other words:

- **lower quartile(Q_1)** = splits lowest 25% of data
- **second quartile** (Q_2) = cuts data set in half = also known as the median
- **third quartile** (Q_3) = **upper quartile** = splits highest 25% of data, or lowest 75%

The difference between the upper and lower quartiles is called the ***interquartile range***

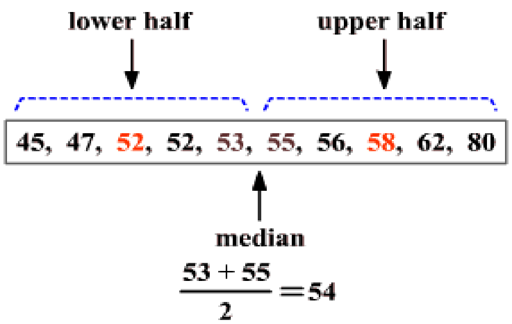


Example 6.11

For the set given below, identify the median, the middle value of the 1st set and the middle value of the 2nd set

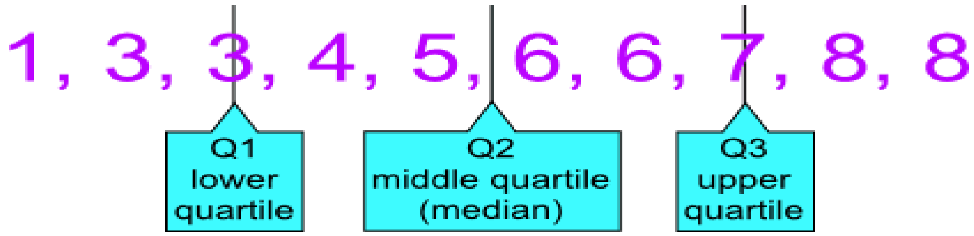
45, 47, 52, 52, 53, 55, 56, 58, 62, 80

Answer



Example 6.12

Identify the median, the upper and lower quartiles



Example 6.13

Find the upper and lower quartile of the following set of data

1, 11, 15, 19, 20, 24, 28, 34, 37, 47, 50, 57

ANSWER

First calculate the median of the data.

Since there is an even number of data values, the median is the mean of the tenth and eleventh values. In other words, the median is:

$$\text{Median} = \frac{24 + 28}{2}$$

$$= \mathbf{26}$$

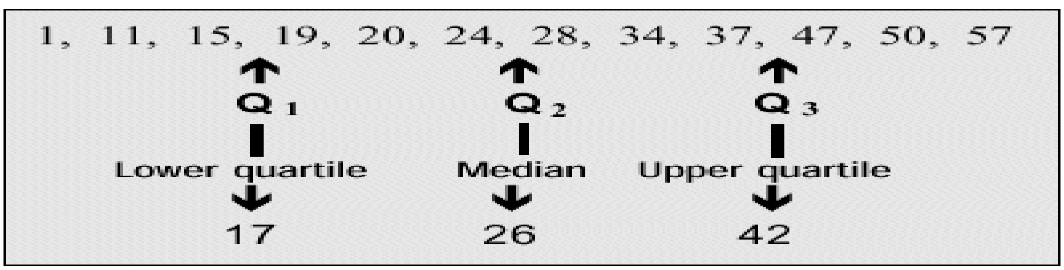
The Lower and Upper Quartiles are as follows

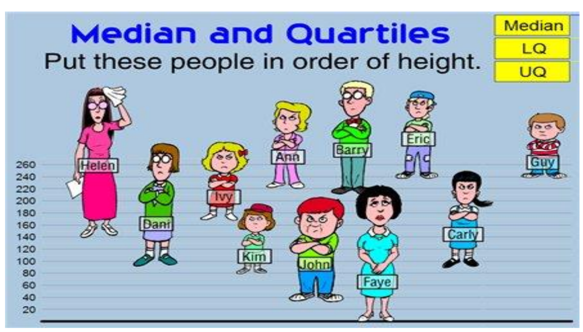
$$\text{Lower quartile} = \frac{15 + 19}{2}$$

$$= \mathbf{17}$$

$$\text{Upper Quartile} = \frac{37 + 47}{2}$$

$$= \mathbf{42}$$





Example 6.14

7, 1, 3, 6, 3, 7 may be ordered as 1, 3, 3, 6, 7, 7

i. The sample median is : 1, 3, **3, 6**, 7, 7

= 3 + 6

= 9 ÷ 2 = **4 .5**

ii. The upper quartile and lower quartile is:

1**,** **3**, 3, 6, **7**, 7

$\mathbf{Q_1}$ = 3 and $\mathbf{Q_3}$ = 7

iii. The interquartile range is:

1**,** **3**, 3, 6, **7**, 7

= 7 – 3 = **4**

The standard deviation: A measure that does not require sorting of the data and has good statistical properties is the ***standard deviation*.**

The **Standard Deviation** is a measure of how spread out numbers is.

Its symbol is **σ** (the Greek letter sigma)

Exercise 6.4

1. The list below shows the prices of soft drinks at Sunshine Supermarket:

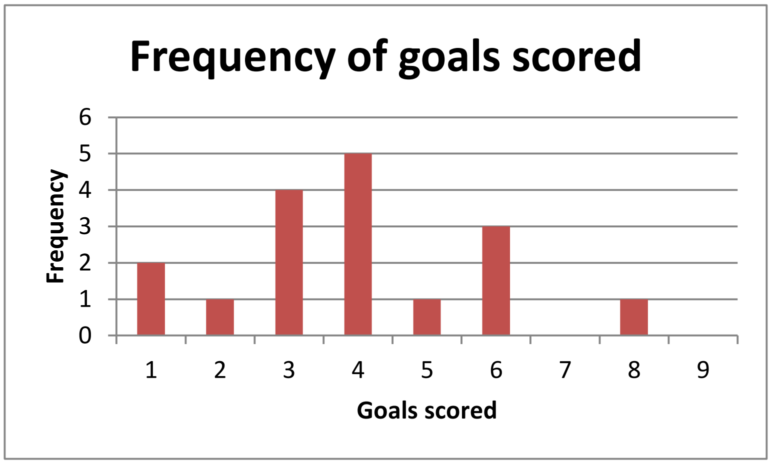
   $1.50, $2.00, $3.20,$3.20, $4.00, $4.80, $2.40

   a. Find the median price of a soft drink
   b. Find the range

c. Find the upper and lower quartiles
d. Find the interquartile range.

2. The diagram below shows the frequency of goals scored by a soccer team in a 2014 tournament.

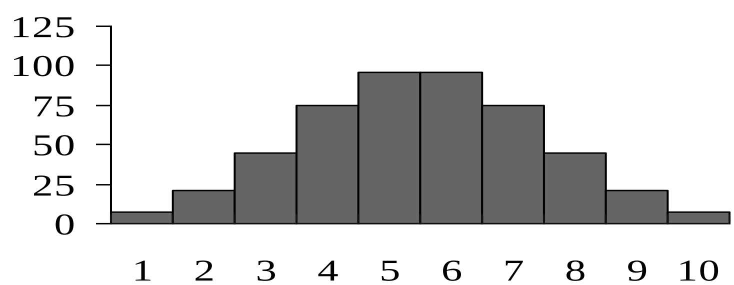


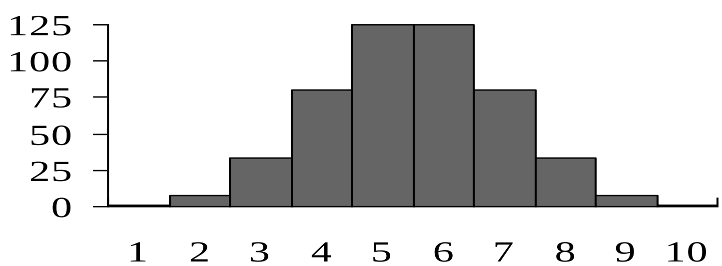
a. How many games did the team play?
b. What was the modal number of goals scored?
c. What was the median?
d. What was the mean?
e. What was the range and the interquartile range?

3. Which of the distributions of scores has the larger dispersion? In Graph I or Graph II? Explain your choice.

Graph I

Graph II





In today's information over-loaded age, statistics is a very useful subject to learn. Newspapers are filled with statistical data and anyone who is ignorant is at a risk of being misled about important real- life decisions as to what to eat, who is leading the polls, the dangers of smoking etc. Knowing a little about statistics will help you to make informed decisions. When used correctly, statistics can also tell us any trends in what happened in the past and can be useful in predicting what may happen in the future.

Exercise 6.5

Find an article from a newspaper or magazine which includes a graphical form of data representation. Read the whole article and study the graph carefully before answering the following questions.

1. What was the investigation about?
2. How is the data represented?
3. Is the presentation used to compare sets of data? If it is, then what is the comparison about?
4. Is the data represented clearly? Give reasons for your answer. Explain any abuses of statistics which you feel are present, if there are any.
5. Discuss the usefulness of the data.
6. What technology (such as computers) do you think was used to collect the data?
7. Will it be a useful exercise for the learners? Why?

Introduction

Probability is simply how likely something is to happen. Whenever we're unsure about the outcome of an event, we can talk about the probabilities of certain outcomes (how likely they are). The analysis of events governed by probability is called statistics.

Probability can be expressed as a ratio, a percentage or as a fraction

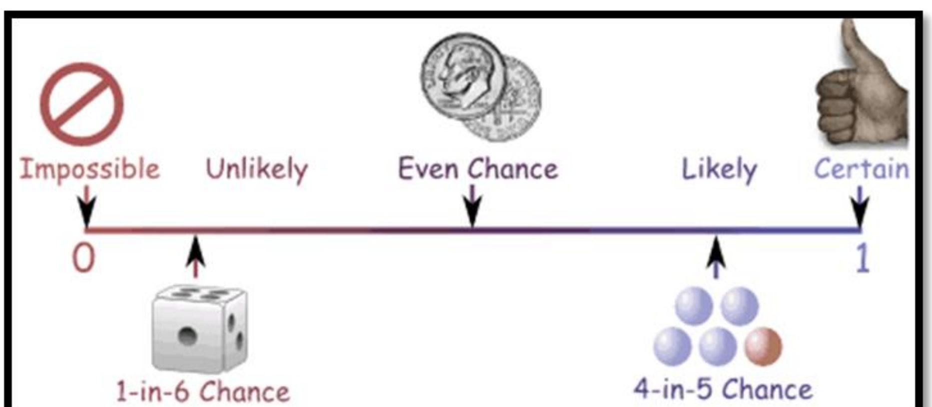
6.4 Probability Experiments

LEARNING OUTCOMES

Students should be able to:

- Define simple probability terms
- Perform simple probability experiments and determine relative frequency outcomes

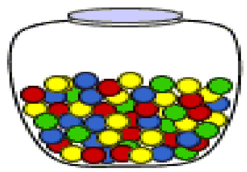
Probability we had said earlier is the likelihood or chance that something will happen. The highest probability an event can have is 1 or 100% and it means that the event will certainly happen. The lowest probability is 0% meaning that the event is certainly not going to happen.



Probability Key Terms

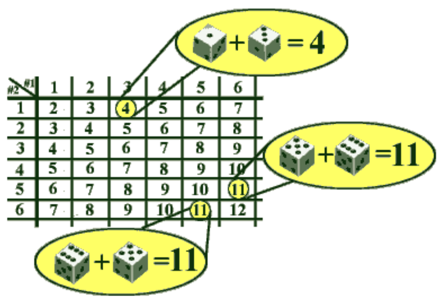
- **Experiment**

  An experiment or a trial in probability is a test to see what will happen in case you do something. A simple example is flipping a coin. When you flip a coin, you are performing an experiment to see what side of the coin you'll end up with.



- **Outcome**

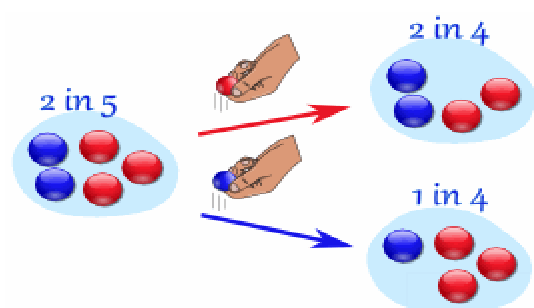
  An outcome in probability refers to a single (one) result of an experiment. In the example of an experiment above, one outcome would be heads and the other would be tails.



Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+an+outcome+in+a+probability

- **Event**

  An event in probability is the set of a group of different outcomes of an experiment. Suppose you flip a coin multiple times, an example of an event would the getting a certain number of heads.



Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+an+event+in+a+probability

- **Sample Space**

A sample space in probability is the total number of all the different possible outcomes of a given experiment. If you flipped a coin once, the sample space **S** would be given by:

$$S = (H, T)$$

If you flipped the coin multiple times, all the different combinations of heads and tails would make up the sample space. A sample space is also defined as a Universal Set for the outcomes of a given experiment.

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+a+sample+space

Exercise 6.6

1. Which of these can you be certain of?Why?
   i. Your height
   ii. Cost of your fare to school
   iii. Exact time of arrival of your school bus
2. Write the following probabilities as percentages:
   i. 0.35
   ii. 2/3
   iii. 1 out of 5
3. List the sample space for:
   i. Throwing a single die
   ii. Throwing two dice
   iii Tossing a coin

Basic Concepts Of Probability

i. ***We can use probability to make decisions.***

For example:

The weather report might say that the probability of rain is 40%. This means that there is a 40% chance that it will rain and a 60% chance that it won't (40% + 60% = 100%). This probability will help us to make decisions regarding activities like picnics or games to organize for the day.

ii. ***We can use probability to make predictions.***

For example:

The probability of tossing a single coin and getting a tail is 0.5. If we toss the coin 30 times how many times will we get tails?

Probability of tails x Number of tosses = Number of tails

0.5 x 30 = 15 tails

iii. ***We can estimate probability from observations.***

Example 6.15

In April 2014, the Levuka Islanders counted 17 sunny days out of a total of 30. They assumed then that the probability of a sunny day in April 2015 will be the same as the percentage of sunny days last April.

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+sunny+day

$$\frac{Sunny\ day}{Total\ days} = \frac{17}{30} = 0\ 57 = 56.7\%$$

Do you think the islanders are correct in saying that the weather in the future will be like in the past?

They are correct in assuming the weather will be like the previous years but they are not correct in counting the number of sunny days from just one year. It would be better to take the number of sunny days over several years.

$$\frac{\text{Sunny days in April over 10 years}}{\text{Total days in April over 10 years}} = \frac{120}{300} = 0.4 = 40\%$$

The more data we use for our estimation the better it will be.

iv. ***We can find the probability of events by calculating the relative frequency.***

Relative Frequency is the number of successful trials divided by the total number of trials.

Relative frequency = no. of successful trials / Total count

The above equation expresses relative frequency as a *proportion.* It is also often expressed as a percentage. Thus, a relative frequency of 0.40 is equivalent to a percentage of 40%.

Example 6.16

Tossing a coin

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+tossing+a+coin+10+times

Raju tosses a coin 20 times and gets a total of 14 heads. The relative frequency of the head is:

Relative frequency = no. of successful trials / Total count

= 14/ 20 = 0.7

Example 6.17

Rolling a Die

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+rolling+dice

a. Sera did an experiment in which she threw a single die 50 times. She found that the relative frequency of a "five" was 0.2. How many "fives" did she get:

Relative frequency = no. of successful trials / Total count

0.2 = x / 50

x = 10 "fives"

b. If Sera throw the same die 150 times, how many "fives" should she expect?

Relative frequency = no. of successful trials / Total count

0.2 = x / 150

x = 30 "fives"

Example 6.18

Drawing a card

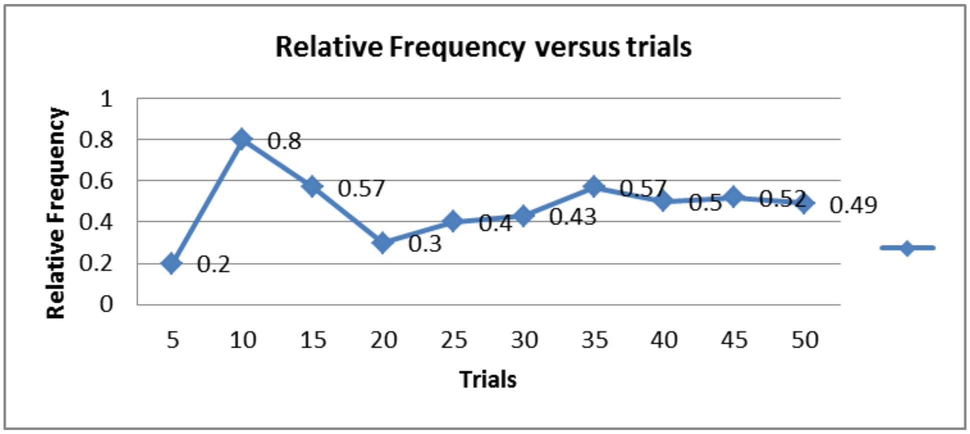
Mani did an experiment to find the probability of drawing a black card from a standard deck of 52 cards.

The table below shows the data of the relative frequencies in various trials:

Trials	Relative Frequencies
5	0.2
10	0.8
15	0.57
20	0.3
25	0.4
30	0.43
35	0.57
40	0.50
45	0.52
50	0.49

a. What is the limit of relative frequency?

We plot the data given above on a graph.



The graph above shows that the relative frequency starts very low then goes very high and then comes closer and closer to 0.5. If we keep increasing the number of trials, it will keep getting closer to 0.5. So, the limit of relative frequency is 0.5.

b. What is the probability of drawing a black card?
The probability of an event is equal to the limit of its relative frequency.
So, the probability of drawing a black card is 0.5.

Exercise 6.7

1.a) If you toss a fair coin, the probability of getting "heads" is 0.5

i. Write this probability as a percentage.

ii. If John tosses the coin 28 times how many times would you expect to get heads?

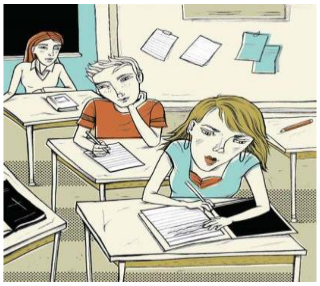
iii. Will you always get tails this many times when anyone does this experiment? Explain.

2. When a child is born the probability that it will be a girl is equal to that it will be a boy: both probabilities are 0.5

i. If 300 babies were born in CWMH how many would you expect to be girls?

ii. How many would you expect to be boys?

3. The probability of getting a surprise Accounting test in Year 10 is 0.25.



Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+cartoon+students+sitting+tests

If there are 180 school days in the year, how many surprise tests can students expect?

Is it certain that students will have this many surprise tests? Explain.

4. The table below shows the percentage of Rosi High School Year 10 students who passed their mathematics test over the last 5 years.

Year	1	2	3	4	5
Pass rate	90%	99%	100%	87%	91%

i. Find the average pass rate of students.

ii. What is the probability of a randomly selected student passing this year?

iii. If there are 260 students in Year 10, how many are likely to pass?

5. Some students in Year 10 investigated the probability of a box landing on its side as shown below.

i. Use the data below to calculate the relative frequency for each set of toss.

Trials	Side	Trials	Side
5	3	30	8
10	2	35	18
15	0	40	6
20	9	45	10
25	14	50	19

ii. On graph paper draw the graph of the relative frequency.

iii. What seems to be the limit of relative frequency?

iv. What is the probability of the matchbox landing on its side?

v. Is this reasonable? Explain in terms of the shape of the matchbox.

6.5 Events Of Probability

LEARNING OUTCOMES

Students should be able to:

- Identify and define the different types of events
- Describe the probability

Life is full of random events! We need to get a "feel" for them to be smart and successful people.

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+life

The throw of a dice, toss of a coin and lottery draws are all examples of random events.

When we say "Event" we mean one (or more) outcomes.

An event therefore is defined as any outcome that can occur.

Types Of Probability Events

An event can include several outcomes:

- Choosing a "King" from a deck of cards (any of the 4 Kings) is an event
- Rolling an "even number" (2, 4 or 6) is **also** an event
- Getting a Tail when tossing a coin is an event
- Rolling a "5" is an event.

Let's look at each of those types of events.

- **Random event**

In a random event all the outcomes have the same probability.

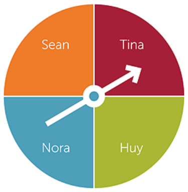
Example 6.19

Tossing a coin is a random event because "heads" and "tails" both have the same chance of occurring.

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=IMAGES+OF+FIJI+COINS

- **Equally likely outcome**

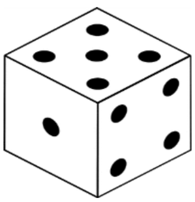
Equally likely outcomes are outcomes that have the same probability of occurring.



Source: http://amsi.org.au/teacher_modules/Chance_years_1-3.html

Example 6.20

A single die is rolled*:*



What is the probability of each outcome?

What is the probability of rolling an even number? of rolling an odd number?

Outcomes: The possible Outcomes of this experiment are 1, 2, 3, 4, 5 and 6

Probabilities:

$$P(1) = \frac{\text{\# of ways to roll a 1}}{\text{total \# of sides}} = \frac{1}{6}$$

$$P(2) = \frac{\text{\# of ways to roll a 2}}{\text{total \# of sides}} = \frac{1}{6}$$

$$P(3) = \frac{\text{\# of ways to roll a 3}}{\text{total \# of sides}} = \frac{1}{6}$$

$$P(4) = \frac{\text{\# of ways to roll a 4}}{\text{total \# of sides}} = \frac{1}{6}$$

$$P(5) = \frac{\text{\# of ways to roll a 5}}{\text{total \# of sides}} = \frac{1}{6}$$

$$P(6) = \frac{\text{\# of ways to roll a 6}}{\text{total \# of sides}} = \frac{1}{6}$$

In this experiment the probability of rolling each number on the die is always one sixth. We say that the outcomes are equally likely to occur.
Knowing that the above are equally likely outcomes we can calculate their probabilities without doing the experiment i.e the probability of anyone of them is equal to 1 divided by the number of outcomes:

$$1 \div 6 = \frac{1}{6}$$

- **Fair event**

Tossing a fair coin is a random event. This means that all the outcomes are equally likely.

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+fair+events

- **Biased event**

This means that the probabilities of the outcomes are not equal.

.

"With careful and prolonged planning, we may reduce or eliminate many potential sources of bias, but seldom will we be able to eliminate all of them. Accept bias as inevitable and then endeavor to recognize and report all exceptions that do slip thought the cracks."

Good and Hardin (2006) *Common Errors in Statistics (and How to Avoid Them)*, p. 113

Biased samples can occur two ways:

1. If the sample selected is not representative of the population to be surveyed.

Example:

The city council is going to vote on whether to keep its leash laws.

Sample 1 ~ survey dog owners

Sample 2 ~ survey people who don't have pets

Sample 3 ~ surveying some people who own dogs and some who don't.

Which sample shows the least bias?

Answer

Sample 3 ~ equal chances of yes or no answers

2. If the survey questions are worded in ways meant to lead the people surveyed to

Example:

Wording 1 ~ Should the mayor approve a bill requiring dogs that are outdoors to be on leashes at all times, instead of running over people's property?

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+dogs

Wording 2 ~ Should the mayor approve a bill requiring dogs that are outdoors to be on leashes at all times, instead of running freely to exercise?

Wording 3 ~ Should the mayor approve a bill requiring dogs that are outdoors to be on leashes at all times?

Which wording shows the least bias?

Answer

Wording 3 ~ the other two wordings make it sound like either a bad idea or good idea to let the dogs run freely.

Exercise 6.8

1. Identify more likely, less likely, equally likely, sure and impossible events:

   i. Selection of a white ball from a box with 5 white balls, 8 red balls and 10 yellow balls.
   ii. Selection of a black card from a deck of cards
   iii. Occurrence of even number when a die is rolled.
   iv. Selection of red marble from a box with 12 red marbles.
   v. Selection of red marble from a box with 12 white balls.
   vi. Selecting a boy for a field trip from a group of 35 students with 12 girls.

2. A fair dice is tossed:

   i. Write down the set of possible outcomes.

   ii. Are the outcomes equally likely?

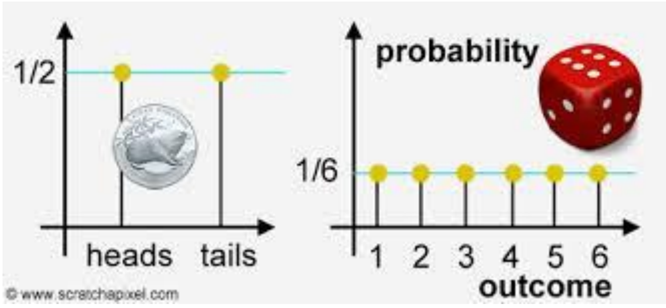
   iii. What must the sum of the probabilities of all the possible outcomes

   iv. Find the probability of each of the outcomes listed in iii.

3. State whether or not a sample is biased. If so, identify the source of the bias in the following scenarios.

(a). A movie theater surveyed patrons 45 years of age and older and asked what type of movie they prefer.
(b). The student body wants to survey the 8th grade class to choose a theme for the winter dance. They surveyed Mr. Walker's 3rd period art class.
(c). The patrons of a pet store were asked what their favorite type of pet is.
(d). Students are asked, "Are you willing to waste weekend time volunteering at an animal shelter?"
(e). Parents are asked, "Should we develop hand-eye coordination in our children by teaching them how to play computer games?"

Determining Probability From An Experiment



LEARNING OUTCOME

Students should be able to:

- Determine the probability from an experiment consisting of equally likely outcomes for example; drawing a colored ball from a box.

Experimental probability refers to the probability of an event occurring when an experiment was conducted.

For example, if a dice is rolled 6000 times and the number '5' occurs 990 times, then the experimental probability that '5' shows up on the dice is 990/6000 = 0.165.

On the other hand, theoretical probability is determined by noting all the possible outcomes theoretically, and determining how likely the given outcome is. Mathematically,

Exercise 6.9

1. A bag contains 7 red marbles, 3 blue marbles, and 1 green marble. What is the probability of choosing a marble that is not blue?

   A. 11/8 B. 3/11

   C. 3/8 D. 8/11

2. A bag contains 6 red, 3 blue, 2 green and 1 white balls. A ball is picked at random. What is the probablity that it is:
   1. red
   2. blue or white
   3. not green
   4. not green or red?

3. A card is selected randomly from a pack of 52. What is the probability that it is:
   1. the 2 of hearts
   2. a red card
   3. a picture card
   4. an ace
   5. a number less than 4?

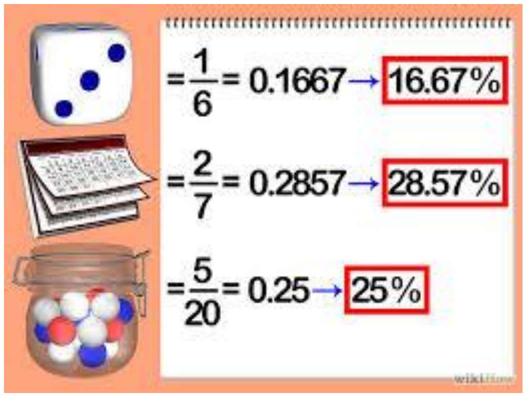
4. Even numbers from 2 -100 are written on cards. What is the probability of selecting a multiple of 5, if a card is drawn at random?

6.6 Probability Formula

LEARNING OUTCOMES

Students should be able to:

- Solve the probability of an event P (E)
- Determine the expected number of occurrences in a sequence of independent trials



Probability formula is the ratio of number of favorable outcomes to the total number of possible outcome.

Probability of an event X is symbolized by P(X). Probability of an event X is lies between $0 \leq P(X) \leq 1$.

Probability of an event = (number of ways it can happen) / (total number of outcomes)

P(X) = (no. of ways X can happen) / (Total number of outcomes)

How to find the probability:

Step 1: List the outcomes of the experiment.

Step 2: Count the number of possible outcomes of the experiment.

Step 3: Count the number of favorable outcomes.

Step 4: Use the probability formula.

Example 1: Rolling dice

When rolling a single die, there are six different outcomes: {1,2,3,4,5,6}

i. What's the probability of rolling a '1'?

ii. What's the probability of rolling a '1' or a '6'?

iii. What's the probability of rolling an even number? (i.e. rolling a 2, 4 or 6)

Using the formula from above:

P(A) = (no. of ways A can happen) / (Total number of outcomes)

- The probability of rolling a '1' is $\frac{1}{6}$
- The probability of rolling a '1' or a '6' is $\frac{2}{6}$ or $\frac{1}{3}$
- The probability of rolling an even number is $\frac{3}{6}$

Example 6.21

Two dice are rolled once. Calculate the probability that the sum of the numbers on the two dice is 6.

Solution:
Possible outcomes (Sample Space) = {(1, 1), (1, 2),...............,(1, 6), (2, 1), (2, 2),................,

(2, 6), (3, 1), (3, 2),...........,(3, 6), ...,(4, 1), (4, 2),..........,(4, 6), (5, 1), (5,2),...............,(5, 6), (6, 1), (6, 2),......................,(6, 6)}

Total possible outcomes = **36**

No. of outcomes of the experiment that are favorable to the event that a sum of two events is 6

=> Favorable outcomes are: (1, 5), (2, 4), (3, 3), (4, 2) and (5, 1)

Number of favorable outcomes = **5**

Use, probability formula = *Number of favorable outcomes/Total number of possible outcomes*

$$= \frac{5}{36}$$

The probability of a sum of 6 is $\frac{5}{36}$

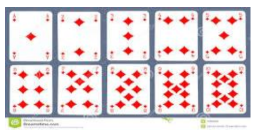
Note:

- *The probability of event A is often written as P(A)*
- *If P(A) > P(B) then event A has a higher chance of occurring than event B*
- *If P(A) = P(B) then events A and B are equally likely to occur*

Exercise 6.10

Using the above formula find:

i. A card is drawn at random from a deck of cards. Find the probability of getting the 5 of diamond.



ii. Two dice are rolled, find the probability that the sum is

a) equal to 1

b) equal to 5

c) less than 13

iii. A jar contains 4 red marbles, 6 green marbles and 10 white marbles. If a marble is drawn from the jar at random, what is the probability that this marble is white?

6.7 Properties Of Probability

LEARNING OUTCOMES

Students should be able to:

- Find out why the sum of all probabilities for an experiment is equal to one
- Examine and explain why the probability of an event is between zero and one
- Apply the above properties to real life situations

i. The probability of an event can only be between 0 and 1.

$$0 \leq P\,(A) \leq 1$$

In other words, the highest probability an event can have is 1 or 100% (certain to happen) and the lowest probability is 0(certain not to happen). Probabilities between 0 and 1 mean that the event might happen; the higher the probability, the greater the chance that it will happen.

ii. The sum of the probabilities of an event and its complementary is 1, so the probability of the complementary event is:

$$p(\overline{A}) = 1 - p(A)$$

iii. The probability of an impossible event is zero.

$$p(\varnothing) = 0$$

Exercise 6.11

1. Which of the following events has the highest probability of occurring?

   a. The probability of rain on Christmas day is 65%.

   b. The probability of tossing a coin and getting a “tail” is 0.5%

   c. The probability of the Fiji Rugby team winning the rugby game in Dubai is 2 out of 15

   d. The probability of rolling two dice and getting a total of 8 is 5/36

2. A lollie was picked from a bag that contains 1 blue lollie, 1 red lollie and 1 orange lollie.

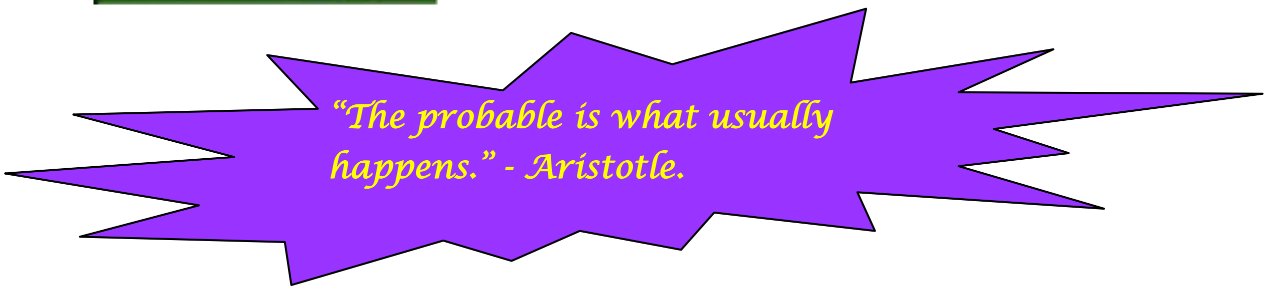
   a. List the sample space

   b. What is the probability of picking a red lollie?

   c. What is the probability of picking a lollie that is not red?

   d. What is the probability of picking a white lollie?

Application Of Probability

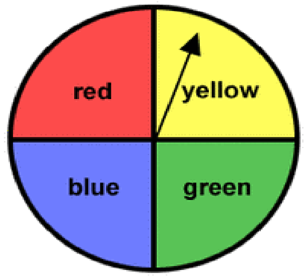


It is true that the future cannot be predicted but mathematics can help determine its probability and how likely something might occur or not. In our daily lives we often use probability to make better decisions.

Here are some of the common categories to discuss:

- **Sports** – be it basketball or football and coin is tossed and both teams have 50/50 chances of winning it, a basketball player takes a free throw judging his past performance it can be determined if he will make it or not.

- **Board Games** – a game spinner with four sections, there is a 25% chance of it landing blue, since one of the 4 sections is blue. Similarly the odds of rolling one die and getting and odd number there is a 50% chance since three of the six numbers on a die are odd.

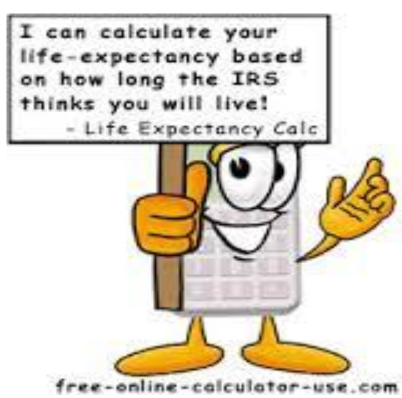


Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+SPINNERS

- **Medical Decisions** – When a patient is advised to undergo surgery, they often want to know the success rate of the operation which is nothing but a probability rate. Based on the same the patient takes a decision whether or not to go ahead with the same.

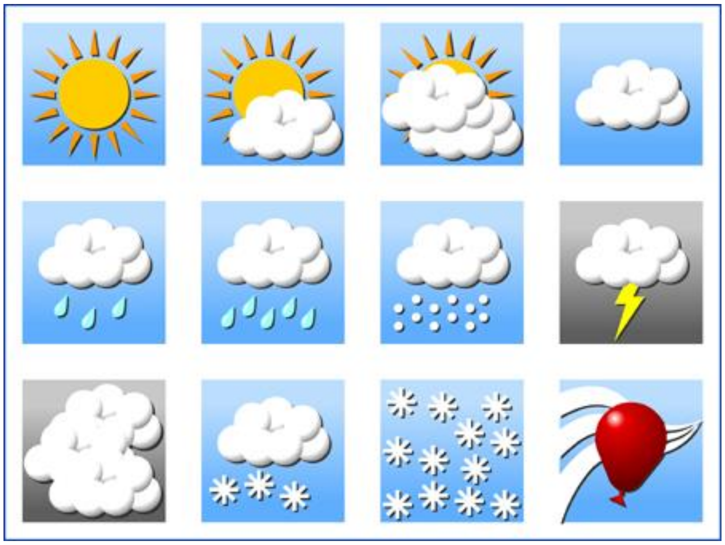
Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf
8#q=cartoon+images+of+hospital+patients

- **Life Expectancy** – this is based on the number of years the same groups of people have lived in the past. "These ages are used as guidelines by entities such as financial advisers to help clients prepare for their retirement years." – ehow.com



Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+life+expectancy

- **Weather** – when planning an outdoor activity, people generally check the probability of rain. Meteorologists also predict the weather based on the patterns of the previous year, temperatures and natural disasters are also predicted on probability and nothing is ever stated as a surety but a possibility and an approximation.



Therefore, even though we do not realize the use of mathematical probabilities in everyday life, subconsciously we use it in every step that we take.

Exercise 6.12

1. Calculate the probability of obtaining the following outcomes when rolling a die:

i. An even number.
ii. A multiple of three.
iii. A five or a six.

2. Find the probability of obtaining the following outcomes when flipping two coins:

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+cartoon+boy+tossing+a+coin

i. Two heads.
ii. Two tails.
iii. One head and one tail.

4. There are 20 raffle tickets in an envelope. Eight are for a car, and the rest are blank. Find the probability of drawing a ticket for the car if:

i. Only one raffle ticket is extracted.
ii. Two raffle tickets are extracted.
iii. Three raffle tickets are extracted.

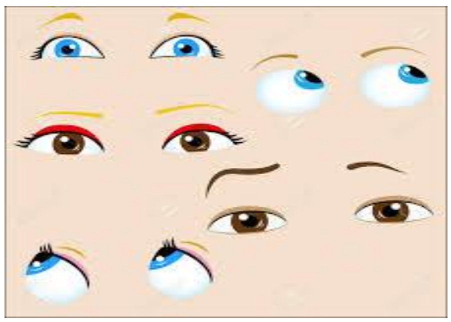
4. Students A and B have probabilities of failing an exam of 1/2 and 1/5 respectively. The probability of them both failing the examination is 1/10. Determine the probability that at least one of the two students fail.

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+cartoon+students+failing+exams

5. A father and a son go hunting. The father kills an average of 2 animals every 5 shots and the son kills one animal every 2 shots. If the two fired at the same animal at the same time, what is the probability that they will obtain a kill?

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+cartoon+people+go+hunting

6. A class consists of 10 boys and 20 girls. Exactly half of the boys and half of the girls have brown eyes. Determine the probability that a randomly selected student will be a boy or a student with brown eyes.



Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+cartoon+students+who+have+brown+eyes

7. The probability that a husband and wife will still be living in 20 years is ¼ for the husband and is 1/3 for the wife. Calculate the probability that:

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+husband+and+wife+

- Of them both being alive in 20 years.
- The husband living in 20 years but not the wife.
- The husband and the wife dying before 20 years.

GLOSSARY		
1	Bar graph	A chart that uses bars to show comparisons between categories of data. The bars can be either horizontal or vertical
2	Central tendency	A measure of central tendency is a measure that tells us where the middle of a bunch of data lies.
3	Complementary	Two events are complementary if they are the only two possible outcomes
4	Dispersion	Denotes how stretched or squeezed a distribution is
5	Event	It is any collection of outcomes of an experiment.
6	Experiment	It is any procedure that can be infinitely repeated and has a well-defined set of possible outcomes, known as the sample space
7	Frequency	The number of times the event occurred in an experiment
8	Histogram	It represents a frequency distribution by means of rectangles whose widths represent class intervals and whose areas are proportional to the corresponding frequencies: the height of each is the average frequency density for the interval
9	Interquartile range	Equal to the difference between the upper and lower quartile
10	Lower quartile	The median of the lower half of the data
11	Mean	Sum total of all the observations divided by the number of observations.
12	Median	If the data is arranged in the increasing or decreasing order then the middlemost value of the data
13	Mode	Observation has appeared the most number of times
14	Outcome	A single (one) result of an experiment
15	Pictogram	Visual presentation of data using icons, pictures, symbols
16	Pie chart	A circular statistical graph, which is divided into sectors to illustrate numerical proportion
17	Probability	The relative possibility that an event will occur, as expressed by the ratio of the number of actual occurrences to the total number of possible occurrences
18	Quartile	A set of data values are the three points that divide the data set into four equal groups
19	Random sample	It is a subset of individuals (a sample) chosen from a larger set (a population). Each individual is chosen randomly and entirely by chance, such that each individual has the same probability of being chosen at any stage during the sampling process, and each subset of *k* individuals has the same probability of being chosen for the sample as any other subset of *k* individuals
20	Range	Difference between the highest value and the lowest value
21	Relative frequency	The ratio of the actual number of favourable events to the total possible number of events
22	Sample space	The total number of all the different possible outcomes of a given experiment
23	Standard deviation	It is a measure of how numbers are spread out
24	Statistics	A branch of mathematics dealing with the collection, analysis, interpretation, and presentation of masses of numerical data
25	Upper quartile	The median of the **upper** half of the data.

Corrections in Year 10 Mathematics

1. Page 8: The learning outcomes should be stated as
   Students should be able to:
   - describe linear and quadratic functions.
   - identify and describe the domain and range of functions.
   - calculate the values of functions using function notations.
   - generating domain and range of functions as ordered pairs.

2. Page 19: The learning outcomes should be stated as
   Students should be able to:
   - calculate intercepts and gradient of the linear equations in the form y=mx + c.
   - draw graphs of linear equation.
   - identify the intercepts from the graph of linear equations.
   - determine and shade regions indicated by inequations.

3. Page 27 Example 1.10: the gradient of the line is 3. Therefore the equation of the line is y=3x+3.

4. Page 31 Example 1.12: the shading should be above the graph.

5. Page 37: The learning outcomes should be stated as
   Students should be able to:
   - factorise algebraic expressions by applying:
   - ➢ common Factor method.
   - ➢ grouping method.
   - ➢ difference of two squares method.
   - ➢ perfect square method.

6. Page 42. The symbol $\sqrt{\ }$ means positive square root so $\sqrt{4}$ = 2 and $\sqrt{25}$ = 5. Also the learning outcome should read as "Students should be able to
   - ➢ find square root of numbers.
   - ➢ simplify expressions involving square roots.

7. Page 59: The learning outcomes should be stated as

   Students should be able to:
   - define indices.
   - write numbers given in expanded form in the base index form and vice versa.

8. Page 83: The learning outcomes should be stated as
   Students should be able to:
   - describe the three basic trigonometric functions.
   - calculate sine, cosine and tangent values of theta and vice versa.
   - use SOH, CAH, TOA to find unknown side and angle of right angle triangle.

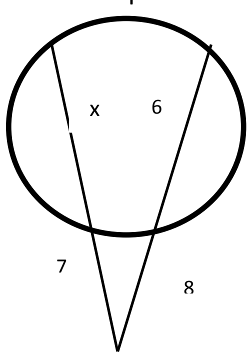
9. Page 86 Step 3 should be H=x.

10. Page 91.The angle should be $75°$.

11. Page 99 definition of clinometer: "an instrument for measuring angles of elevation and depression".

12. Page 114 Question 5 part a should be "Which segment is the angle bisector of angle TPR?" and part b should read as " P is the mid-point of which two segments?".

13. Page 130 Example 4.18.The diagram should be labeled as



14. Page 140. In Fiji any individual earning up to <u>16000</u> does not have to pay tax. (http://www.frca.org.fj/residents-tax-rates)

15. Page 188 exercise 6.8
    Question 2. A fair die is tossed.
    i. Write down the set of all possible outcomes.
    ii. Are the outcomes equally likely?
    iii. What must be the sum of the probabilities of all the possible outcomes?
    iv. Find the probability of each of the outcomes listed in part i.